

Popis elementarnih funkcija

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- apsolutna vrijednost
- racionalne funkcije
- eksponencijalne funkcije
- logaritamske funkcije
- opće potencije i korijeni
- trigonometrijske funkcije
- arkus funkcije

Polinomi

Polinom stupnja n ($n \in \mathbb{N}_0$)

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \quad a_0, \dots, a_n \in \mathbb{R}, \quad a_n \neq 0$$

Domena: \mathbb{R}

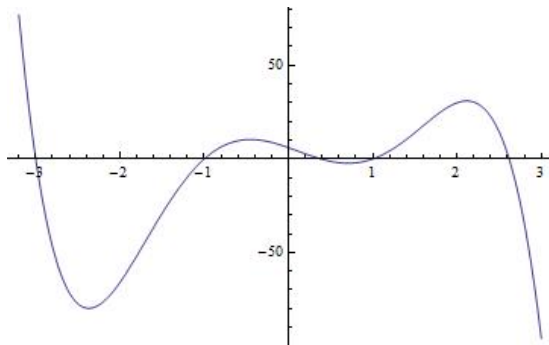
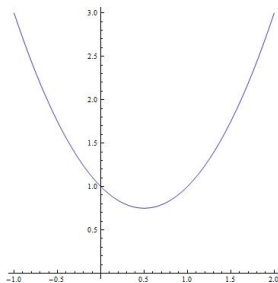
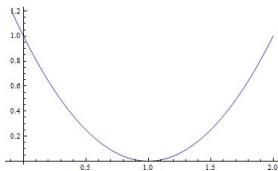


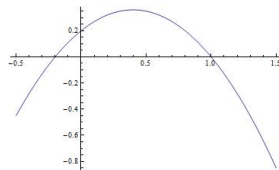
Figure: $p(x) = -2x^5 + 18x^3 - 6x^2 - 16x + 6$



(a) $p(x) = x^2 - x + 1$



(b) $p(x) = x^2 - 2x + 1$



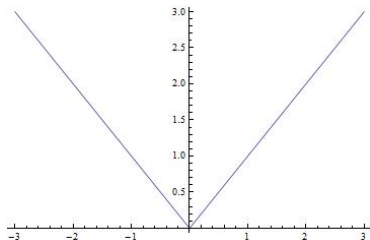
(c) $p(x) = -x^2 + 0.8x + 0.2$

Figure: Primjeri grafa kvadratne funkcije

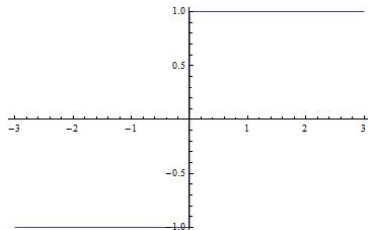
Apsolutna vrijednost i signum

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0 \end{cases}, \quad \operatorname{sgn} x = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0 \end{cases}$$

Domena: \mathbb{R}



(a) $f(x) = |x|$



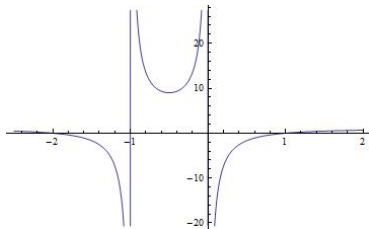
(b) $f(x) = \operatorname{sgn} x$

Figure: Apsolutna vrijednost i signum

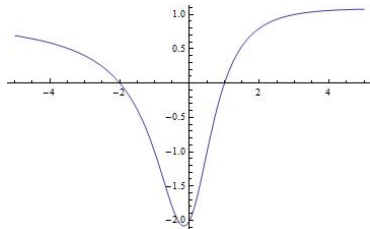
Racionalne funkcije

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ su polinomi}$$

Domena: $\mathbb{R} \setminus \{x \in \mathbb{R} : q(x) = 0\}$



(a) $f(x) = \frac{(x-1)(x+2)}{x(x+1)}$



(b) $f(x) = \frac{(x-1)(x+2)}{x^2+1}$

Figure: Primjeri grafa racionalne funkcije

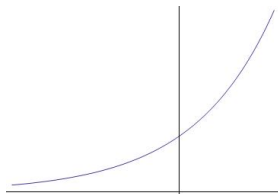
Eksponecijalne funkcije

$$f(x) = a^x, \quad a > 0, a \neq 1$$

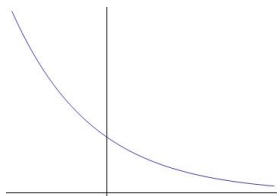
Vrijedi: $a^x a^y = a^{x+y}$, $(a^x)^y = a^{xy}$

Domena: \mathbb{R}

Slika: $\mathbb{R}^+ = \{x > 0\}$



(a) $f(x) = a^x, a > 1$,
strogo raste



(b)
 $f(x) = a^x, 0 < a < 1$,
strogo pada

Figure: Grafovi eksponencijalne funkcije

Eksponecijalne funkcije

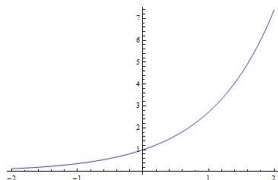
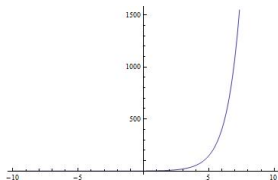
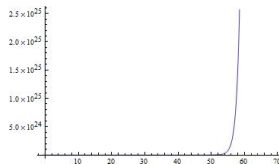
(a) e^x na $[-2, 2]$ (b) e^x na $[-10, 10]$ (c) e^x na $[-1, 70]$

Figure: Rast funkcije $f(x) = e^x$

$e = 2.718281828459045235360287471352662497757247093699959575 \dots$

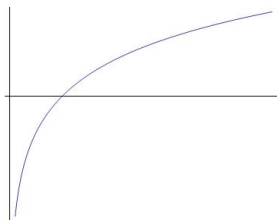
Logaritamske funkcije

$$f(x) = \log_a(x), \quad a > 0, \quad a \neq 1$$

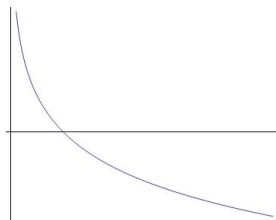
Oznake: $\log_e(x) = \ln x$ (prirodni logaritam), $\log_{10}(x) = \log x$

- $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$,
- $\log_a(x^y) = y \log_a(x)$,
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Domena: \mathbb{R}^+ , **Slika:** \mathbb{R}



(a) $f(x) = \log_a(x)$,
 $a > 1$, strogo raste



(b) $f(x) = \log_a(x)$,
 $0 < a < 1$, strogo pada

Logaritamska funkcija

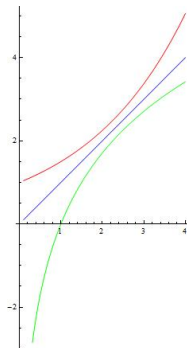


Figure: $f(x) = a^x$, $y = x$, $f^{-1}(x) = \log_a(x)$

Graf funkcije $f(x)$ i njoj inverzne funkcije $f^{-1}(x)$ je zrcalno simetričan s obzirom na pravac $y = x$.

Opće potencije i korijeni

$$f(x) = x^a = e^{a \cdot \ln x}, \quad a \in \mathbb{R}$$

Domena: \mathbb{R}^+

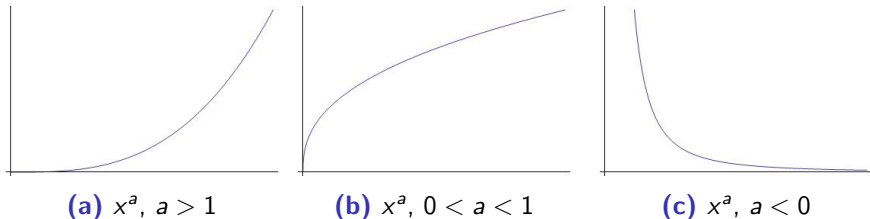


Figure: Grafovi opće potencije

Dogovor: domena funkcije $x \mapsto x^n, n \in \mathbb{N}$, i funkcije $x \mapsto x^{1/n} = \sqrt[n]{x}$, $n \in \mathbb{N}$ neparan, je čitav \mathbb{R} !

Sinus, kosinus

$$x \mapsto \sin x, \quad x \mapsto \cos x$$

Domena: \mathbb{R} , **Slika:** $[-1, 1]$, **Period:** 2π

$\sin(-x) = -\sin x$ (neparna), $\cos(-x) = \cos x$ (parna)

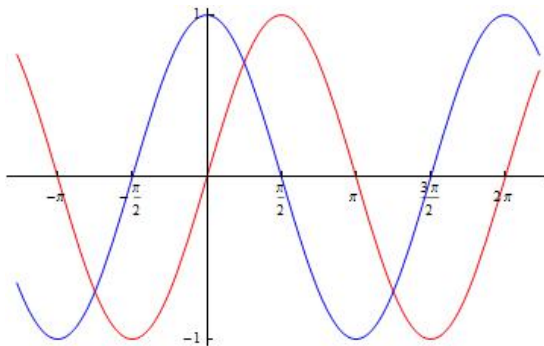


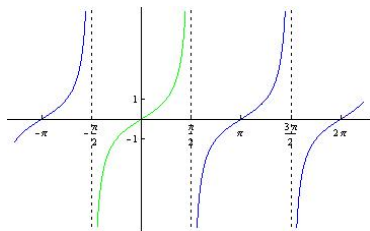
Figure: $\sin x$, $\cos x$

Tangens, kotangens

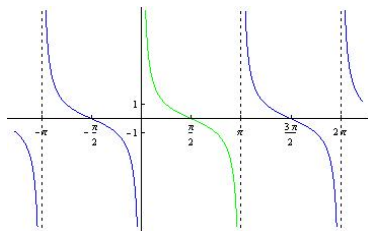
$$\operatorname{tg} x = \frac{\sin x}{\cos x}, \quad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = \frac{\cos x}{\sin x}$$

Domena: $D(\operatorname{tg}) = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\}$, $D(\operatorname{ctg}) = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$

Slika: \mathbb{R} , **Period:** π



(a) $\operatorname{tg} x$, $\operatorname{Tg} x$



(b) $\operatorname{ctg} x$, $\operatorname{Ctg} x$

Figure: Tangens i kotangens

Osnovne relacije

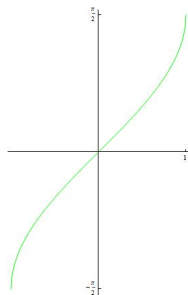
- $\sin^2 x + \cos^2 x = 1,$
- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y,$
 $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- $\sin 2x = 2 \sin x \cos y, \cos 2x = \cos^2 x - \sin^2 x,$
- $\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right),$
 $\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right),$
- $\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right),$
 $\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right).$

Arkus funkcije

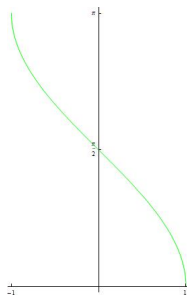
Inverzne funkcije (*restringiranih*) trigonometrijskih funkcija.

- $\text{Sin}^{-1}x = \arcsin x$, $\text{Sin } x = \sin x / [-\pi/2, \pi/2]$,
 $\arcsin: [-1, 1] \rightarrow [-\pi/2, \pi/2]$,
- $\text{Cos}^{-1}x = \arccos x$, $\text{Cos } x = \cos x / [0, \pi]$, $\arccos: [-1, 1] \rightarrow [0, \pi]$,
- $\text{Tg}^{-1}x = \arctg x$, $\text{Tg } x = \text{tg } x / (-\pi/2, \pi/2)$, $\arctg: \mathbb{R} \rightarrow (-\pi/2, \pi/2)$,
- $\text{Ctg}^{-1}x = \text{arcctg } x$, $\text{Ctg } x = \text{ctg } x / (0, \pi)$, $\text{arcctg}: \mathbb{R} \rightarrow (0, \pi)$.

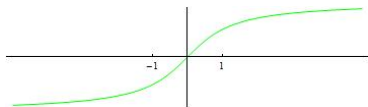
Arkus funkcije



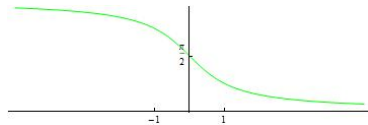
(a) $\arcsin x$



(b) $\arccos x$



(c) $\arctan x$



(d) $\operatorname{arccot} x$