Limesi funkcija - teorijska pitanja i zadatci

$$\lim_{x \to \pm\infty} \left(1 + \frac{\alpha}{x}\right)^x = e^\alpha \quad \bigg| \quad \lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \bigg| \quad \lim_{x \to 0} \frac{e^x - 1}{x} = 1 \quad \bigg| \quad \lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

1. Što znače sljedeći zapisi: $\lim\limits_{x \to x_0} f(x) = L$, $\lim\limits_{x \to x_0+} f(x) = L$ i $\lim\limits_{x \to x_0-} f(x) = L$.

2. Ima li smisla računati $\lim\limits_{x \to -1} \ln x$?

3. Što znače sljedeći zapisi: $\lim\limits_{x \to \infty} f(x) = L$, $\lim\limits_{x \to -\infty} f(x) = L$.

4. Navedite primjer funkcije za koju vrijedi $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to -\infty} f(x) = \infty$.

5. Dokažite, po definiciji limesa, da vrijedi $\lim\limits_{x \to 3} \dfrac{x - 1}{x + 1} = \dfrac{1}{2}$.

6. Izračunajte sljedeće limese

    (a) $\lim\limits_{x \to \infty} (x - \sqrt{x^2 - 2x + 3})$ (b) $\lim\limits_{x \to -\infty} (x - \sqrt{x^2 - 2x + 3})$ (c) $\lim\limits_{x \to -\infty} (\sqrt{x^2 - 3} - \sqrt{x^2 - x})$

    (d) $\lim\limits_{x \to \infty} \left(\dfrac{x}{x + 1}\right)^{4x}$,   (e) $\lim\limits_{x \to 1} \dfrac{1 - x}{1 - \sqrt[3]{x}}$ (f) $\lim\limits_{x \to 0} \dfrac{\ln(1 + x)}{x}$

    (g) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$ (h) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x}$ (i) $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x}{x}$

    (j) $\lim\limits_{x \to 1} \dfrac{\sin(x - 1)}{x^2 - 1}$ (k) $\lim\limits_{x \to 0} \dfrac{\operatorname{arctg} x}{x}$

7. Izračunajte sljedeće limese

    (1) $\lim\limits_{x \to 0^+} \dfrac{2}{1 + 2^{\frac{1}{x}}}$ (2) $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$ (3) $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x - \sin x}{x^3}$

    (4) $\lim\limits_{x \to 0^+} \dfrac{\sin x}{\sqrt{x}}$ (5) $\lim\limits_{x \to 0} \dfrac{\operatorname{tg} x}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}$ (6) $\lim\limits_{x \to 0} \dfrac{\arcsin 2x}{\arcsin 3x}$

    (7) $\lim\limits_{x \to 0^+} \dfrac{\sin \sqrt{x}}{x}$ (8) $\lim\limits_{x \to 0} \dfrac{\arctan 2x}{x}$ (9) $\lim\limits_{x \to \infty} \left(\dfrac{x^2 - x}{x^2 + 3x - 1}\right)^x$

    (10) $\lim\limits_{x \to \infty} \left(\dfrac{x^2 - x + 7}{x^2 + 4x + 4}\right)^{6x - 2}$ (11) $\lim\limits_{x \to 0^+} \ln\left((1 - 4x)^{\frac{x+1}{x}}\right)$ (12) $\lim\limits_{x \to 0^+} (\cos x)^{\operatorname{ctg} x}$

    (13) $\lim\limits_{x \to \infty} \dfrac{x + \sin x}{x + \cos x}$ (14) $\lim\limits_{x \to 0^+} (1 - \cos x)\operatorname{ctg} x$,   (15) $\lim\limits_{x \to \infty} \left(\dfrac{\sqrt{x} + 2}{\sqrt{x} - 1}\right)^{2\sqrt{x}}$

    (16) $\lim\limits_{x \to \infty} \left(\dfrac{x + 3}{x - 2}\right)^{\sqrt{x}}$ (17) $\lim\limits_{x \to \infty} \left(\dfrac{\sqrt{x} + 3}{\sqrt{x} - 2}\right)^x$ (18) $\lim\limits_{x \to 0} \dfrac{\sin x + \operatorname{tg} 2x}{\sin 3x + \operatorname{tg} 4x}$.

8. Odredite asimptote sljedećih preslikavanja

    a) $f(x) = \dfrac{x}{x - 2}$,   b) $f(x) = \sqrt{1 + x^2}$,   c) $f(x) = \dfrac{x^2 - x - 6}{x - 1}$,

    d)$f(x) = 2\operatorname{arctg} x + 1$,   e) $f(x) = \dfrac{\cos x}{x^2 - 1}$.