TEHNIČKO VELEUČILIŠTE U ZAGREBU

GRADITELJSKI ODJEL

TEHNIČKA MEHANIKA

skripta

mr. sc. Ljerka Kopričanec – Matijevac, viši predavač

PREDGOVOR

Skripta **Tehnička mehanika** sadrže gradivo koje se obrađuje na vježbama iz predmeta Tehnička mehanika na stručnom studiju graditeljstva Tehničkog veleučilišta u Zagrebu.

Bolonjski proces je donio puno mogućnosti i povlastica, ali isto tako je donio i puno obveza, kako za studente, tako i za njihove nastavnike.

Sa željom za postizanjem što boljih rezultata, ali i za što većim usvojenim znanjem, prionula sam stvaranju ovih skripata. Zahvaljujem kolegi Josipu Ataliću, dipl. ing. građ. koji je prvi započeo s prijepisom mojih pisanih bilješki te demonstratorima Vedranu Katiću i Dariju Vukoviću koji su prepisali i precrtali najveći dio materijala koje sam koristila na vježbama iz Tehničke mehanike.

Skripta se sastoje iz tri dijela. Svaki dio pokriva gradivo za jedan kolokvij. Postupno se obrađuju primjeri i zadaci potrebni za usvajanje osnovnih znanja, a na kraju svakog dijela nalaze se zadaci s rješenjima za samostalni rad.

Prvi dio sadrži gradivo prvog kolokvija, tj. obrađuje osnovne pojmove mehanike, statiku točke u ravnini te statiku tijela u ravnini.

Drugi dio sadrži gradivo za drugi kolokvij, tj. određivanje unutarnjih sila odnosno crtanje dijagrama unutarnjih sila za ravninske sustave.

Treći dio sadrži gradivo trećeg kolokvija, tj. obrađuje karakteristike poprečnih presjeka (površina, težište, statički moment površine i moment inercije ravnih ploha), jednoosno i dvoosno homogeno polje naprezanja, Mohrovu kružnicu naprezanja, naprezanje u proizvoljnom presjeku štapa te koso savijanje.

Studenti će biti spremni za izlazak na kolokvij iz pojedinog gradiva u onom trenutku kada samostalno riješe zadatke, ali u određenom vremenu. Za svladavanje gradiva i rješavanje zadataka iz Tehničke mehanike, osim znanja, potrebno je steći i kondiciju jer do točnog rješenja treba doći u vremenu od 90 minuta, koliko i traje pisanje pojedinog kolokvija.

Osim primjera i zadataka u ovim skriptama, studenti mogu koristiti i riješene zadatke s ispitnih rokova koje mogu naći na www.ljerka.com/dokumenti.html.

SADRŽAJ

1. DIO – primjeri i zadaci za 1. kolokvij

UVOD

MEHANIKA znanost o općim zakonima gibanja i ravnoteže (materijalnih) tijela izloženih djelovanju sila.

Prema ***značajkama i zadaćama*** koje se proučavaju i rješavaju (klasična) mehanika se dijeli na: **statiku**, kinematiku i dinamiku,

Statika je dio mehanike u kojem se opisuje ponašanje (nepomičnog) materijalnog tijela (ili konstrukcije) na koje djeluju sile neovisno o vremenu.

Kinematika je dio mehanike u kojem se **proučava geometrija gibanja tijela**. Pod gibanjem tijela podrazumijeva se promjena njegovog položaja tijekom vremena u odnosu na neko drugo tijelo. Značajka je kinemtike da se *ne uzimaju u obzir mase (inercije) tijela niti sile koje na njih djeluju.*

Dinamika je dio mehanike u kojemu **se proučavaju zakoni gibanja materijalnih tijela** na koje djeluju **sile ovisne o vremenu**, a u *obzir se uzima i inercija masa koje se ubrzavaju.*

Prema ***materijalnim i geometrijskim svojstvima*** promatranog medija mehanika se dijeli na : **mehaniku materijalne točke**, mehaniku krutih tijela, mehaniku deformabilnih tijela, mehaniku tekućina i mehaniku plinova.

Klasična se mehanika danas smatra aproksimacijom koja vrijedi za tijela koja su velika u odnosu na elementarne čestice te za brzine male u odnosu na brzinu svjetlosti.

MATERIJALNA TOČKA je idealizirani model čvrstog tijela, nema dimenzije, ali ima masu jednaku masi tijela za koje je ona idealizirani model. Položaj materijalne točke u prostoru ili ravnini određen je koordinatama koje su identične koordinatama težišta tijela za koje je model napravljen. *(U našim konstrukcijama točke se pojavljuju kod rešetkastih nosača – Proračun konstrukcija II. semestar).*

Znamo da se MATERIJALNA TOČKA (točka) kao model za neko tijelo može uzeti samo onda ako dimenzije tijela nemaju utjecaj u provedenoj mehaničkoj analizi.

Za razliku od materijalne točke **KRUTO TIJELO** je idealizirani model realnog čvrstog tijela kod kojeg udaljenosti pojedinih čestica tijela ostaju nepromijenjene pri bilo kojem djelovanju sila.

Prema tome KRUTO TIJELO se pri opterećenju ne deformira pa mu oblik i volumen ostaju nepromijenjeni.

.

OSNOVNI POJMOVI MEHANIKE

OSNOVNI ZAKONI MEHANIKE (NEWTONOVI AKSIOMI)

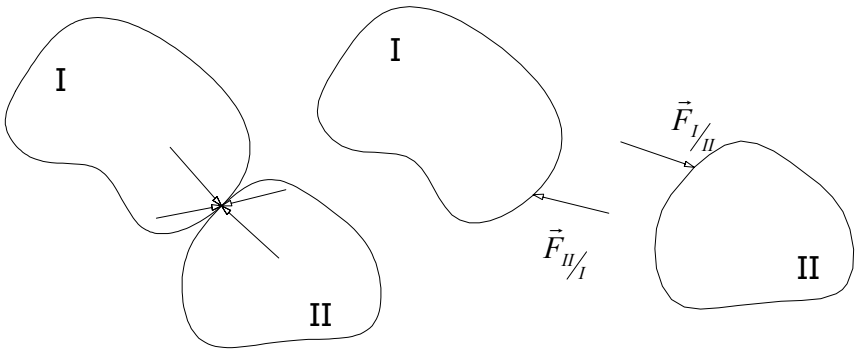
ZAKON INERCIJE (I. Newtonov zakon mehanike)
Svako materijalno tijelo ostaje u stanju mirovanja ili jednolikog gibanja po pravcu dok na njega ne djeluje sila.

Otpor mijenjanju stanja mirovanja ili jednolikog gibanja po pravcu je svojstvo materije i naziva se **inercija**.

ZAKON GIBANJA (II. Newtonov zakon mehanike)
Promjena količine gibanja u vremenu proporcionalna je sili i ima smjer sile.

Odnosno: Sila je jednaka umnošku mase i akceleracije F = m·a (slijedi: 1 Newton = 1 kgm/sek)

ZAKON AKCIJE I REAKCIJE (III. Newtonon zakon mehanike)
Na svaku silu akcije javlja se jednaka i suprotno usmjerena sila reakcije.



$\vec{F}_{I/II} = -\vec{F}_{II/I}$

$\vec{F}_{I/II}$ - ukupno djelovanje tijela I na tijelo II

$\vec{F}_{II/I}$ - ukupno djelovanje tijela II na tijelo I

PRIMJERI SILA

Gravitacijska sila, težina. Dvije izolirane materijalne točke privlače se **gravitacijskim silama** veličine $F = 6{,}67 \cdot 10^{-11} \cdot m_1 \cdot m_2 / r^2$ m_1 *i* m_2 su mase točaka u kg, a *r* je udaljenost točaka u metrima. **Težina** je sila kojom planet privlači tijelo.

Unutarnje sile teže održavanju određenog razmaka između čestica tijela u krutom agregatnom stanju. Djeluju samo među neposredno blizim česticama i ako razmak čestica odstupa od osnovnog. Uz povećani razmak sile su privlačne, a uz smanjeni razmak sile su odbojne. Veličina sila raste s povećanjem odstupanja od osnovnog razmaka.

Kontaktne sile predstavljaju otpor prodoru čestica materije u tijelo u krutom agregatnom stanju. Pojavljuju se pri određenoj, vrlo maloj udaljenosti čestica i rastu s daljnjim približavanjem čestica.

PODJELA SILA

Prema **dosegu** sile mogu biti **kratkog** i **dalekog** dosega. Kratkog dosega su unutarnje, kontaktne, tlak u tekućinama i plinovima. Sile dalekog dosega su gravitacijske, magnetne, elektrostatske.

Prema **smislu** djelovanja sile mogu biti **tlačne** ili *odbojne* koje teže razmicanju i **vlačne** ili *privlačne* koje teže približavanju dijelova materije.

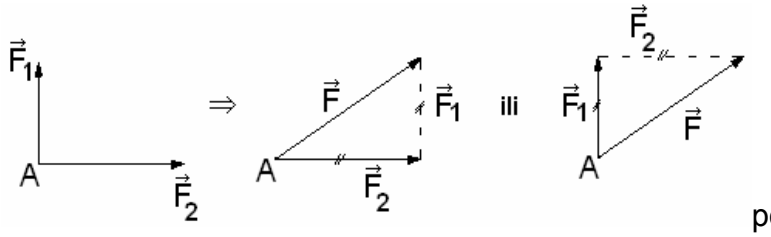
Podjela sila prema području djelovanja može biti na **koncentrirane** koje djeluju na pojedine točke materije i **distribuirane** ili *raspoređene* (*kontinuirane*) koje djeluju na konačne dijelove materije. Distribuirane sile nadalje se dijele na linijske, plošne i volumenske.

DJELOVANJE DVIJE SILE NA MATERIJALNU TOČKU

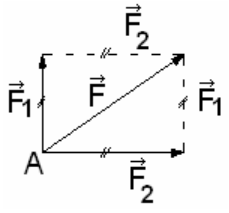
PRAVILO PARALELOGRAMA SILA

Dvije sile koje djeluju na istu točku tijela u različitim smjerovima, imaju isto djelovanje na to tijelo kao i jedna sila koja je jednaka dijagonali paralelograma kojemu su stranice te dvije sile.

Pravilo paralelograma je geometrijsko zbrajanje vektora.



povlačimo paralele sa zadanim silama ili možemo odmah na nacrtu povući sile i dobiti paralelogram



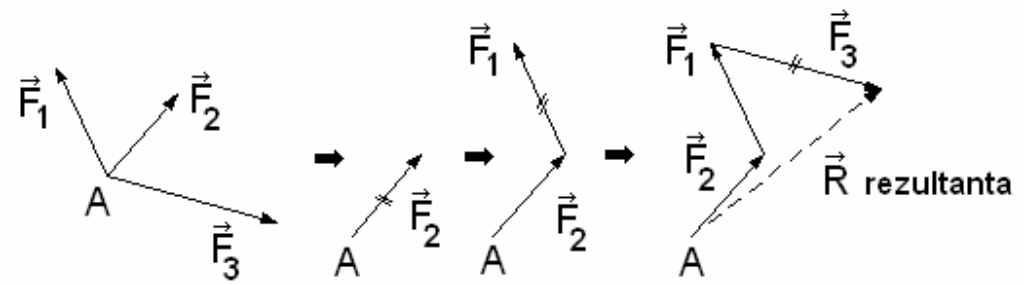
DJELOVANJE VIŠE SILA NA MATERIJALNU TOČKU

PRAVILO VEKTORSKOG POLIGONA

Djelovanje većeg broja sila na materijalnu točku određuje se pravilom **vektorskog poligona**.

Početak prve sile i šiljak zadnje sile određuju silu koja je po djelovanju jednaka zadanim silama, a naziva se zbroj ili **REZULTANTA.**

REZULTANTA – je zamišljena sila čije je djelovanje jednako djelovanju zadanih sila.



1.korak 2.korak 3.korak $\Rightarrow$ konačno rješenje

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{R}$ vektorska jednadžba

$$\left.\begin{aligned} F_{1x} + F_{2x} + F_{3x} &= R_x \\ F_{1y} + F_{2y} + F_{3y} &= R_y \end{aligned}\right\}$$ skalarne jednadžbe $R = \sqrt{R_x^2 + R_y^2}$

RAVNOTEŽA je stanje mirovanja uz djelovanje sila.

Materijalna točka je u ravnoteži ako iščezne rezultanta sila koje na nju djeluju tj. ako je poligon sila zatvoren. Pojam mirovanja je relativan.

UVJET EKVIVALENCIJE

Dva sustava sila koja djeluju na materijalnu točku **su ekvivalentni** ako su **njihovi doprinosi** uvjetima **ravnoteže isti**. Njihovi poligoni moraju imati zajednički početak i zajednički šiljak.

OSNOVNO O VEKTORIMA

skalari – određeni su veličinom, intenzitetom, odnosno samo brojem (masa, temperatura itd.)

vektor – je geometrijski određen početkom i šiljkom ili početkom, pravcem, duljinom i orijentacijom ili smjerom, pravcem i intenzitetom. Vektori se u nacrtima ili skicama označavaju strelicama, a uz strelicu se označava veličina vektora kao poseban ili opći broj. Vektor se označava slovom i strelicom povrh (npr. $\vec{p}$).

Sila je fizikalna veličina kojom se prikazuje uzajamno djelovanje tijela.

Sila koja djeluje na **materijalnu točku** naziva se **koncentriranom silom**.

Koncentrirane sile prikazuju se **vektorima** –, prema tome imaju,

- **brojčanu vrijednost** nazvanu **intenzitetom,**
- **pravac i smjer djelovanja**
- **hvatište**.

Koncentrirana sila je **idealizacija**: svaka sila djeluje kontinuirano raspodijeljena po većoj ili manjoj površini ili po volumenu tijela te je koncentrirana sila rezultanta takvog djelovanja.

U mehanici krutih tijela može se u mnogim slučajevima **sila prikazati kliznim vektorom** – njen utjecaj se ne mijenja pomakne li se sila po pravcu svog djelovanja do nekog drugog hvatišta pa i izvan tijela.

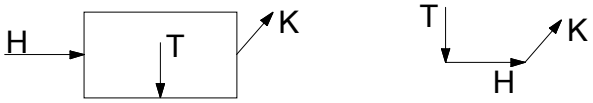
Mjerna jedinica za silu je Newton (N)

POSTUPCI S VEKTORIMA

Postupci s vektorima mogu biti grafički, računski i kombinirani.

GRAFIČKI
na crtežu se prikazuju točka ili tijelo koje se analizira i svi vektori potrebni za postupak. Ako imamo tijelo crtamo ga u **mjerilu položaja**, a vektore u **mjerilu sila**

PLAN POLOŽAJA PLAN SILA



1:500 1cm ::10kN

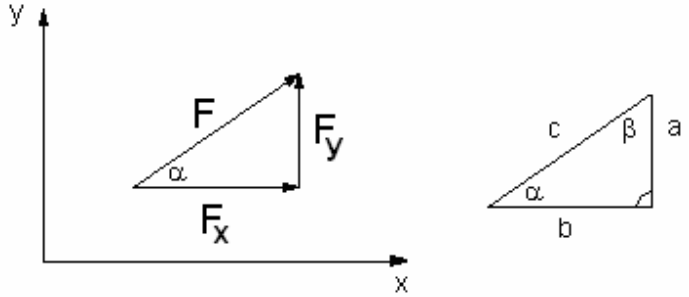
RAČUNSKI POSTUPCI

mogu biti direktni, projekcijski i dekompozicijski.

U **direktnim** postupcima se planimetrijskim i stereometrijskim postupcima određuju podaci o odnosima među vektorima. Npr.: određivanje rezultante dviju sila svodi se na proračun jedne stranice trokuta.

U **projekcijskim** i **dekompozicijskim** postupcima vektori se opisuju pomoću određenog broja skalara i s njima se postupak realizira.

Mi ćemo pokazati samo **projekcijski** postupak koji sadrži plan položaja. ***Svaka sila u ravnini je određena s dvije projekcije.***



$$\alpha + \beta = 90^\circ$$

$$\sin\alpha = \frac{a}{c}$$

$$\cos\alpha = \frac{b}{c}$$

$$tg\alpha = \frac{a}{b}$$

$$\left.\begin{aligned} F_x &= F\cos\alpha \\ F_y &= F\sin\alpha \end{aligned}\right\} \quad F = \sqrt{F_x^2 + F_y^2}$$

STATIKA MATERIJALNE TOČKE U RAVNINI

UVJET RAVNOTEŽE

Ravnoteža je stanje mirovanja uz djelovanje sila.

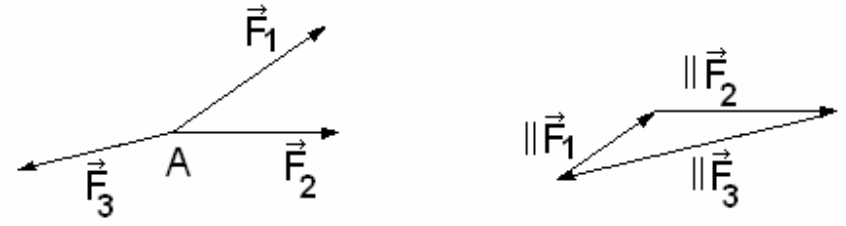
Točka je u **ravnoteži** ako **iščezava vektorski zbroj svih sila** koje djeluju na točku ili je **poligon sila zatvoreni lik**.

Uvjet ravnoteže može se koristiti za provjeravanje i određivanje *uravnotežavajućih sila*.

Uravnotežavajuće sile se zadanim silama pridružuju tako da točka na koju djeluju bude u ravnoteži.

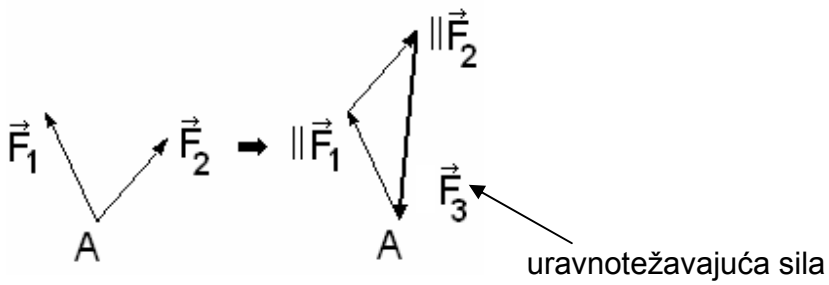
ODREĐIVANJE URAVNOTEŽAVAJUĆE SILE

Ako imamo djelovanje **3 sile** one su u ravnoteži ako je poligon sila zatvoreni lik.



***grafički** : poligon sila zatvoreni lik*

Određivanje jedne uravnotežavajuće sile ($\vec{F}_3$)

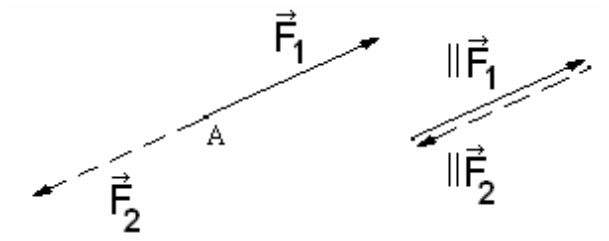


algebarski

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$
$$F_{1x} + F_{2x} + F_{3x} = 0$$
$$F_{1y} + F_{2y} + F_{3y} = 0$$

Uz djelovanje samo **2 sile** na materijalnu točku ona je u ravnoteži ako sile leže na istom pravcu, jednake su po intenzitetu, a suprotnog smjera.

grafički



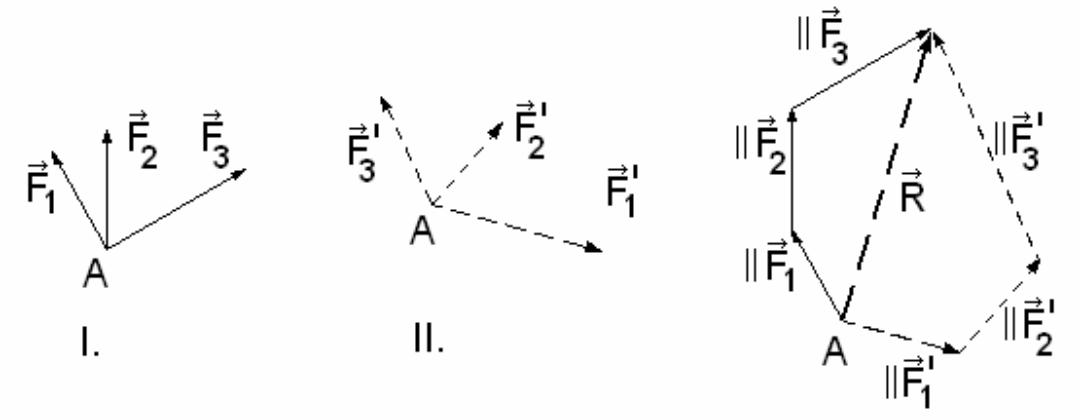
algebarski $\vec{F}_1 + \vec{F}_2 = 0$

$$\vec{F}_1 = -\vec{F}_2$$
$$F_{1x} = -F_{2x}$$
$$F_{1y} = -F_{2y}$$

UVJET EKVIVALENCIJE

Dva sustava sila *(I. i II.)* koji djeluju na materijalnu točku **A su ekvivalentni** ako su **njihovi doprinosi** uvjetima **ravnoteže isti**. Njihovi poligoni moraju imati zajednički početak i zajednički šiljak.

Imaju zajedničku **rezultantu**.



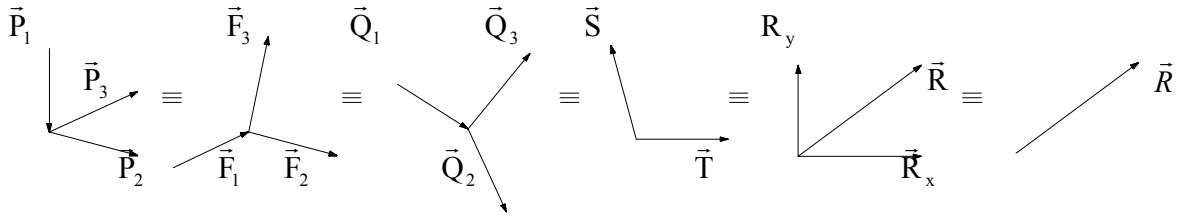
poligoni sila moraju imati zajednički početak i zajednički šiljak

algebarski uvjeti :

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}'_1 + \vec{F}'_2 + \vec{F}'_3 = \vec{R}$$

Jednadžbe ekvivalencije

$$F_{1x} + F_{2x} + F_{3x} = F'_{1x} + F'_{2x} + F'_{3x} = R_x$$
$$F_{1x} + F_{2x} + F_{3x} = F'_{1y} + F'_{2y} + F'_{3y} = R_y$$



$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{Q}_1 + \vec{Q}_2 + \vec{Q}_3 = \vec{S}_1 + \vec{T}_2 = \vec{R}_x + \vec{R}_y = \vec{R}$$

Jednadžbe ekvivalencije :

$$P_{1X} + P_{2X} + P_{3X} = F_{1X} + F_{2X} + F_{3X} = Q_{1X} + Q_{2X} + Q_{3X} = S_X + T_X = R_X$$

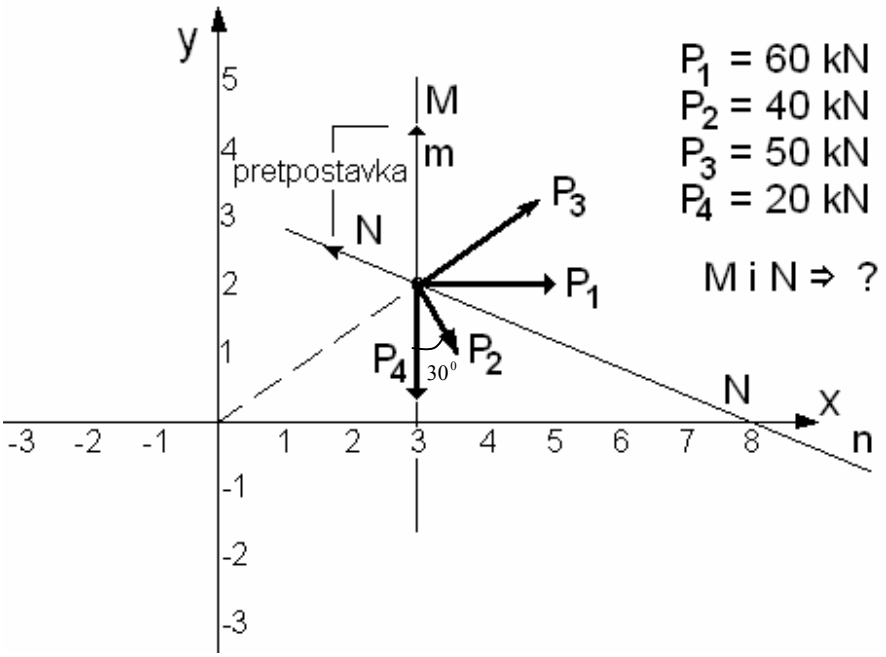
$$P_{1Y} + P_{2Y} + P_{3Y} = F_{1Y} + F_{2Y} + F_{3Y} = Q_{1Y} + Q_{2Y} + Q_{3Y} = S_Y + T_Y = R_Y$$

$$\sum_{i=1}^{N} \vec{P}_i = \sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} \vec{Q}_i = \vec{S} + \vec{T} = \vec{R}$$ - **vektorska jednadžba**

$$\left.\begin{aligned} \sum_{i=1}^{N} P_{ix} = \sum_{i=1}^{N} F_{ix} = \sum_{i=1}^{N} Q_{ix} = S_x + T_x = R_x \\ \sum_{i=1}^{N} P_{iy} = \sum_{i=1}^{N} F_{iy} = \sum_{i=1}^{N} Q_{iy} = S_y + T_y = R_y \end{aligned}\right\}$$ dvije **skalarne jednadžbe**

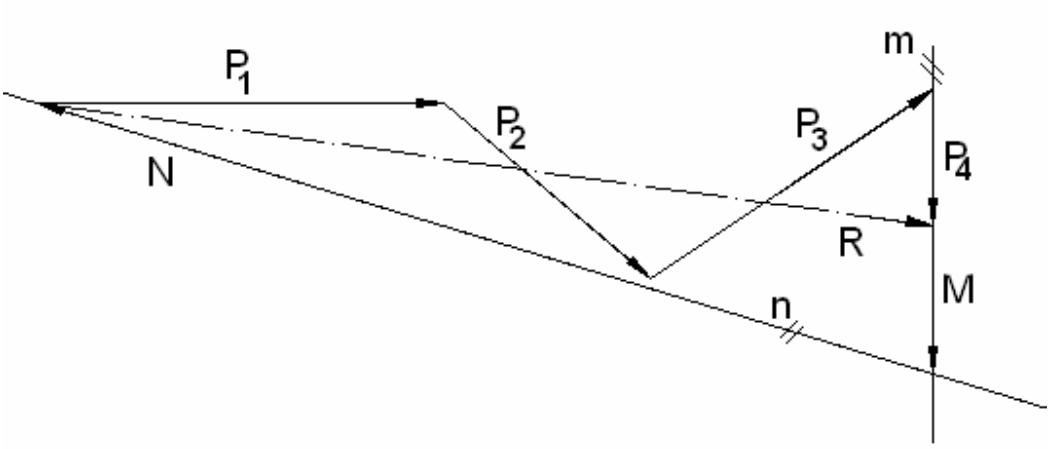
ZADATAK 1: (ravnoteža materijalne točke)

Zadani sustav sila $\vec{P}_1, \vec{P}_2, \vec{P}_3$ i $\vec{P}_4$ potrebno je uravnotežiti silama M i N na pravcima m i n. Riješiti grafički i računski.



Grafički : *poligon sila mora biti zatvoren lik*

Mjerilo sila : 1cm : 10 kN

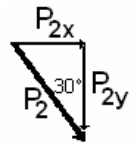


uravnotežimo R sa silama M i N

Očitamo : R = 12,4 cm ⇒ 124 kN
A = 2,2cm ⇒ - 22 kN (negativan predznak zbog "krive" pretpostavke)
B = 13,2 cm ⇒ 132 kN

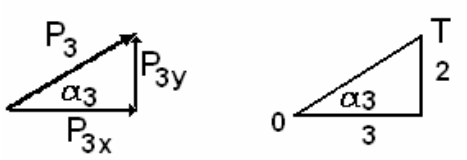
Računski :

1. pretpostavimo smjerove za M i N
2. projiciramo sile na komponente



$P_{2x} = P_2 \times \sin 30° = 40 \times 0{,}5 = 20$ kN

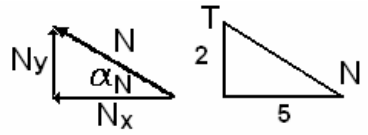
$P_{2y} = P_2 \times \cos 30° = 40 \times 0{,}866 = 34{,}64$ kN



$tg\alpha_3 = \frac{2}{3} = 0{,}667 \quad \alpha_3 = 33{,}69°$

$P_{3x} = P_3 \times \cos\alpha_3 = 50 \times 0{,}832 = 41{,}60$ kN

$P_{3y} = P_3 \times \sin\alpha_3 = 50 \times 0{,}5547 = 27{,}735$ kN



$tg\alpha_N = \frac{2}{5} = 0{,}4 \qquad \alpha_N = 21{,}80°$

$N_X = N \times \cos\alpha_N = 0{,}928N$

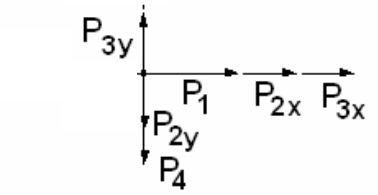
$N_Y = N \times \sin\alpha_N = 0{,}3714$ N

Rezultanta R

$R_X = P_1 + P_{2X} + P_{3X} = 60 + 20 + 41{,}60 = 121{,}6$ kN

$R_Y = P_{3Y} - P_{2Y} - P_4 = 27{,}735 - 34{,}64 - 20 = -26{,}905$ kN

$R = \sqrt{121{,}6^2 + 26{,}905^2} = 124{,}54$ kN



Jednadžbe ravnoteže

$P_{1X} + P_{2X} + P_{3X} + N_X = 0 \quad \Rightarrow \quad$ **Σ x = 0**

$P_{2y} + P_{3y} + P_4 + M + N_Y = 0 \Rightarrow \quad$ **Σ y = 0**

Σ x = 0

$P_{1X} + P_{2X} + P_{3X} - N_X = 0$

60 + 20 + 41,60 - N_X = 0

N_X = 121,6 kN $\Rightarrow N = \frac{N_X}{\cos\alpha_N} = 131{,}03$ Kn

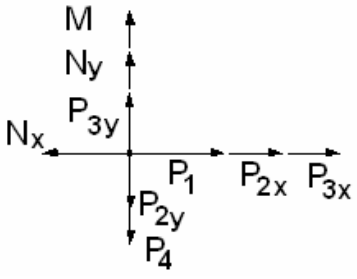
N_Y = N × sin α_N = 48,66 kN

Σ y = 0

$M + N_Y + P_{3Y} - P_{2Y} - P_4 = 0$

M + 48,66 + 27,735 – 34,64 -20 = 0

M + 21,755 = 0 $\Rightarrow$ M = - 21,75 kN

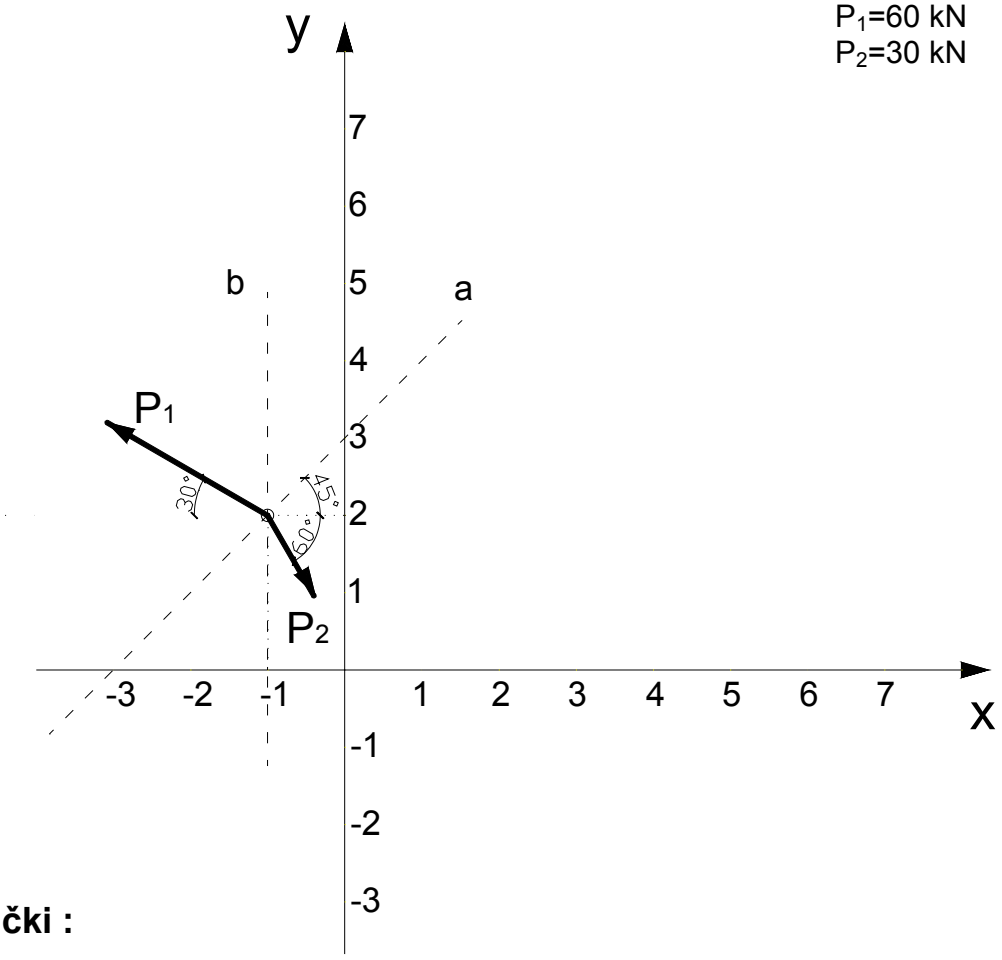


skica sila sa stvarnim djelovanjem na točku T

ZADATAK 2: (ravnoteža materijalne točke)

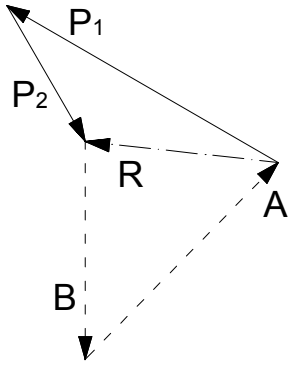
Na pravcima *l* i *k* potrebno je odrediti koncentrirane sile L i K koje će zajedno sa silama P_1 i P_2 držati materijalnu točku T u stanju mirovanja tj. ravnoteži. Riješiti grafički i računski.

P_1=60 kN
P_2=30 kN



Grafički :

Mjerilo sila :
1cm :: 20 kN



OČITANO:

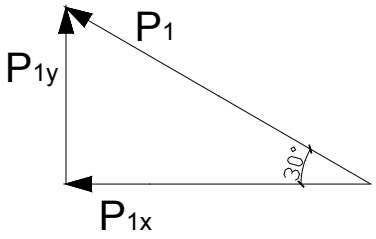
R = 37.2 kN
A = 52.5 kN
B = 41.7 kN

• rezultantu sila u ovom slučaju ne projiciramo na komponente već tražimo dvije sile s kojima će biti u ravnoteži, tj. poligon sila treba biti zatvoren.

Računski :

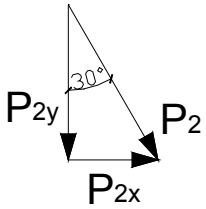
- *sve sile projiciramo na komponente u smjerovima x i y*

P1



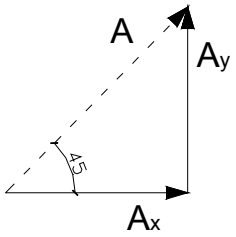
$P_{1x} = P_1 \times \cos\alpha_1$ = 60 × cos 30° = 52.0 kN
$P_{1y} = P_1 \times \sin\alpha_1$ = 60 × sin 30° = 30.0 kN

P2



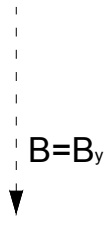
$P_{2x} = P_2 \times \sin\alpha_2$ = 30 × sin 30° = 15,0 kN
$P_{2y} = P_2 \times \cos\alpha_2$ = 30 × cos 30° = 26.0 kN

A



$A_x = A \times \cos\alpha_A$ = A × cos 45° = 0.7×A
$A_y = A \times \sin\alpha_A$ = A × sin 45° = 0.7×A

B



$B_y = B$

Jednadžbe ravnoteže :

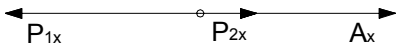
$\vec{P}_1 + \vec{P}_2 + \vec{A} + \vec{B} = 0$ → vektorska jednadžba

$P_{1x} + P_{2x} + A_x = 0$

$P_{1y} + P_{2y} + A_y + B = 0$ → skalarne jednadžbe

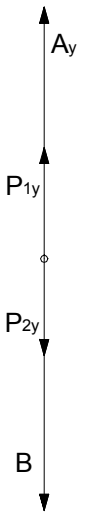
$\sum x = 0$

-52,0 + 15.0 + 0.7A = 0



$\sum y = 0$

30.0 - 26.0 + 0.7A –B = 0

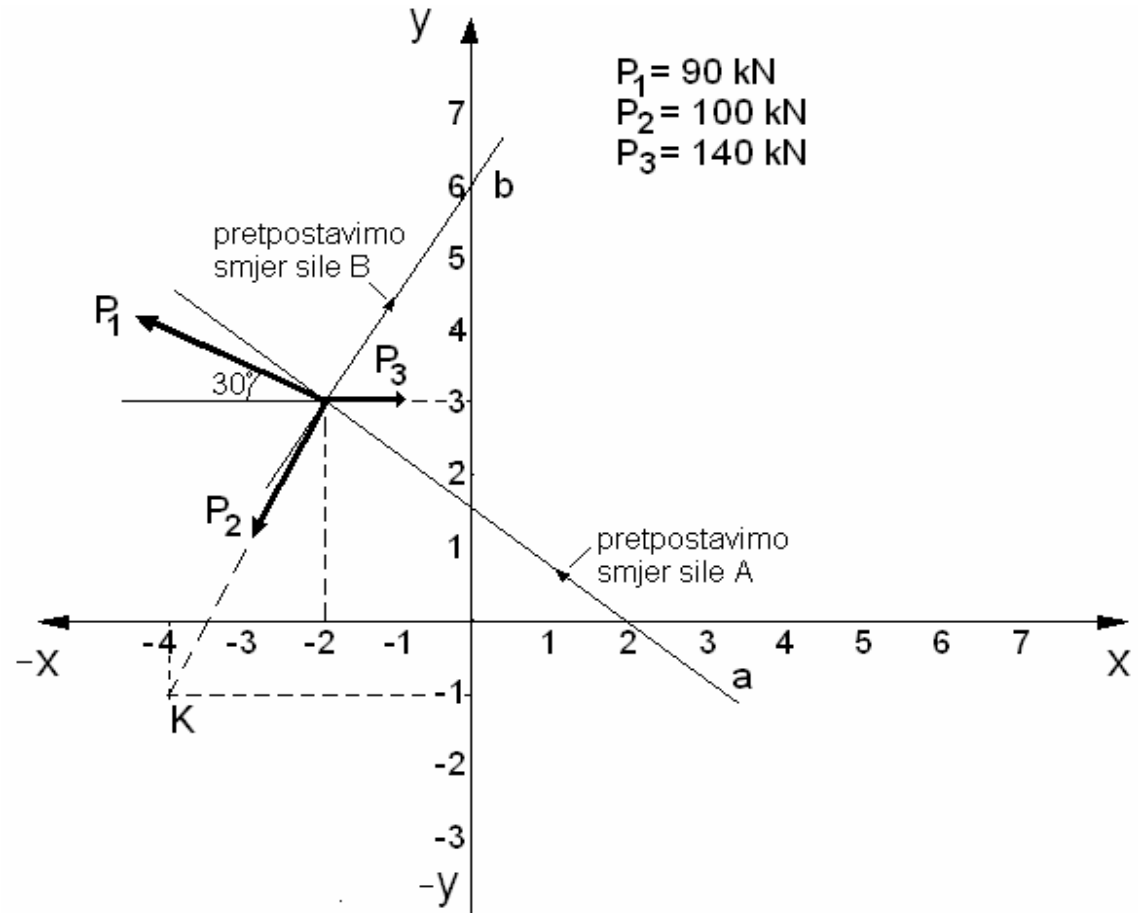


-52,0 + 15.0 + 0.7A = 0 → **A = 52.8 kN**

30.0 - 26.0 + 0.7(52.85) – B = 0 → **B = 41.0 kN**

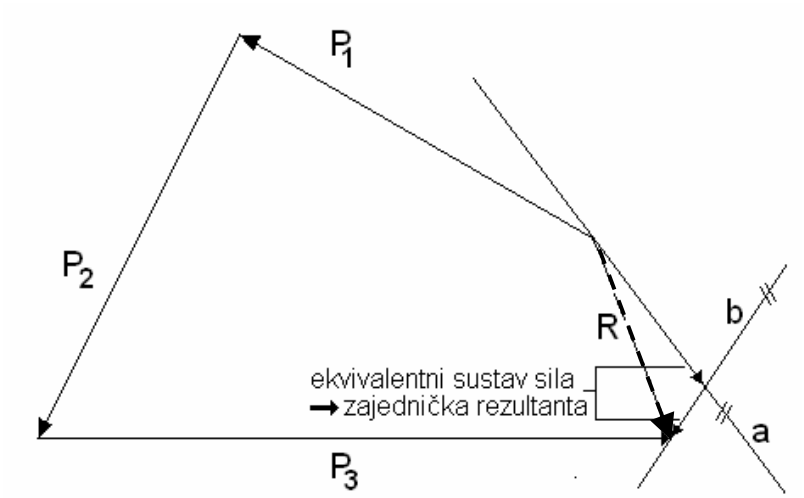
ZADATAK 3: (rezultirajuće → ekvivalentno djelovanje na materijalnu točku)

Za prikazani sustav sila $\vec{P}_1, \vec{P}_2$ i $\vec{P}_3$ potrebno je odrediti rezultantu R te ekvivalentni sustav sila A i B koje djeluje na pravcima a i b. Riješiti grafički i računski

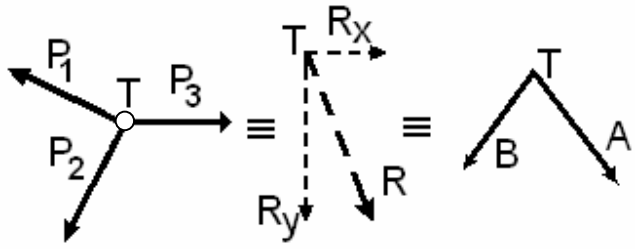


Grafički : Mjerilo sila ⇒ 1 cm = 20 kN

Očitano : R = 2,4 cm ⇒ 48 kN
A = 2,0 cm ⇒ - 40 kN
B = 1,3 cm ⇒ - 26 kN

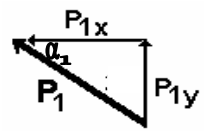


Napomena : Kod očitavanja sila A i B negativan predznak znači da smo mi pretpostavili krivi smjer sila.



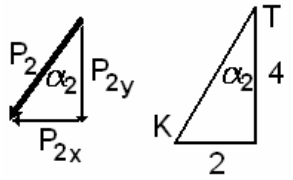
zadani sustav rezultirajuće djelovanje ekvivalentni sustav

Računski : svaku kosu silu treba projicirati na komponente u smjeru osi x i y



$P_{1x} = P_1 \times \cos\alpha_1$ = 90 × cos 30° = 77.94 kN

$P_{1y} = P_1 \times \sin\alpha_1$ = 90 × sin 30° = 45 kN



kut α_2 očitamo iz crteža

$tg\alpha_2 = \frac{2}{4}$ = 0,5

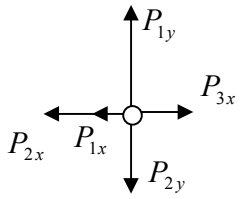
$\alpha = 26{,}565°$

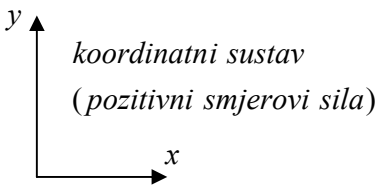
$P_{2y} = P_2 \times \cos\alpha_2$ = 100 × 0,8944 = 89,44 kN

$P_{2x} = P_2 \times \sin\alpha_2$ = 100 × 0,4472 = 44.72kN

⟶ P_3 ⟶ P_{3x} = 140 kN

Određivanje rezultante R





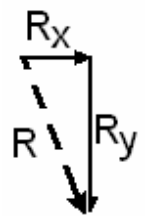
$$\sum_i P_{ix} = R_x \qquad \sum_i P_{iy} = R_y$$

$$-P_{1X} - P_{2X} + P_{3X} = R_X \qquad P_{1Y} - P_{2Y} = R_Y$$

- 77,94 – 44,72 +140 = R_X 45 – 89,44 = R_Y

R_X = 17,34 kN R_Y = - 44,44

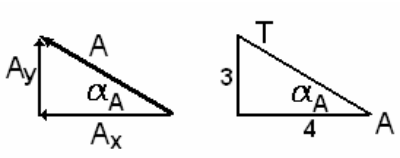
$$R = \sqrt{R_x^2 + R_y^2} \qquad R = \sqrt{17{,}34^2 + (-44{,}44)^2} = 47{,}70 \text{ kN}$$



= usporedimo s grafičkom rezultantom

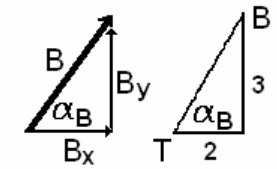
Određivanje ekvivalentnih sila A i B

odredimo komponente sila A i B



$$tg\alpha_A = \frac{3}{4} = 0{,}75$$

$$\alpha_A = 36{,}87°$$



$$tg\alpha_B = \frac{3}{2} = 1{,}5$$

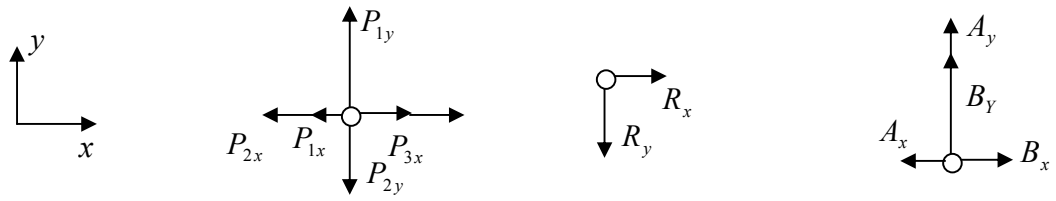
$$\alpha_B = 56{,}31°$$

$A_x = A \times \cos\alpha_A$ = 0,8 A $\qquad B_x = B \times \cos\alpha_B$ = 0,5547 B

$A_y = A \times \sin\alpha_A$ = 0,6 A $\qquad B_y = B \times \sin\alpha_B$ = 0,832 B

Jednadžbe ekvivalencije :

Pozitivni smjerovi: Vanjske sile: Rezultanta: Zamjenjujuće djelovanje (pretpostavka)



$$-P_{1X} - P_{2X} + P_{3X} = R_X = -A_X + B_X$$

17,34 = -0,8 A + 0,5547 B **(1)**

$$P_{1Y} - P_{2Y} = R_Y = A_Y + B_Y$$

- 44,44 = 0,6 A + 0,832 B (2)

imamo sustav od 2 jednadžbe s 2 nepoznanice, kad ih riješimo dobijemo

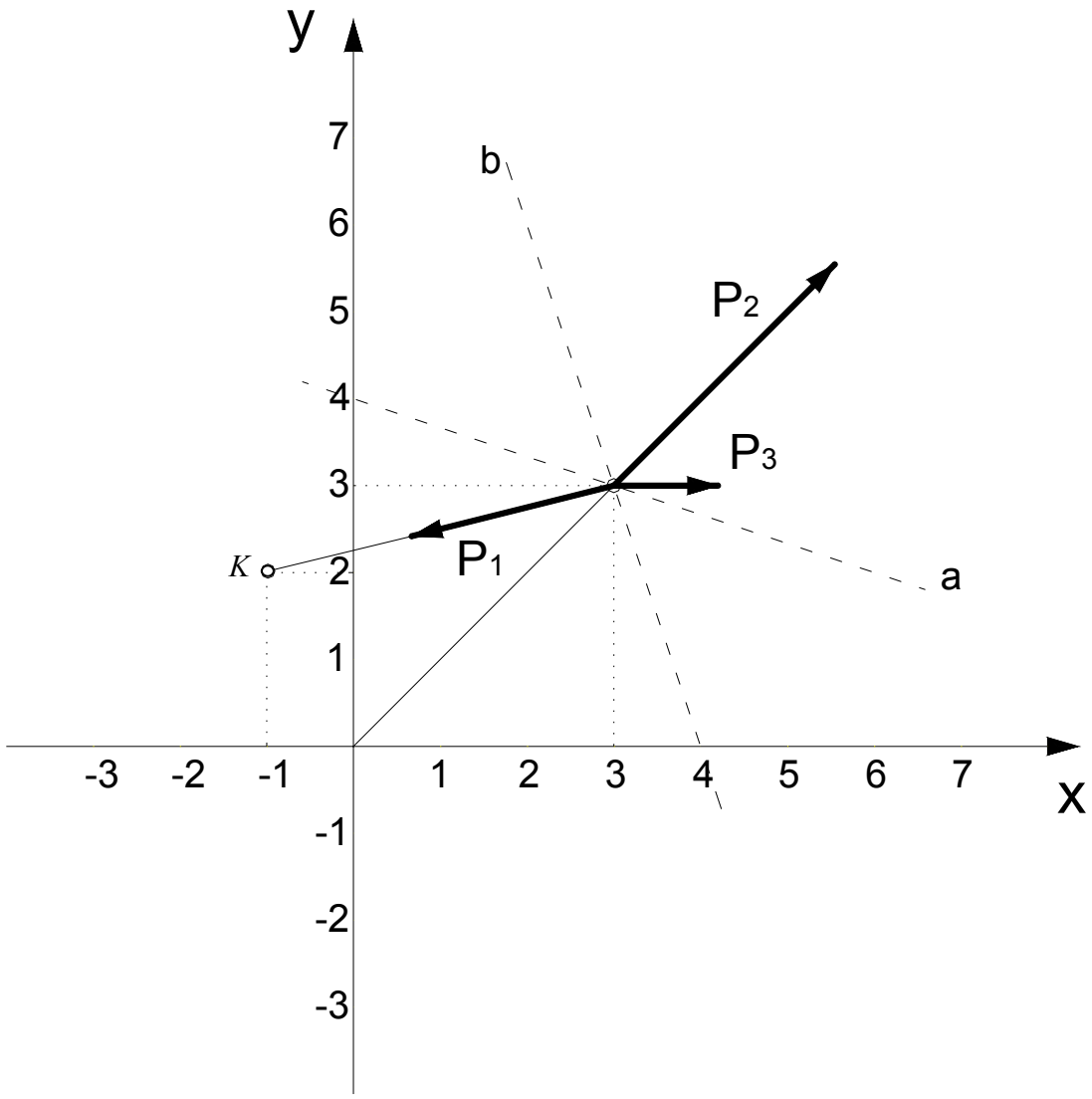
A = -39,14 kN
B = -25,187 kN

Usporedimo rezultat s grafičkim rješenjem. *(Negativan predznak znači da smo pretpostavili krivi smjer djelovanja.)*

ZADATAK 4: (rezultirajuće → ekvivalentno djelovanje na materijalnu točku)

Za prikazani sustav sila $\vec{P}_1, \vec{P}_2$ i $\vec{P}_3$ potrebno je odrediti rezultantu R te ekvivalentni sustav sila A i B koje djeluje na pravcima a i b. Riješiti grafički i računski

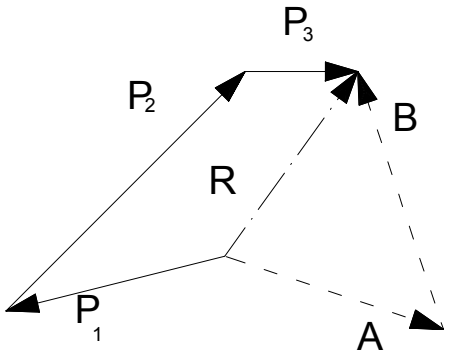
P_1=60 kN
P_2=90 kN
P_3=30 kN



Grafički :

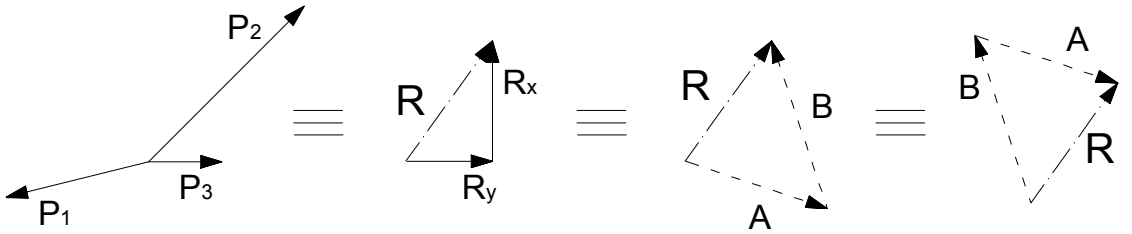
Mjerilo sila : *(kako crtate tako i očitavate)*
1cm :: 20 kN

OČITANO:
R = 60,5 kN
A = 61,0 kN
B = 72,0 kN



* *primjenjujemo pravilo vektorskog poligona*

Zadane sile: Rezultanta : Zamjenjujuće (ekvivalentno) djelovanje :

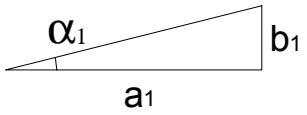


(*nije bitno u koju točku postavljamo pojedini pravac glavno je da su paralelni sa smjerom sile*)

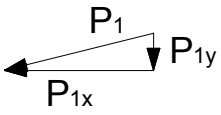
Računski :

- *sve sile projiciramo na komponente u smjerovima x i y*
- *kuteve nagiba pravaca sila očitavamo iz koordinatnog sustava*
- *smjerove sila su na prethodnoj stranici*

P_1



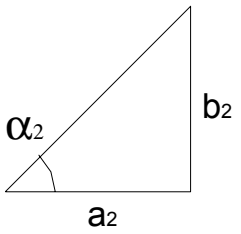
$$\mathrm{tg}\alpha_1 = \frac{b_1}{a_1} = \frac{1}{4} \Rightarrow \alpha = 14°$$



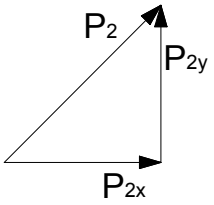
$P_{1x} = P_1 \times \cos\alpha_1$ = 60 × cos 14° = 58.2 kN

$P_{1y} = P_1 \times \sin\alpha_1$ = 60 × sin 14° = 14.5 kN

P_2



$$tg\alpha_1 = \frac{b_1}{a_1} = \frac{3}{3} \Rightarrow \alpha = 45°$$



$P_{2x} = P_2 \times \cos\alpha_2$ = 90 × cos 45° = 63.6 kN

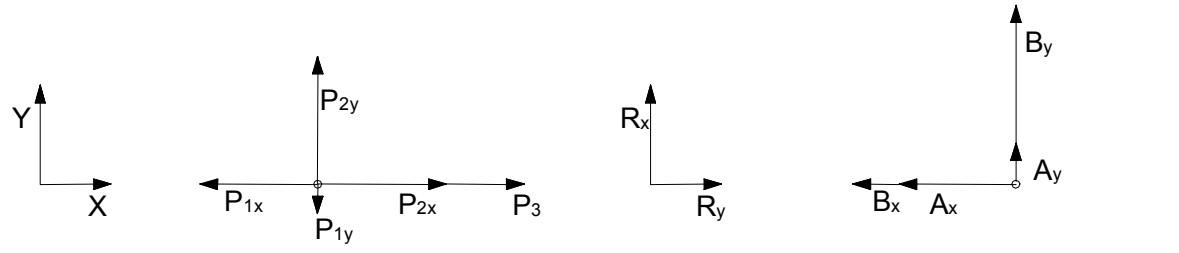
$P_{2y} = P_2 \times \sin\alpha_2$ = 90 × sin 45° = 63.6 kN

P_3

$P_{3x} = P_3$ = 30 kN

<u>Komponente sila za računski dio:</u>

Pozitivni smjerovi: Vanjske sile: Rezultanta: Zamjenjujuće djelovanje (pretpostavka)



Rezultanta :

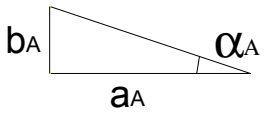
$$\sum P_{ix} = R_x \qquad \sum P_{iy} = R_y \qquad \rightarrow \qquad R = \sqrt{R_x^2 + R_y^2}$$

$$-P_{1x} + P_{2x} + P_{3x} = R_x \qquad -P_{1y} + P_{2y} = R_y$$

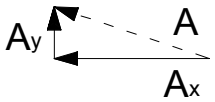
-58.2+63.6+30=35.4kN -14.5+63.6=49.1 kN

$$R = \sqrt{35.4^2 + 49.1^2} = \mathbf{60.5\ kN}$$

Ekvivalentni sustav sila A i B :

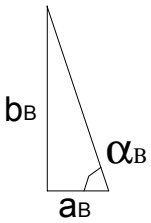


$$\mathrm{tg}\alpha_A = \frac{b_A}{a_A} = \frac{1}{3} \Rightarrow \alpha = 18.4°$$

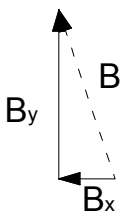


$A_x = A \times \cos\alpha_A$ = A × cos 18.4° = 0.95×A
$A_y = A \times \sin\alpha_A$ = A × sin 18.4° = 0.31×A

** pretpostavljeni smjer sile A*



$$\mathrm{tg}\alpha_B = \frac{b_B}{a_B} = \frac{3}{1} \Rightarrow \alpha = 71.6°$$



$B_x = B \times \cos\alpha_B$ = B × cos 71.6° = 0.31×B
$B_y = B \times \sin\alpha_B$ = B × sin 71.6° = 0.95×B

** pretpostavljeni smjer sile B*

Rješavanje sustava (supstitucijom) :

$$\sum P_{ix} = R_x = -A_x - B_x \qquad \sum P_{iy} = R_y = A_y + B_y$$

➔

$$35.4 = -A_x - B_x \qquad 49.1 = A_y + B_y$$

(pozitivni smjerovi kao u koordinatnom sustavu)

➔

$$35.4 = -0.95A - 0.31B \qquad 49.1 = 0.31A + 0.95B$$

$$A = \frac{49.1 - 0.95B}{0.31} = 158.4 - 3.1B \quad \text{➔} \quad 35.4 = -0.95 \times (158.4 - 3.1B) - 0.31B$$

$$35.4 = -150.5 + 2.9B - 0.31B \quad \text{➔} \quad \textbf{B = 71.7 kN}$$

$$49.1 = 0.31A + 0.95(71.7) \quad \text{➔} \quad \textbf{A = - 61.5 kN}$$

(sila A je negativna što znači da je suprotnog smjera od pretpostavljenog)

STATIKA TIJELA U RAVNINI

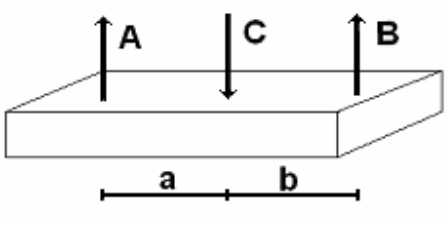
OSNOVNI POJMOVI

hvatište – točka u kojoj djeluje koncentrirana sila ili koncentrirani moment
pravac djelovanja sile – određen je vektorom sile i hvatištem

ZAKON POLUGE

Tijelo na koje djeluju 3 paralelne sile biti će u **ravnoteži** ako :

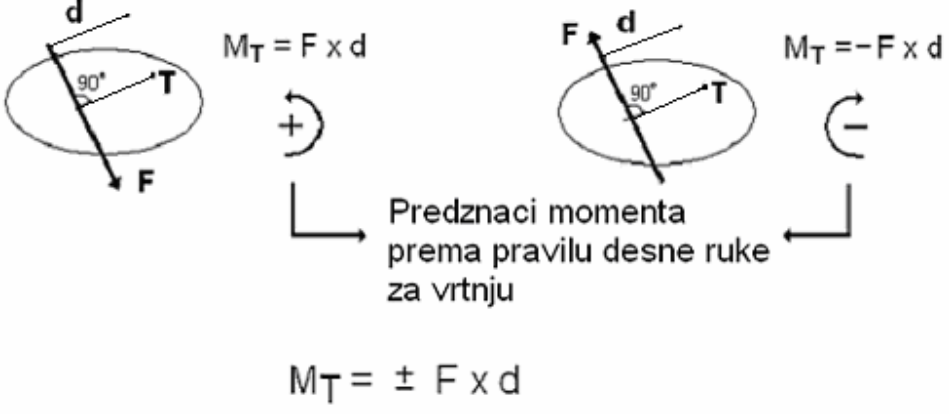
a.) sile leže u istoj ravnini
b.) je srednja sila suprotno usmjerena od rubnih
c.) je veličina srednje sile jednaka zbroju veličina rubnih sila
d.) je umnožak veličine rubnih sila s pripadajućim udaljenostima od srednje sile jednak, tj. **A×a = B×b**



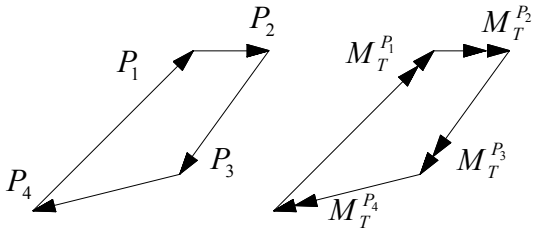
VEKTOR MOMENTA SILE NA TOČKU

Za određivanje tog vektora potrebno je odabrati proizvoljnu točku u ravnini, tzv. **točku redukcije.**

Vektor momenta sile na točku može se direktno geometrijski definirati: Određuje se **udaljenost** pravca djelovanja sile od točke redukcije i to nazivamo **krak sile**. Pravac vektora momenta okomit je na ravninu određenu točkom redukcije i pravcem djelovanja sile. Duljina vektora momenta jednaka je **umnošku veličine sile i kraka sile**.



GEOMETRIJSKA FORMULACIJA UVJETA RAVNOTEŽE TIJELA NA KOJE DJELUJU KONCENTRIRANE SILE

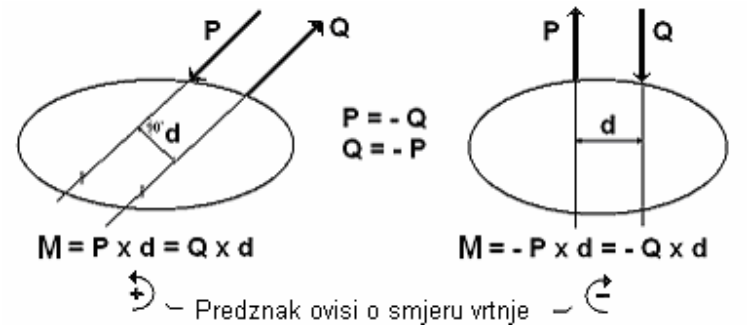


Tijelo na koje djeluju koncentrirane sile bit će u ravnoteži ako je poligon sila zatvoren i ako je poligon vektora momenata zatvoren.

Zakon poluge je specijalni slučaj općeg uvjeta.

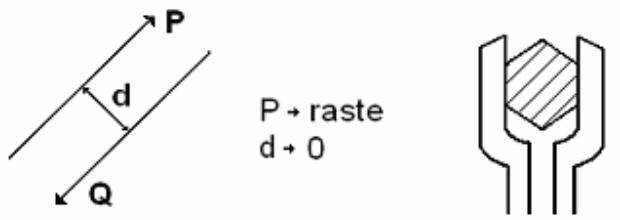
SPREG SILA

čine dvije sile koje djeluju na tijelo na paralelnim pravcima, smjerovi djelovanja su im suprotni, a veličine jednake. Doprinos sprega sila poligonu sila iščezava, a doprinos sprega sila poligonu momenata je konstantan, tj. ne ovisi o odabiru točke redukcije.

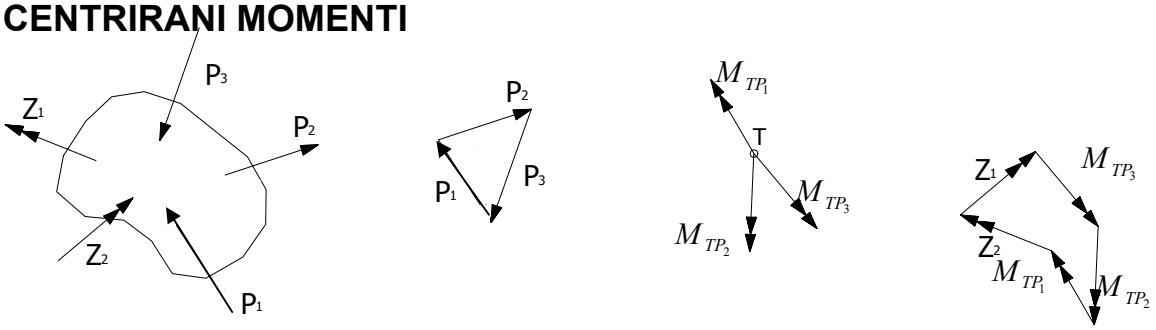


KONCENTRIRANI MOMENT (bolje koncentrirani spreg)

- je ekvivalent spregu sila koje su proizvoljno velike , a krak dovoljno malen, pri čemu je vektor momenata konstantan.



UVJETI RAVNOTEŽE TIJELA NA KOJEG DJELUJU KONCENTRIRANE SILE I KONCENTRIRANI MOMENTI



Tijelo je u ravnoteži ako je zatvoren poligon sila i ako je zatvoren poligon kojega čine koncentrirani momenti i vektori momenata sila na istu točku redukcije.

Ako na tijelo djeluju samo spregovi i koncentrirani momenti, poligon sila iščezava i uvjet se svodi na zahtjev zatvorenosti momentnog poligona.

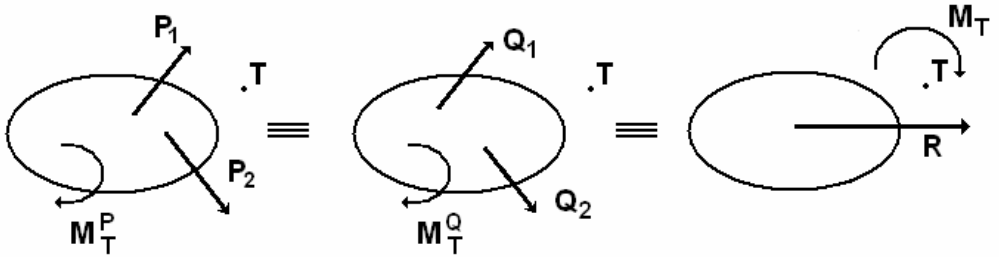
SILA je KLIZNI vektor jer njezin doprinos uvjetima ravnoteže ne ovisi o položaju na pravcu djelovanja.

KONCENTRIRANI MOMENT je SLOBODNI vektor jer njegov doprinos uvjetima ravnoteže ne ovisi o položaju njegovog hvatišta.

STATIČKA EKVIVALENCIJA

Dva sustava sila i koncentriranih momenata su statički ekvivalentni ako su njihovi doprinosi uvjetima ravnoteže jednaki. Poligoni sila moraju imati iste početke i iste šiljke te poligoni momenata moraju također imati zajedničke početke i šiljke.

Rezultirajuće (**ekvivalentno**) djelovanje čine jedna zamišljena koncentrirana sila koja se naziva **REZULTANTNA sila** i jedan zamišljeni koncentrirani moment koji se naziva **REZULTIRAJUĆI moment.**



$\Sigma x_P = \Sigma x_Q = R_X$

$\Sigma y_P = \Sigma y_Q = R_Y$ jednadžbe ekvivalencije

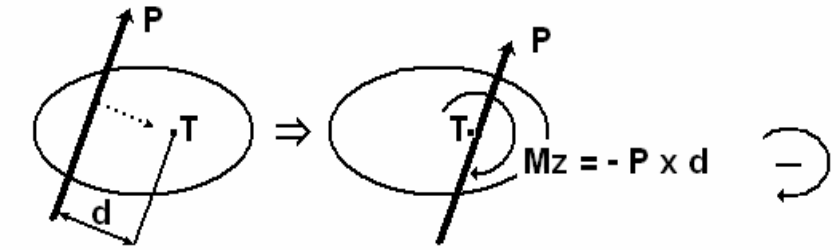
$\Sigma M^P_{(T)} = \Sigma M^Q_{(T)} = M_T$

REZULTANTNO (REZULTIRAJUĆE) djelovanje – tražimo rezultantu sila i rezultantu momenata i na kraju sve svedemo na 1 silu R (*redukcija sile na točku*)

Da bismo mogli naći samo jednu silu R kao rezultirajuće djelovanje potrebno je poznavati

REDUKCIJA SILE NA TOČKU

– umjesto zadane uvodi se zamišljena sila u zadanoj točki , a ekvivalencija je moguća samo ako se uvede još i odgovarajući " zamišljeni " moment.



JEDNADŽBE

Kod tijela kao i kod točke možemo tražiti :

a.) **ekvivalentno** djelovanje tj. **rezultantu** sile.

Jednadžbe ekvivalencije :

$$\sum x_P = \sum x_Q = R_x$$

zadano ekviv.

$$\sum y_P = \sum y_Q = R_y$$

b.) **ravnotežno** djelovanje – **zatvoren poligon sila**

Jednadžbe ravnoteže :

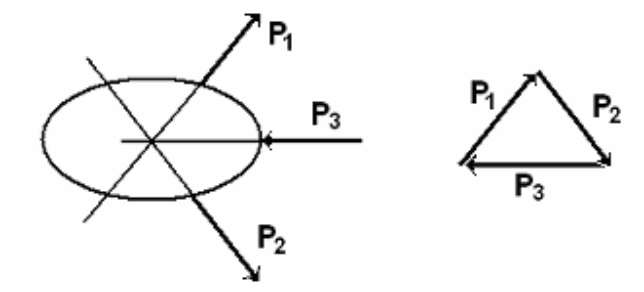
$$\sum x_P + \sum x_Q = 0 \qquad \sum x = 0$$

zadano traženo

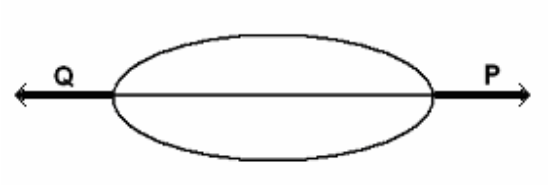
$$\sum y_P + \sum y_Q = 0 \qquad \sum y = 0$$

STATIKA TIJELA NA KOJE DJELUJU SAMO KONCENTRIRANE SILE ČIJI SE PRAVCI SIJEKU U JEDNOJ TOČKI

To je specijalni slučaj statike kod kojeg se uvjet ravnoteže svodi na uvjet ravnoteže točke.
(ako na tijelo djeluju 3 sile čiji se pravci sijeku u 1 točki → isto kao statika točke).

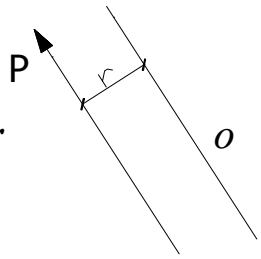


Ako na tijelo djeluju samo 2 sile – ravnoteža je moguća samo ako sile leže na istom pravcu , iste su po veličini, a suprotnog smjera djelovanja.



moment oko osi → konstantan, ne ovisi o izboru čvrste točke.

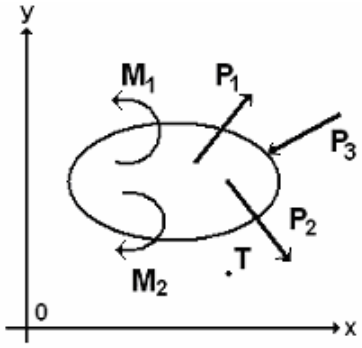
$$M_o = P \times r$$



ALGEBARSKI UVJETI RAVNOTEŽE

OSNOVNA FORMULACIJA

Tijelo na koje djeluju koncentrirane sile i koncentrirani momenti u ravnini bit će u ravnoteži ako iščezava zbroj projekcija svih sila na svaku od dvije neparalelne osi u toj ravnini i ako iščezava zbroj momenata svih sila i koncentriranih momenata na istu, proizvoljno odabranu točku redukcije.



* 2 osi (x i y) i 1 točka redukcije (T)

$\sum x = 0$

$\sum y = 0$

$\sum M_T = 0$

prostor (općenito)

$\Sigma x = 0 \quad \Sigma M_X = 0$

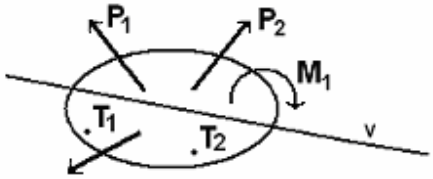
$\Sigma y = 0 \quad \Sigma M_Y = 0$

$\Sigma z = 0 \quad \Sigma M_Z = 0$

ravnina **Σ x = 0**
Σ y = 0
Σ M = 0

FORMULACIJA S JEDNOM OSI I DVIJE TOČKE REDUKCIJE

Tijelo na koje djeluju koncentrirane sile i koncentrirani momenti u ravnini bit će u ravnoteži ako iščezava zbroj projekcija svih sila na jednu os u toj ravnini i ako iščezavaju zbrojevi momenata svih sila i svih koncentriranih momenata na dvije različite točke čija spojnica nije okomita na os projekcija.



* 1 os (v) i 2 točke redukcije (T_1 i T_2)

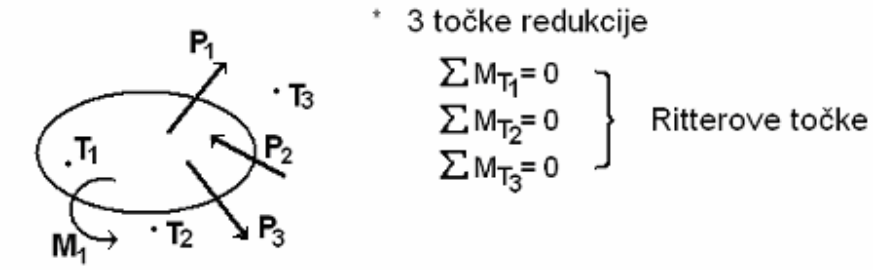
$\sum v = 0$

$\sum M_{T_1} = 0$

$\sum M_{T_2} = 0$

FORMULACIJA S TRI TOČKE REDUKCIJE

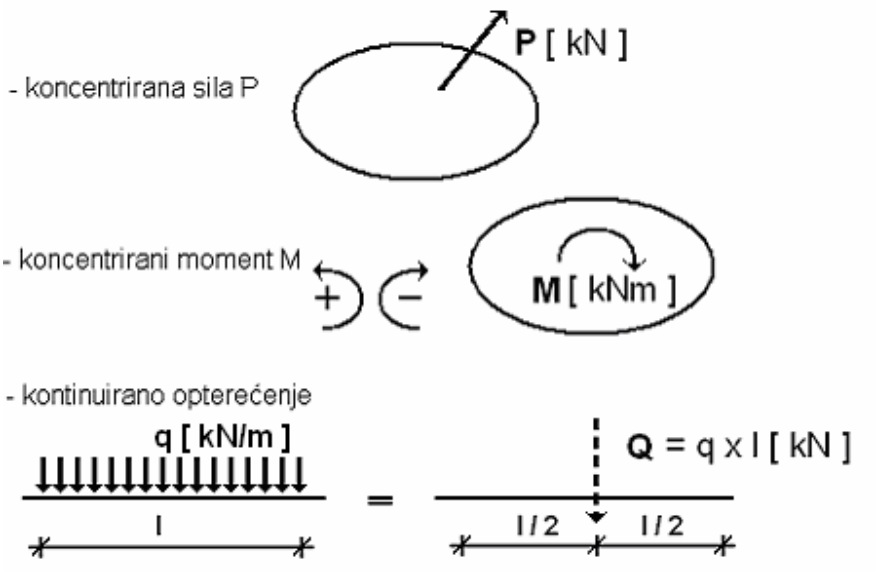
* Tijelo na koje djeluju koncentrirane sile i momenti ravnini bit će u ravnoteži ako iščezava zbroj momenata od svih sila i svih koncentriranih momenata na tri točke redukcije koje leže na istom pravcu.



GEOMETRIJSKI UVJETI RAVNOTEŽE

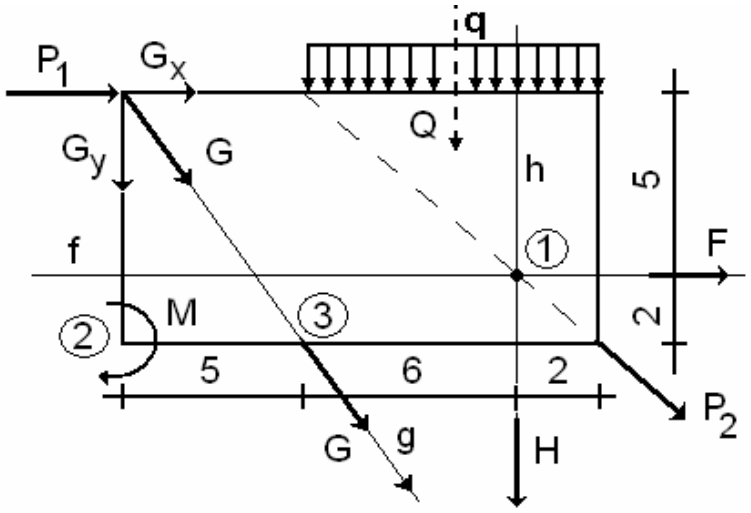
Tijelo je u ravnoteži ako je poligon sila zatvoreni lik, te ako je poligon momenata kojeg čine vektori momenata sila oko neke proizvoljne točke te vektori koncentriranih momenata također zatvoreni poligon.

VRSTE OPTEREĆENJA



ZADATAK 5: – ravnoteža tijela

Zadano opterećenje tijela potrebno je uravnotežiti silama F, G i H na pravcima f , g i h .



g = 20 kN/m

$\Rightarrow$ **Q = 8 × 20 = 160 kN**

P_1 = 50kN

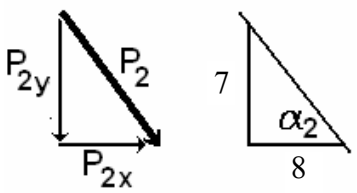
P_2 = 60kN

M = 320kNm

Rješenje:

1 .**pretpostavimo smjerove** za sile F, G i H na pravcima f, g i h

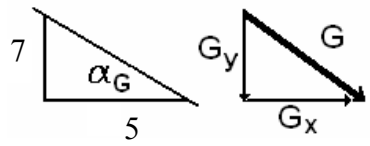
2. **projiciramo sile** P_2 i G



$tg\alpha_2 = \frac{7}{8} = 0{,}875 \quad \alpha_2 = 41.186^0$

$P_{2x} = P_2 \cdot \cos\alpha_2 = 45{,}1545$ kN

$P_{2y} = P_2 \cdot \sin\alpha_2 = 39{,}51$ kN



$tg\alpha_G = \frac{7}{5} = 1{,}4$; $\alpha_G = 54{,}462^\circ$

$G_x = G \cdot \cos\alpha_G = 0{,}5812\ G$

$G_y = G \cdot \sin\alpha_G = 0{,}8137\ G$

3. **Jednadžbe ravnoteže** : **Σ x = 0 ; Σ y = 0 ; Σ M_T = 0**

Ako raspišemo jednadžbu **Σx = 0** vidjet ćemo da nam se pojavljuju **2 nepoznate** sile F i G_x
Ako raspišemo jednadžbu **Σy = 0** vidjet ćemo da imamo opet **2 nepoznate** sile G_y i H
Zato **odabiremo točku (1)** gdje nam se sijeku pravci sila F i H, dakle **imamo samo 1 nepoznatu** silu G.

****(sila je klizni vektor pa smo pretpostavili da sila G djeluje gore – lijevo)*

Σ $M_{(1)}$ = 0

-M - P_1 · 5 + (q · 8) · (4-2) - G_x · 5 + G_y · 11 + P_{2x} · 2 - P_{2y} · 2 = 0

-320 – 50 · 5 + 160 · 2 – 0,5812 G · 5 + 0,8137 G · 11 + 45,15 · 2 – 39,51 · 2 = 0

6,0447 G = 238,71

G = 39,49 kN → G_x = 39,49 · 0,5812 = 22,95 kN

G_y = 39,49 · 0,8137 = 32,13 kN

$\sum$ **y = 0**

$- Q - P_{2y} - G_y – H = 0$

H $= - Q - P_{2y} - G_y = - 160 – 39,51 – 32,13$ **= -231.6 kN**

$\sum$ **x = 0**

$P_1 + P_{2x} + G_x + F = 0$

F $= - P_1 - P_{2x} - G_x = - 50 – 45,15 – 22,9$ **= -118,1 kN**

KONTROLA!

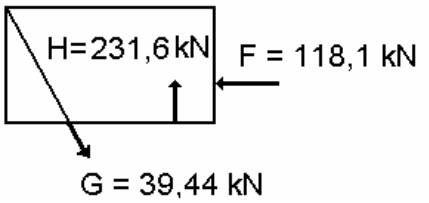
Odabiremo proizvoljnu točku (2)

$- M – H \cdot 11 – F \cdot 2 – (G_y \cdot 5)^{**} - Q \cdot 9 – P_{2y} \cdot 13 – P_1 \cdot 7 = 0$

$- 320 - (- 231,6) \cdot 11 - (- 118,1) \cdot 2 – 32,13 \cdot 5 - 160 \cdot 9 – 39,51 \cdot 13 - 50 \cdot 7 = - 0,28 \approx 0$

*** sada smo zamislili da sila G ima hvatište u točki (3)*

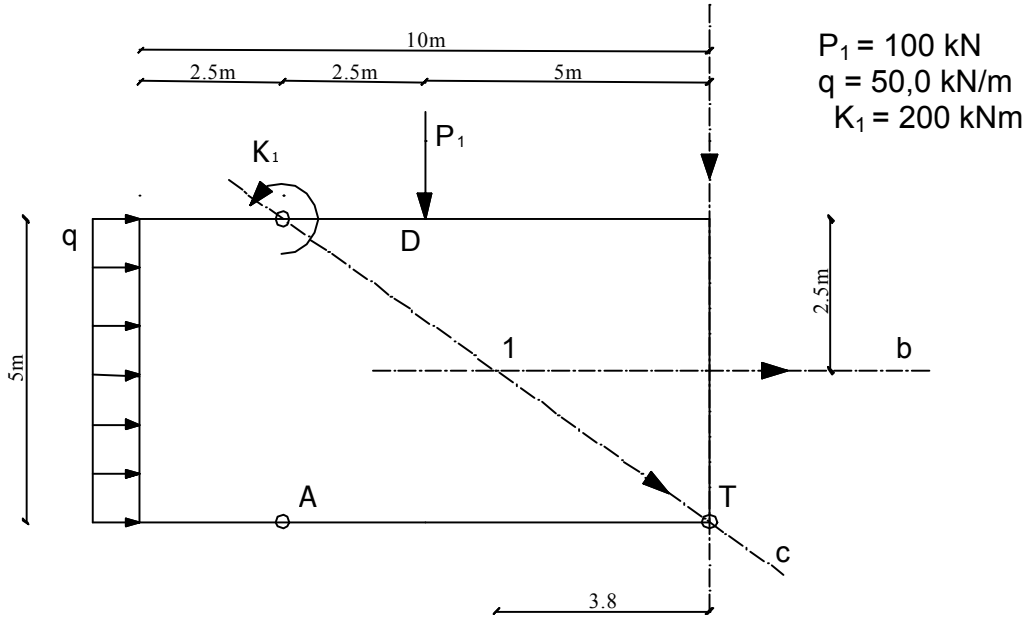
Skica stvarnih djelovanja



F	118,1 kN
G	39,44 kN
H	231,6 kN

ZADATAK 6: – ravnoteža tijela

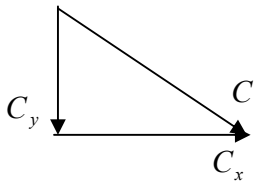
Zadano opterećenje potrebno je uravnotežiti silama A, B i C .na pravcima a, b i c



Rješenje:

Projiciramo silu C :

$$tg\alpha_c = \frac{5}{7.5} = 0.66 \quad \alpha_c = 33.7°$$



$C_x = C \times \cos\alpha_c$ = C × cos 33.7° = 0.83×C

$C_y = C \times \sin\alpha_c$ = C × sin 33.7° = 0.55×C

(pozitivni smjerovi kao kod koordinatnih osi, a pretpostavljeni pozitivni moment suprotno od kazaljke na satu)

Jednadžbe ravnoteže :

$\sum x = 0$ $\sum y = 0$ $\sum M_{(T)} = 0$ → jednadžbe postavljamo tako da dobijemo što manje nepoznanica,

$\sum M_{(1)} = 0$

$$\frac{2.5}{\text{tg}\alpha} = \frac{2.5}{0.66} = 3.78 = \text{n}$$

$P_1 \times (5-3.8) + K_1 - A \times 3.8 = 0$
$100 \times (5-3.8) + 200 - A \times 3.8 = 0$

A = 84.2 kN

$\sum y = 0$

$-P_1 - A - C_y = 0$
$100 + 84.2 + C_y = 0$

C_y = -184.2

C_x = $\frac{-184}{tg\alpha} = \frac{-184}{0.67} =$ **-278.8 kN**

(lakše je poslužiti se projiciranjem)

C = $\frac{-184}{\sin\alpha} = \frac{-184}{0.55} =$ **-334.5 kN**

$\sum x = 0$

$q \times 5 + B + C_x = 0$
$50 \times 5 + B - 278.8 = 0$

B = 28.8 kN

Kontrola :

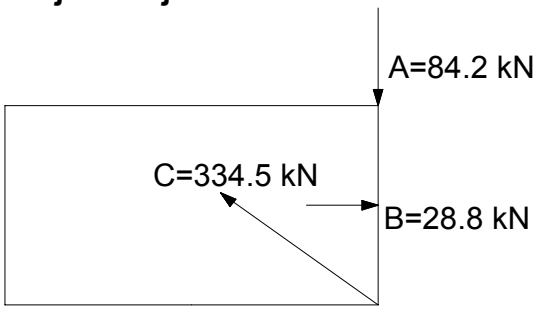
$\sum M_{(A)} = 0$

$-P_1 \times 2.5 + K_1 - A \times 7.5 - B \times 2.5 + C_y \times 7.5 - q \times 5 \times 2.5 = 0$

$-100 \times 2.5 + 200 - 84.2 \times 7.5 - 28.8 \times 2.5 + 184.2 \times 7.5 - 50 \times 5 \times 2.5 =$ **3 ≈ 0**

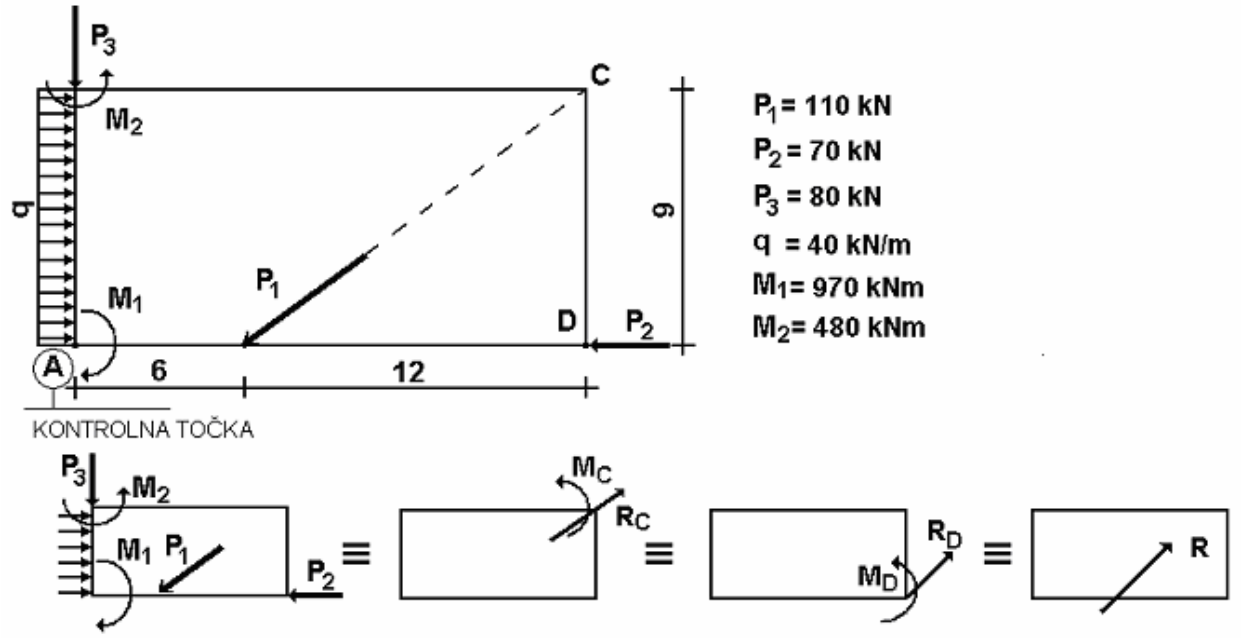
(*greška do 5,0 % je prihvatljiva)*

Skica stvarnih djelovanja :



ZADATAK 7: - ekvivalentno djelovanje u proizvoljnim točkama te rezultanta R

Za prikazani sustav potrebno odrediti u točkama C i D ekvivalentno (rezultirajuće) djelovanje te rezultantu R zadanog djelovanja sila i momenata.



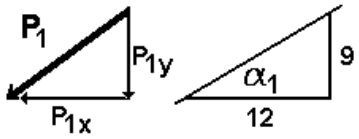
Rješenje:

Prvo silu P_1 projiciramo na komponente

$tg\alpha_1 = \frac{9}{12} = 0{,}75 \qquad \alpha_1 = 36{,}87°$

$P_{1x} = P_1 \cdot \cos\alpha_1 = 0{,}8\ P_1 = 88$ kN

$P_{1y} = P_1 \cdot \sin\alpha_1 = 0{,}6\ P_1 = 66$ kN



<u>Rezultirajuće djelovanje za točku C</u>

1) $\sum P_x = R_x^C \qquad -P_{1x} - P_2 + q \cdot 9 = R_x^C$

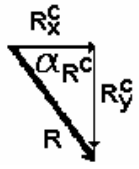
$-88 - 70 + 40 \cdot 9 = R_x^C \quad \Rightarrow \mathbf{R_x^C = 202\ kN}$

2) $\sum P_y = R_y^C \qquad -P_3 - P_{1y} = R_y^C$

$-80 - 66 = R_y^C \quad \Rightarrow R_y^C = -146$ kN

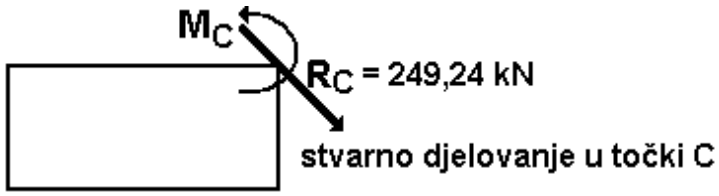
$\mathbf{R^C} = \sqrt{200^2 + 146^2}\ \mathbf{^2 = 249{,}24\ kN}$

$tg\alpha_{Rc} = \frac{R_y^C}{R_x^C} = \frac{-146}{202} = -0{,}7228 \qquad \alpha_{Rc} = 360° - 35{,}86° = 324{,}14°$



3) $\sum M_{©} = M^C$ (*suma svih momenata oko točke C daje nam rezultirajući moment za točku C* M^C)

$-P_2 \cdot 9 + P_3 \cdot 18 - M_1 + M_2 + (q \cdot 9) \cdot 4{,}5 = M^C$
$-70 \cdot 9 + 80 \cdot 18 - 970 + 480 + (40 \cdot 9) \cdot 4{,}5 = M^C \Rightarrow M^C = 1940$ kNm



KONTROLA !!! (**obavezno**)

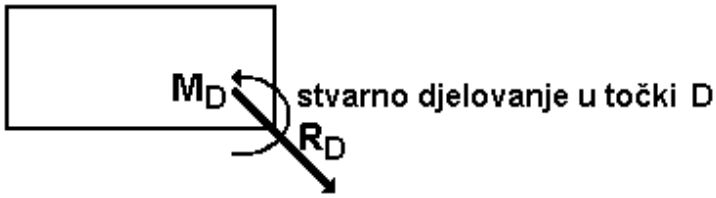
$\sum M_{(A)} = \sum M_{(A)}$ (suma momenata zadanih sila oko točke A mora biti jednaka
zadano rezult. sumi momenata rezultirajućeg djelovanja u točki C)

$-P_{1y} \cdot 6 - (q \cdot 9) \cdot 4{,}5 - M_1 + M_2 = - R_x^C \cdot 9 - R_y^C \cdot 18 + M^C$
$-66 \cdot 6 - (40 \cdot 9) \cdot 4{,}5 - 970 + 480 = - 202 \cdot 9 - 146 \cdot 18 + 1940$
$- 2506 = - 2506$
(*kontrolu smo mogli napraviti oko bilo koje proizvoljno odabrane točke*)

<u>Rezultirajuće djelovanje za točku D</u>

$\sum P_x = R_x^D = 202$ kN
$\sum P_y = R_y^D = -146$ kN $\Rightarrow$ **$R^D = 249{,}24$ kN** **Rezultante sila su iste!!!**

$\sum M_{(D)} = \sum M^D$
$-(q \cdot 9) \cdot 4{,}5 - M_1 + M_2 + P_3 \cdot 18 + P_{1y} \cdot 12 = M^D$
$-40 \cdot 9 \cdot 4{,}5 - 970 + 480 + 80 \cdot 18 + 66 \cdot 12 = M^D \Rightarrow M^D = 122$ kNm



KONTROLA *!*

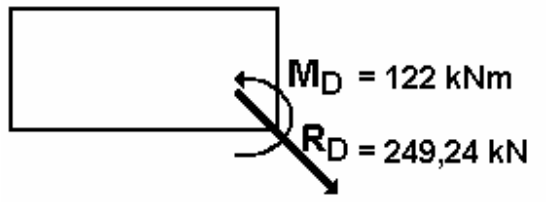
$\sum M_{(A)} = \sum M_{(A)}$
zadano rezult.

$- 2506 = - R_y^D \cdot 18 + 122$
$- 2506 = -146 \cdot 18 + 122$
$- 2506 = -2506$

Vidimo da su $R^C = R^D = R$ ali $M^C \neq M^D$

Rezultanta sila je ista za bilo koju odabranu točku jer uvijek imamo
$\boldsymbol{\sum P_x = R_x}$
$\boldsymbol{\sum P_y = R_y}$

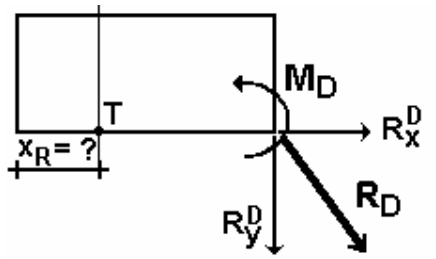
Izračun rezultantnog djelovanja, tj. R^D i M^D svodimo samo na jednu silu tj, rezultantu R



Da bi dobili samo **R** potrebno je R^D translatirati toliko da se poništi moment M^D (***redukcija sile na točku***)

Moment M^D je pozitivan ↺ što znači da rezultantu trebamo translatirati prema točki s lijeve strane kako bi nastao negativni moment ↻ - .

Odabiremo proizvoljno točku **T** kroz koju će prolaziti rezultanta **R** i svojom translacijom izazvati moment u iznosu od –122 kNm



$$\sum M_{(T)} = 0$$

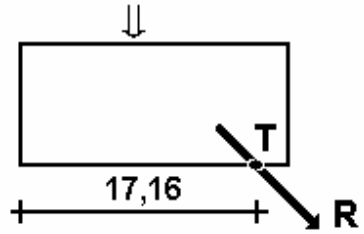
$$-R_y^D \cdot (18 - x_R) + M^D = 0$$

$$-146 \cdot (18 - x_R) + 122 = 0$$

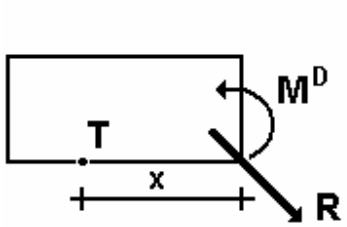
$$-2628 - 146 \cdot x_R + 122 = 0$$

$$X_R = \frac{2628 - 122}{146} = 17{,}16 \text{ m}$$

ovo je točka u kojoj smo (R^D+M^D) zamijenili samo s jednom silom i to rezultantom **R**



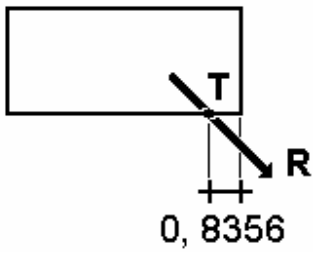
mogli smo i ovako (*čini mi se jednostavnije* ☺)



$$\sum M_{(T)} = 0$$

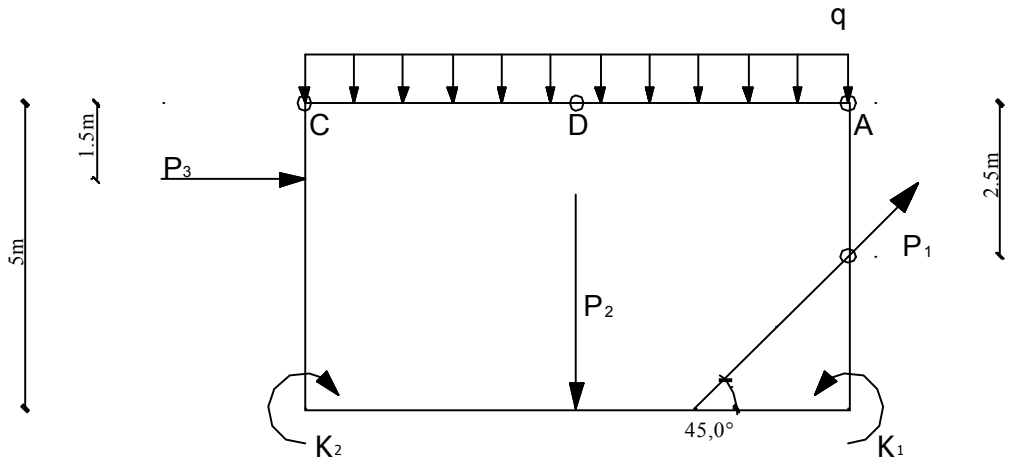
$$M^D - R_y \cdot x = 0$$

$$x = \frac{M^D}{R_y} = 0{,}8356$$



ZADATAK 8: - ekvivalentno djelovanje u proizvoljnim točkama te rezultanta R

Za prikazani sustav potrebno odrediti u točkama C i D ekvivalentno (rezultirajuće) djelovanje te rezultantu R zadanog djelovanja sila i momenata.

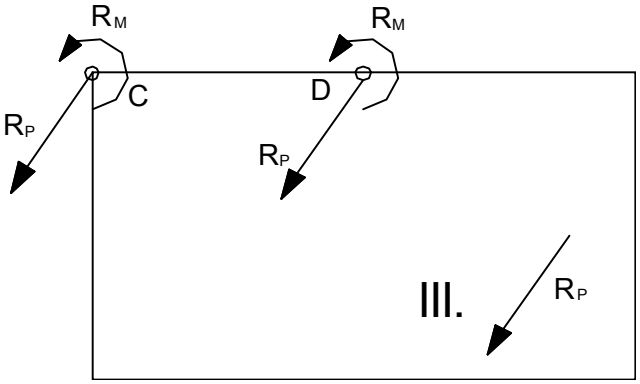


P_1 = 100 kN
P_2 = 75 kN
P_3 = 50 kN
q = 10,0 kN/m
K_1 = 80 kNm
K_2 = 40 kNm

Tražimo:

(*smjerovi pretpostavljeni)*

I. II.

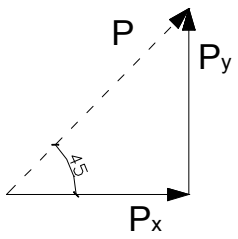


(pokazat kako se moment smanjuje)

Rješenje:

Projiciramo silu P :

P



$P_x = P \times \cos 45° = 70.7$ kN

$P_y = P \times \sin 45° = 70.7$ kN

pozitivni smjerovi kao kod koordinatnih osi, a pretpostavljeni pozitivni moment suprotno od kazaljke na satu)

Rezultirajuće djelovanje na točku C

$\sum_{zad} x = \sum_{traž} x = R_x^c$ → $P_{1x} + P_3 = 70.7 + 50.0$ → R_x^c = **120.7 kN**

$\sum_{zad} y = \sum_{traž} y = R_y^c$ → $P_{1y} - P_2 - q \times l = 70.7 - 75.0 - 10 \times 10$ → R_y^c = **-104,3 kN**

==========

$$R_P^C = \sqrt{120.7^2 + 104.3^2} = \mathbf{159.5\ kN}$$

$$tg\alpha_r^c = \frac{R_x}{R_y} = \frac{120.7}{-104.7} \rightarrow tg\alpha_r^c = 49°$$

$\sum_{zad} M_{(C)} = \sum_{traž} M_{(C)} = R_M^C$

$P_{1x} \times 2.5 + P_{1y} \times 10 + P_3 \times 1.5 - P_2 \times 5 - q \times 10 \times 5 + K_1 - K_2 =$

$70.7 \times 2.5 + 70.7 \times 10 + 50 \times 1.5 - 75 \times 5 - 10 \times 10 \times 5 + 80 - 40$ → R_M^C = **123.75 kN**

Kontrola :

$\sum_{zad} M_{(A)} = \sum_{traž} M_{(A)}$

$P_{1x} \times 2.5 + P_3 \times 1.5 + P_2 \times 5 + q \times 10 \times 5 + K_1 - K_2 = R_{1y} \times 10 + R_M$

$70.7 \times 2.5 + 50 \times 1.5 + 75 \times 5 + 10 \times 10 \times 5 + 80 - 40 = -104,3 \times 10 + 123.75$

1166.7kN = 1166,7kN

Rezultirajuće djelovanje na točku D

$\sum_{zad} x = \sum_{traž} x = R_x^D$ → $P_{1x} + P_3 = 70.7 + 50.0$ → R_x^D = **120.7 kN**

$\sum_{zad} y = \sum_{traž} y = R_y^D$ → $P_{1y} - P_2 - q \times l = 70.7 - 75.0 - 10 \times 10$ → R_y^D = **-104,3 kN**

==========

$$R_P^D = \sqrt{120.7^2 + 104.3^2} = \mathbf{159.5\ kN}$$

$$tg\alpha_r^D = \frac{R_x}{R_y} = \frac{120.7}{-104.7} \rightarrow tg\alpha_r^D = 49°$$

$\sum_{zad} M_{(D)} = \sum_{traž} M_{(D)} = R_M^D$

$P_{1x} \times 2.5 + P_{1y} \times 5 + P_3 \times 1.5 + K_1 - K_2 =$

$70.7 \times 2.5 + 70.7 \times 5 + 50 \times 1.5 + 80 - 40$ → R_M^D = **645.2 kN**

Kontrola :

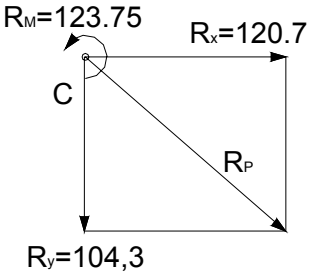
$\sum_{zad} M_{(A)} = \sum_{traž} M_{(A)}$

$P_{1x} \times 2.5 + P_3 \times 1.5 + P_2 \times 5 + q \times 10 \times 5 + K_1 - K_2 = R_{1y} \times 10 + R_M$

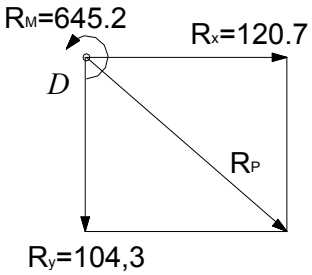
$70.7 \times 2.5 + 50 \times 1.5 + 75 \times 5 + 10 \times 10 \times 5 + 80 - 40 = -104,3 \times 5 + 645.2$

1166.7kN = 1166,7kN

I.



II.



Ako tražimo rezultantu ona je ista u bilo kojoj točci → $R_P^C = R_P^D = R$, ali momenti su različiti $R_M^C \neq R_M^D$.
Zaključuje se da djelovanjem rezultante u povoljnoj točci možemo anulirati djelovanje momenta.

<u>Rezultirajuće djelovanje R</u>

$$\sum_{zad} M_{(T)} = \sum_{traž} M_{(T)} = 0$$

Rezultantu moramo pomaknuti u lijevo (translacijom nastaje negativan moment) tako da poništimo rezultirajući moment koji je pozitivan.
a)

$-R_{Py} \cdot x + R_M = 0$

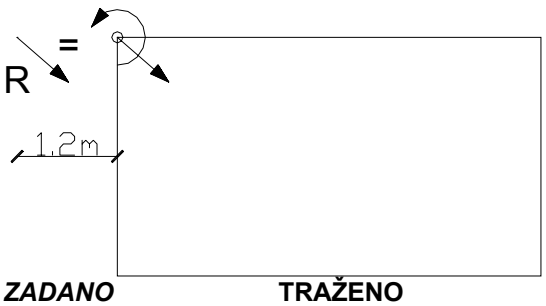
$-104,3 \cdot x + 123.75 = 0$ ➔ **<u>x = 1.2m</u>**

b)

$k_D = R_y \times x$ ➔ $x = \frac{k_D}{R_Y} = \frac{123.75}{104,3}$ **= <u>1.2m</u>**

(paziti na smjerove)

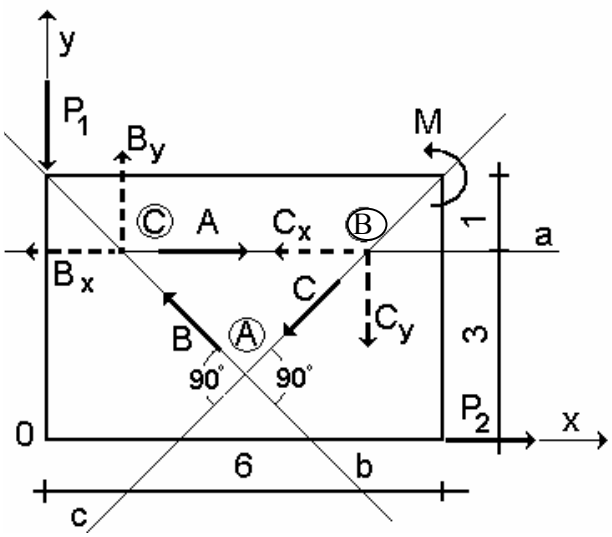
Kontrola :
$P_{1x} \times 2.5 + P_{1y} \times 11.2 + P_3 \times 1.5 - P_2 \times (5+1.2) - q \times 10 \times (5+1.2) + K_1 - K_2$
$70.7 \times 2.5 + 70.7 \times 11.2 + 50 \times 1.5 - 75 \times 6.2 - 10 \times 10 \times 6.2 + 80 - 40$ ➔ **0**



ZADANO	**TRAŽENO**	
sila (R^P)	*sila (**R**)*	*za silu je svejedno gdje se nalazi*
+ ➔	➔	
moment	***0** = moment – sila $\times$ krak*	*novim položajem sile (**R**) anuliramo moment*

Zadatak 9: (Ritterova metoda)

(ne preskačite ovaj zadatak, trebat će vam bezbroj puta na ispitu ili na nekom stručnom predmetu)

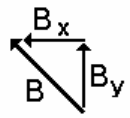


Na pravcima a , b i c odredite sile A , B i C tako da zajedno sa zadanim djelovanjem budu u ravnoteži.

$P_1 = 70$ kN
$P_2 = 80$ kN
M = 200 kNm

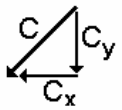
Rješenje:

1. **pretpostavimo** smjerove sila A , B i C
2. **projiciramo** kose sile



$B_x = B \cdot \cos\alpha = 0{,}707\ B$

$B_y = B \cdot \sin\alpha = 0{,}707\ B$



$C_x = C \cdot \cos\alpha = 0{,}707\ C$

$C_y = C \cdot \sin\alpha = 0{,}707\ C$

3. **jednadžbe ravnoteže**

Σ x = 0 $P_2 - B_x - C_x + A = 0 \Rightarrow$ **3 nepoznate** sile

Σ y = 0 $-P_1 + B_y - C_y = 0 \quad \Rightarrow$ **2 nepoznate** sile

Tražimo točku u kojoj ćemo računati sumu momenata i imati **samo 1 nepoznanicu**.

Odabiremo točke (A) , (B) i (C).

$\sum M_{(A)} = 0 \Rightarrow A$
$\sum M_{(B)} = 0 \Rightarrow B$ **metoda triju točaka - Ritterova metoda**
$\sum M_{©} = 0 \Rightarrow C$

$x_{(A)} = 3$ m $\quad y_{(A)} = 1$ m $\quad$ **$\sum M_{(A)} = 0$**

$P_1 \cdot 3 + P_2 \cdot 1 - A \cdot 2 + M = 0 \Rightarrow$ **A = 245 kN**

$x_{(B)} = 5$ m $\quad y_{(B)} = 3$ m $\quad$ **$\sum M_{(B)} = 0$**

$P_1 \cdot 5 + P_2 \cdot 3 + M - B_y \cdot 4 = 0 \Rightarrow B_y = 197{,}5$ kN

B $= \dfrac{B_y}{\cos\alpha} =$ **279,31 kN** $\quad \Rightarrow B_x = 197{,}5$ kN

$x_{©} = 1$ m $\quad y_{©} = 3$ m $\quad$ **$\sum M_{©} = 0$**

$P_1 \cdot 1 + P_2 \cdot 3 + M - C_y \cdot 4 = 0 \Rightarrow C_y = 127{,}5$ kN

C $= \dfrac{C_y}{\cos\alpha} =$ **180 kN** $\quad \Rightarrow C_x = 127{,}5$ kN

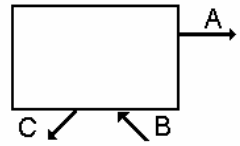
KONTROLA ! $\Sigma x = 0$ $A - B_x - C_x + P_2 = 0$

$245 - 197{,}5 - 127.5 + 80 = 0 \Rightarrow$ zadovoljava

$\Sigma y = 0$ $-P_1 + B_y - C_y = 0$

$- 70 + 197{,}5 - 127{,}5 = 0 \Rightarrow$ zadovoljava

Skica stvarnog djelovanja

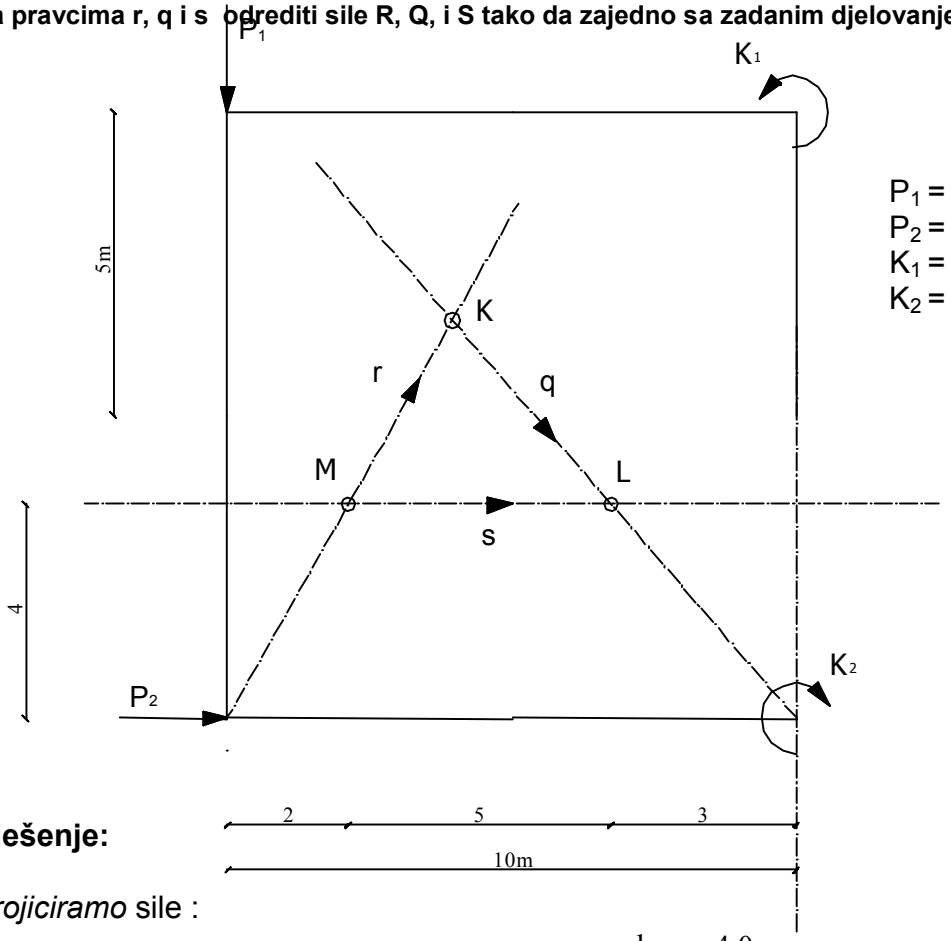


A	245 kN
B	279,3 kN
C	180 kN

Zadatak 10: (Ritterova metoda)

(ne preskačite niti ovaj zadatak, trebat će vam bezbroj puta na ispitu ili na nekom stručnom predmetu)

Na pravcima r, q i s odrediti sile R, Q, i S tako da zajedno sa zadanim djelovanjem budu u ravnoteži.

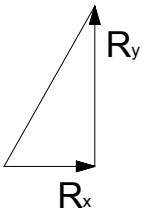


$P_1 = 100$ kN
$P_2 = 200$ kN
$K_1 = 300$ kNm
$K_2 = 400$ kNm

Rješenje:

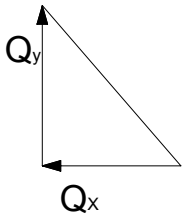
Projiciramo sile :

$$\operatorname{tg}\alpha_R = \frac{b_R}{a_R} = \frac{4.0}{2.0} \Rightarrow \alpha_R = 63.4°$$



$$R_x = R \times \cos\alpha_R = R \times \cos 63.4° = 0.44 \times R$$
$$R_y = R \times \sin\alpha_R = R \times \sin 63.4° = 0.89 \times R$$

$$\operatorname{tg}\alpha_Q = \frac{b_Q}{a_Q} = \frac{4.0}{3.0} \Rightarrow \alpha_Q = 53.1°$$



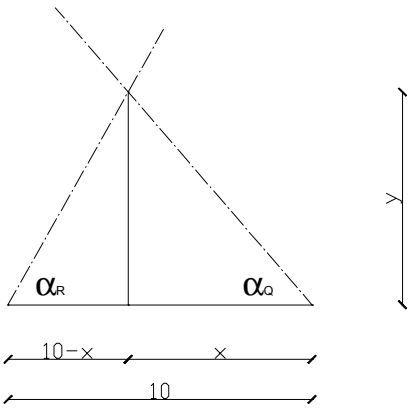
$$Q_x = Q \times \cos\alpha_Q = Q \times \cos 53.1° = 0.6 \times Q$$
$$Q_y = Q \times \sin\alpha_Q = Q \times \sin 53.1° = 0.8 \times Q$$

(pozitivni smjerovi kao kod koordinatnih osi, a pretpostavljeni pozitivni moment suprotno od kazaljke na satu)

Jednadžbe ravnoteže :

ako provjerimo $\sum x = 0$, $\sum y = 0$ i $\sum M_{(T)} = 0$ primijetit ćemo da smo dobili 3 jednadžbe sa tri nepoznanice. Taj slučaj pokušavamo izbjeći birajući točke da pri svakoj jednadžbi imamo smo jednu nepoznanicu.

$\sum M_{(K)} = 0$ ➔ S



$\alpha_Q \times x = \alpha_R \times (10 - x)$

$1.33 \times x = 2 \times (10 - x)$

x = 6,0m

y = $tg\alpha_Q \times x$ = 8m

$P_1 \times 4 + P_2 \times 8 + K_1 - K_2 + S \times 4 = 0$

$100 \times 4 + 200 \times 8 + 300 - 400 + S \times 4 = 0$

S = -475 kN

$\sum M_{(L)} = 0$ ➔ R

$P_1 \times 7 + P_2 \times 4 + K_1 - K_2 - R_{1y} \times 5 = 0$

$100 \times 7 + 200 \times 4 + 300 - 400 - R_{1y} \times 5 = 0$

R_{1y} = 280.0 kN

R = $\frac{280}{0.89} =$ **314.1 kN**

R_{1x} = $314.1 \times 0.44 =$ **140 kN**

$\sum M_{(M)} = 0$ ➔ Q

$P_1 \times 2 + P_2 \times 4 + K_1 - K_2 - Q_{1y} \times 5 = 0$

$100 \times 2 + 200 \times 4 + 300 - 400 - Q_{1y} \times 5 = 0$

Q_{1y} = 180 kN

Q_{1x} = $\frac{180.0}{tg\alpha} = \frac{180}{1.33} =$ **135.4 kN**

Q = $\sqrt{135{,}4^2 + 180^2}$ **= 225.0 kN**

Kontrola :

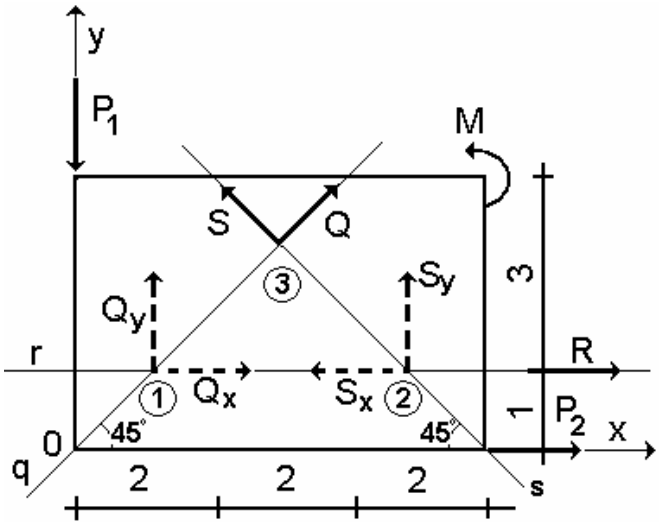
$\sum x = 0$ $P_2 + R_{1x} + Q_{1x} + S = 200 + 140 + 135.4 - 475 =$ **0**

$\sum y = 0$ $-P_1 - Q_{1y} + R_{1y} = -100 - 180 + 280 =$ **0**

Zadatak 11: (Ritterova metoda)

(riješite i ovaj, a nemojte ga samo gledati ☺)

Na pravcima r , g i s odredite sile R , Q i S tako da sa zadanim djelovanjem budu u ravnoteži.

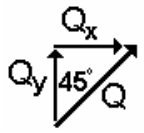


$P_1 = 70$ kN
$P_2 = 80$ kN
M = 200 kNm

Rješenje:

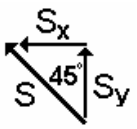
1. **Pretpostavimo** smjerove sila R , Q i S

2. **Projiciramo** sile



$Q_x = Q \cdot 0{,}707$ kN
$Q_y = Q \cdot 0{,}707$ kN

$S_x = S \cdot \cos\alpha = 0{,}707\ S$
$S_y = S \cdot \sin\alpha = 0{,}707\ S$



3. **Jednadžbe ravnoteže**

Σ x = 0

$R + P_2 + Q_x - S_x = 0$ **3 nepoznanice**

Σ y = 0

$-P_1 + S_y + Q_y = 0$ **2 nepoznanice** $\Rightarrow$ tražimo točku u kojoj ćemo napisati sumu momenata $\Sigma M_T = 0$ i imati samo 1 nepoznanicu

$\Sigma M_{(1)} = 0 \Rightarrow S$
$\Sigma M_{(2)} = 0 \Rightarrow Q$
$\Sigma M_{(3)} = 0 \Rightarrow R$

metoda Rittera (metoda triju točaka)

$x_{(1)} = 1$ m $y_{(1)} = 1$ m **$\Sigma M_{(1)} = 0$**

$P_1 \cdot 1 + P_2 \cdot 1 + S_y \cdot 4 + M = 0$
$70 + 80 + 4 \cdot 0{,}7071 + 200 \cdot S \Rightarrow$ **S = -123,71 kN**
$S_x = S \cdot \cos\alpha = -87{,}5$ kN $S_y = S \cdot \sin\alpha = -87{,}5$ kN

$x_{(2)} = 5$ m $y_{(2)} = 1$ m **$\Sigma M_{(2)} = 0$**

$P_1 \cdot 5 + P_2 \cdot 1 + M - Q_y \cdot 4 = 0$
$70 \cdot 5 + 80 \cdot 1 + 200 - 0{,}707 \cdot Q \cdot 4 = 0 \Rightarrow$ **Q = 222,74 kN**
$Q_x = Q \cdot \cos\alpha = 157{,}5$ kN $Q_y = Q \cdot \sin\alpha = 157{,}5$ kN

$x_{(3)} = 3$ m $y_{(3)} = 3$ m **$\Sigma M_{(3)} = 0$**

$P_1 \cdot 3 + P_2 \cdot 3 + M + R \cdot 2 = 0 \Rightarrow$ **R = -325 kN**

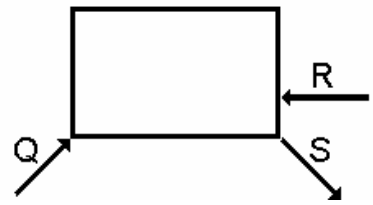
KONTROLA ! $\Sigma x = 0$ $\quad P_2 + R + Q_x - S_x = 0$

$80 - 325 + 157{,}50 + 87{,}50 = 0 \quad \Rightarrow$ zadovoljava

$\Sigma y = 0$ $\quad -P_1 + Q_y + S_y = 0$

$- 70 + 157{,}5 - 87{,}5 = 0 \quad \Rightarrow$ zadovoljava

Skica stvarnog djelovanja

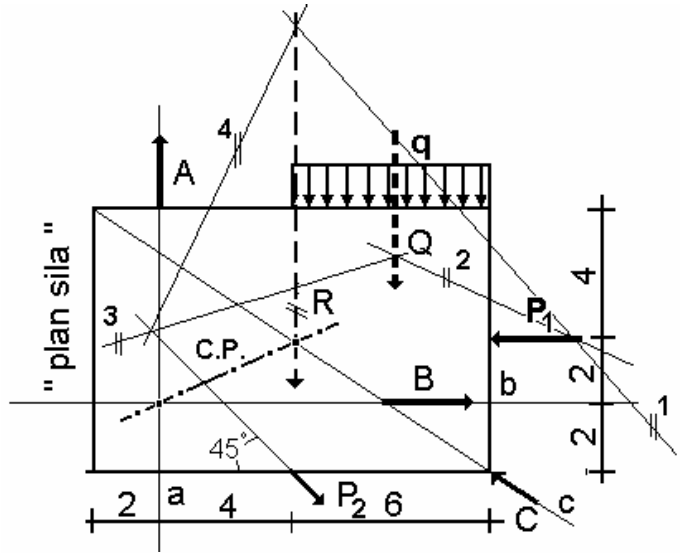


R	325 kN
Q	222,7 kN
S	123,7 kN

Zadatak 12: (Culmannova metoda)

(zadaci koji ne dolaze često, ali kad se pojave, zbog preskakanja istih kod učenja, izazovu nepovoljne reakcije ☺)

Na pravcima a , b i c odredite sile A , B i C tako da sustav bude u ravnoteži . Zadatak riješiti grafičkim postupkom



g = 15 kN/m $\Rightarrow$ Q = 6 $\times$ 15 = 90 kN

P_1 = 60 kN

P_2 = 90 kN

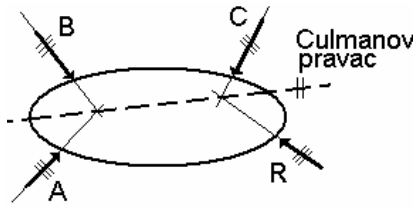
Rješenje:

$$\underbrace{\vec{P}_1 + \vec{P}_2 + \vec{Q}} + \vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{R} \qquad + \vec{A} + \vec{B} + \vec{C} = 0$$

(treba uravnotežiti 1 poznatu silu $\vec{R}$ i 3 nepoznate $\vec{A}, \vec{B}$ i $\vec{C}$)

****Culmannova metoda je grafička metoda koja uravnotežuje 4 sile u ravnini***

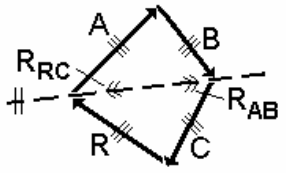


· prvo nađemo rezultantu vanjskog djelovanja $\vec{R} = \vec{P}_1 + \vec{P}_2 + \vec{Q}$ i njezin položaj na tijelu

· dobili smo 4 sile i tražimo gdje se sijeku 2 i 2 sile

$\underbrace{\vec{A} + \vec{B}}$ **+** $\underbrace{\vec{C} + \vec{R}}$ **= 0**

$\vec{R}_{AB}$ $\vec{R}_{RC}$ $\Rightarrow$ dobili smo dvije *nove* sile koje će biti u ravnoteži samo onda ako se nalaze na istom pravcu



(**Culmannovom pravcu**) , ako su istog intenziteta, a suprotnog smjera.

▪ rezultantu R nađemo pomoću poligona sila

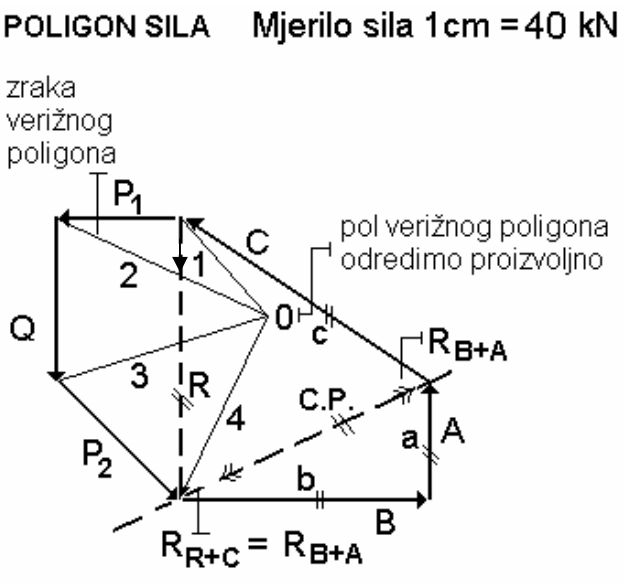
$\vec{P}_1 \rightarrow \vec{Q} \rightarrow \vec{P}_2$

▪ položaj R nalazimo pomoću verižnog poligona

$\vec{P}_1 + \vec{Q} + \vec{P}_2 = \vec{R}$

$\underbrace{\vec{1} + \vec{2}} + \underbrace{(-\vec{2} + \vec{3})} + \underbrace{(-\vec{3} + \vec{4})} = \underbrace{\vec{1} + \vec{4}}$

$\vec{P}_1$ $\vec{Q}$ $\vec{P}_2$ $\vec{R}$



Zrake verižnog poligona su djelomične rezultante

na plan sila "prenesemo" zrake verižnog poligona .

Ako spojimo prvu i zadnju zraku (1 i 4) dobijemo točku kroz koju prolazi R

▪ odredim točke Culmannovog pravca (sjecište a i b te R i c) i povučemo pravac

▪ "vratimo" se u poligon sila i radimo zatvoreni poligon

$$\vec{P}_1 + \vec{P}_2 + \vec{Q} + \vec{A} + \vec{B} + \vec{C} = 0$$

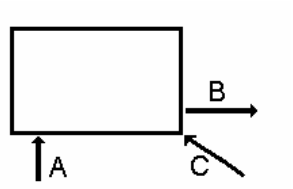
OČITAMO: R = 3,7 cm $\Rightarrow$ 148 kN

A = 1,5 cm $\Rightarrow$ 60 kN

B = 3,3 cm $\Rightarrow$ 132 kN

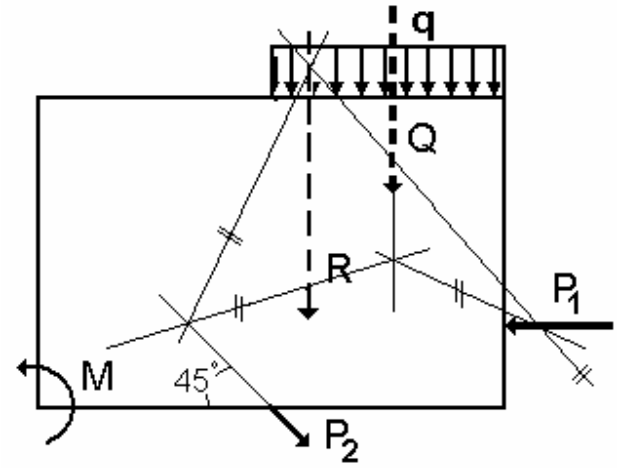
C = 4,1 cm $\Rightarrow$ 164 kN

Skica stvarnog djelovanja



Zadatak 13: (Culmannova metoda)
(još malo nezgodniji od prethodnog zadatka)

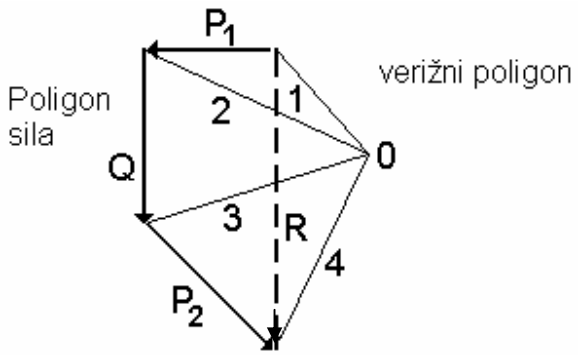
Isto opterećenje kao u prethodnom zadatku samo je **dodan još koncentrirani moment M.**



Q = 15 kN/m ⇒ Q = 90 kN
P_1 = 60 kN
P_2 = 90 kN
M = 330 kNm

Rješenje: prvo nađemo R pomoću poligona sila

▪ Poligon sila



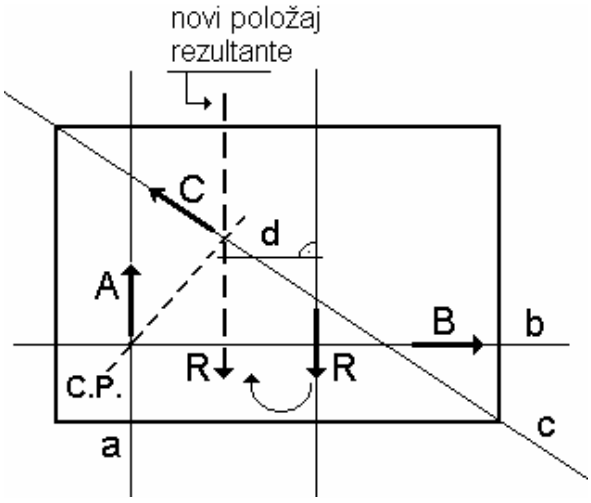
Mj. Sila 1cm : 37 kN

Očitamo : R = 4 cm ⇒ 148 kN

▪ nakon toga moramo translatirati R tako da poništimo moment M .(***redukcija sile na točku***)

M = R · d

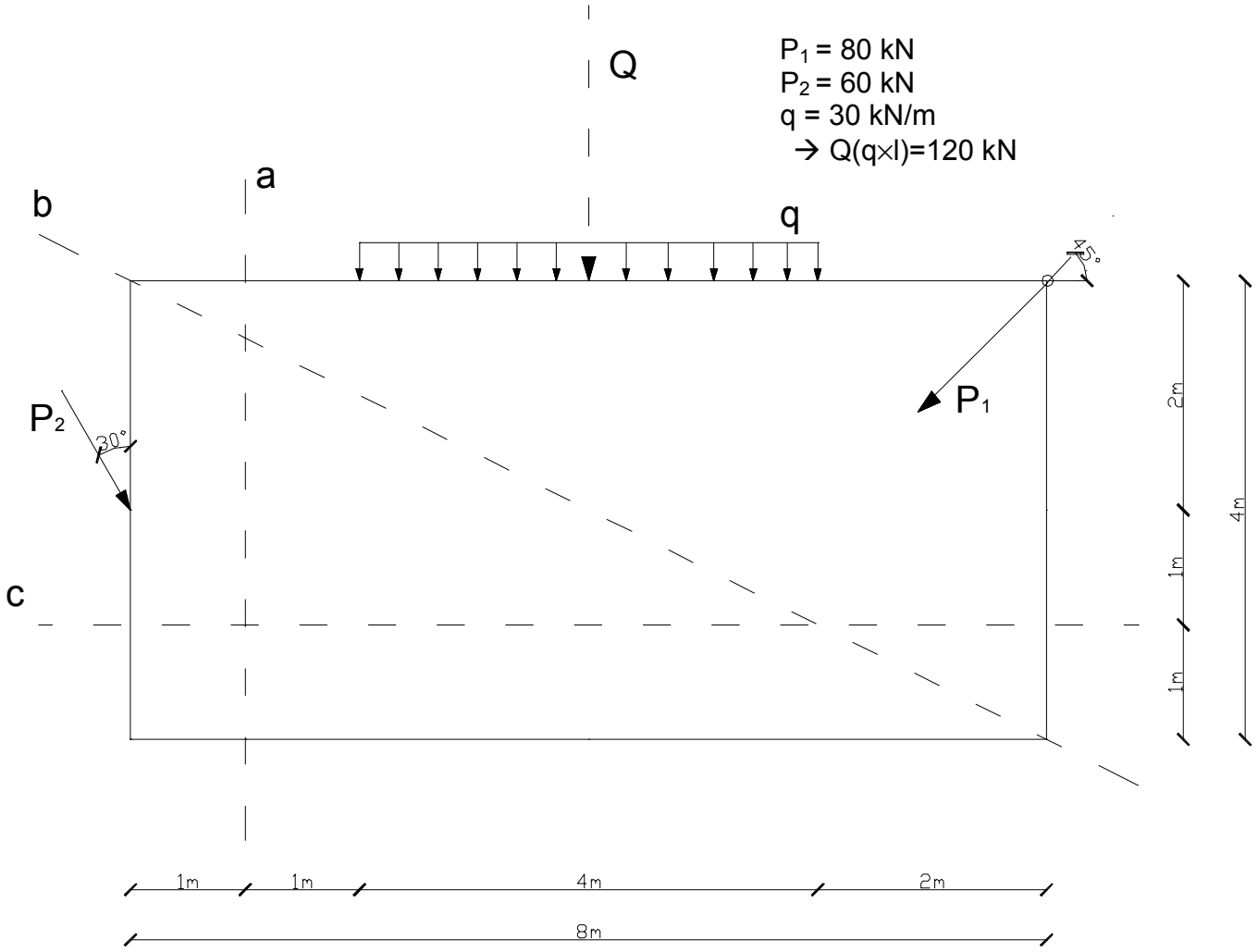
$d = \frac{M}{R} = \frac{330}{148}$ = 2,30 m



▪ kad smo našli novi položaj R radimo Culmannov postupak kao u prethodnom zadatku $\vec{R} + \vec{A} + \vec{B} + \vec{C} = 0$

Zadatak 14: (Culmannova metoda)

Na pravcima a, b, c odrediti koncentrirane sile A, B i C, tako da sustav bude u ravnoteži. Zadatak riješiti grafičkim postupkom.



Tražimo zadovoljenje uvjeta ravnoteže :

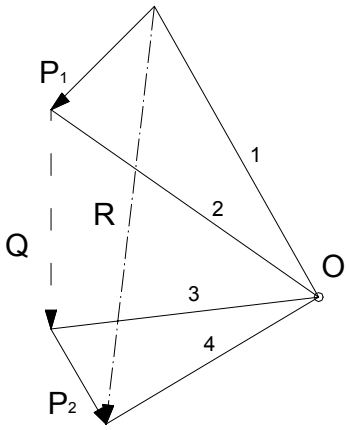
$$\vec{P}_1 + \vec{P}_2 + \vec{Q} + \vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{R} + \vec{A} + \vec{B} + \vec{C} = 0$$

Coulmannova metoda (grafičko uravnoteženje 4 sile u ravnini.)

1.) tražimo rezultantu vanjskog djelovanja : $\vec{R} = \vec{P}_1 + \vec{P}_2 + \vec{Q}$

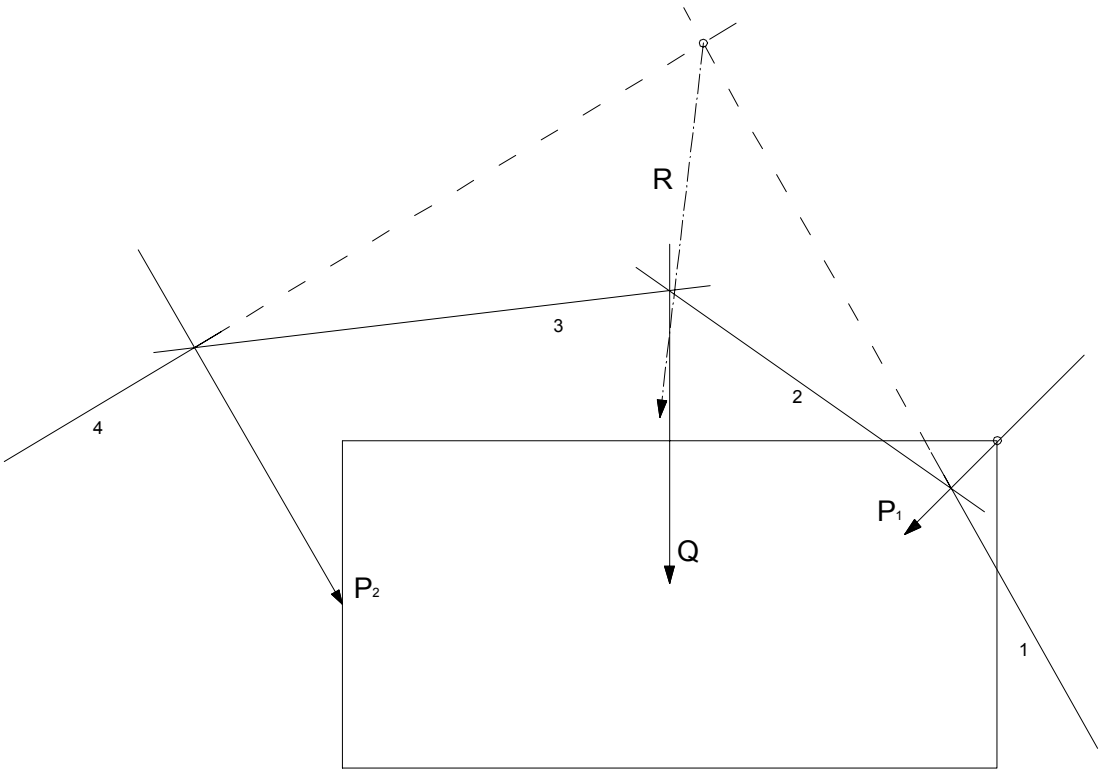
mjerilo SILA izaberemo **1 cm :: 40 kN**



$$\vec{R} = \vec{P}_1 + \vec{Q} + \vec{P}_2$$
$$(\vec{1}+\vec{4}) = (\vec{1}+\vec{2}) + (\vec{2}+\vec{3}) + (\vec{3}+\vec{4})$$

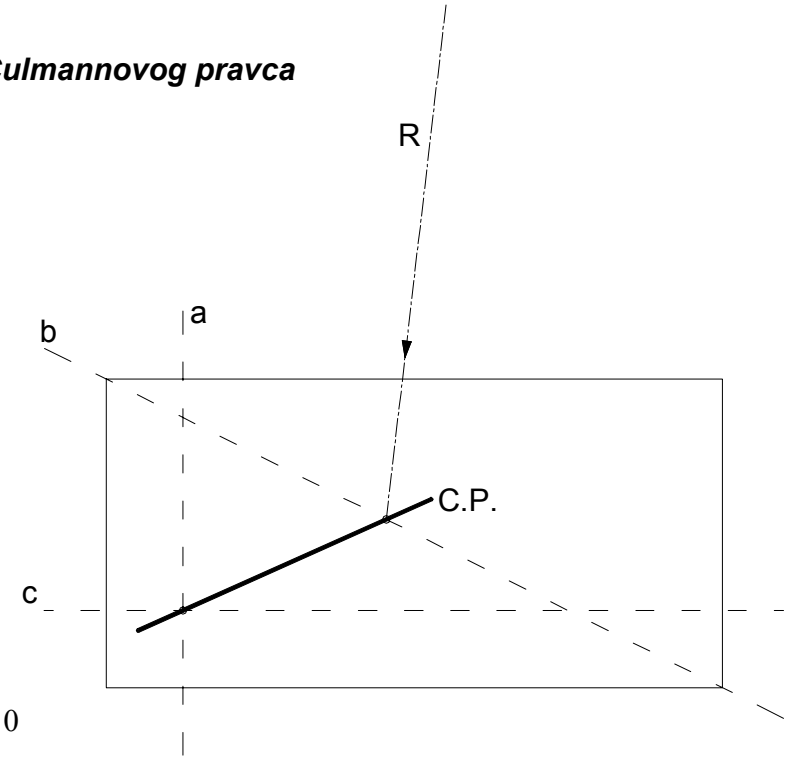
- pol O odabiremo proizvoljno
- zrake verižnog poligona su djelomične rezultante
- svaka zadana sila je projicirana na dvije "zrake"

2.) tražimo položaj rezultante vanjskog djelovanja



- iz plana sila dobivene zrake nanosimo (paralelno) na plan položaja
- između dvije zrake imamo istu silu kao i na planu sila
- spojimo prvu i zadnji pravac dobijemo točku kroz koju prolazi Rezultanta

3.) konstrukcija Culmannovog pravca

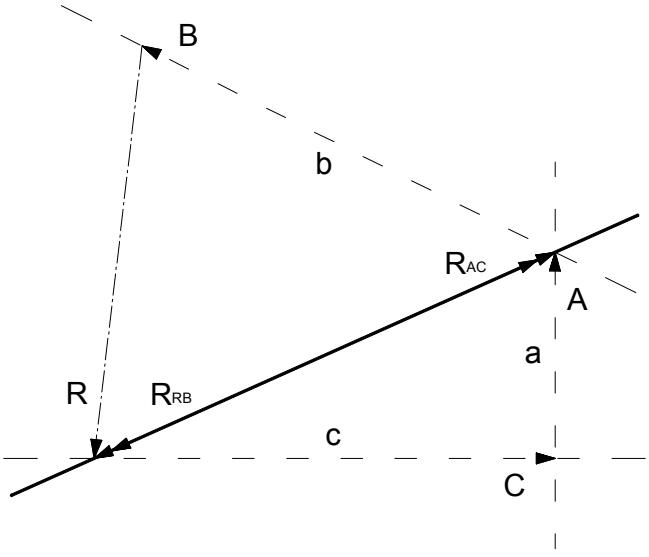


- $\underbrace{\vec{R}+\vec{A}}_{\vec{R}_{RB}}+\underbrace{\vec{B}+\vec{C}}_{\vec{R}_{AC}}=0$

➔ **dvije sile su u ravnoteži ako leže na istom pravcu, iste su veličine, a suprotnog smjera**

- točka sjecišta pravca sile R i pravca b, spajamo s točkom sjecišta pravaca a i c, te dvije točke definiraju Culmannov pravac

4.) očitavanje traženih veličina



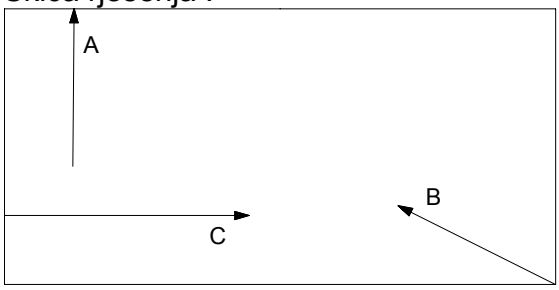
- rezultanti R poznajemo smjer i veličinu te smjerove pravaca a, b i c
- rezultanta R zajedno sa pravcem b i daje rezultantu sila R i B
- okrenemo smjer rezultante i nanosimo iz vrhova pravce a i c

OČITAMO :

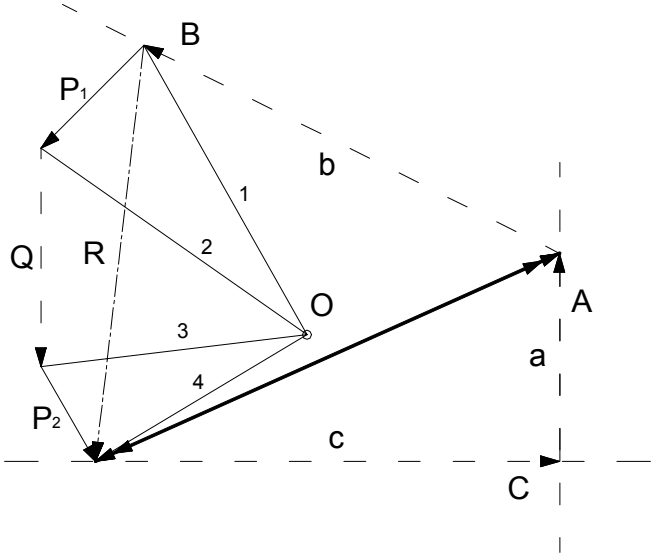
R = 5.7 cm = 230,0 kN
A = 2.9 cm = 256,0 kN
B = 6.4 cm = 255,0 kN
C = 6.4 cm = 120,0 kN

- pogledati zatvoren poligon sila

Skica rješenja :

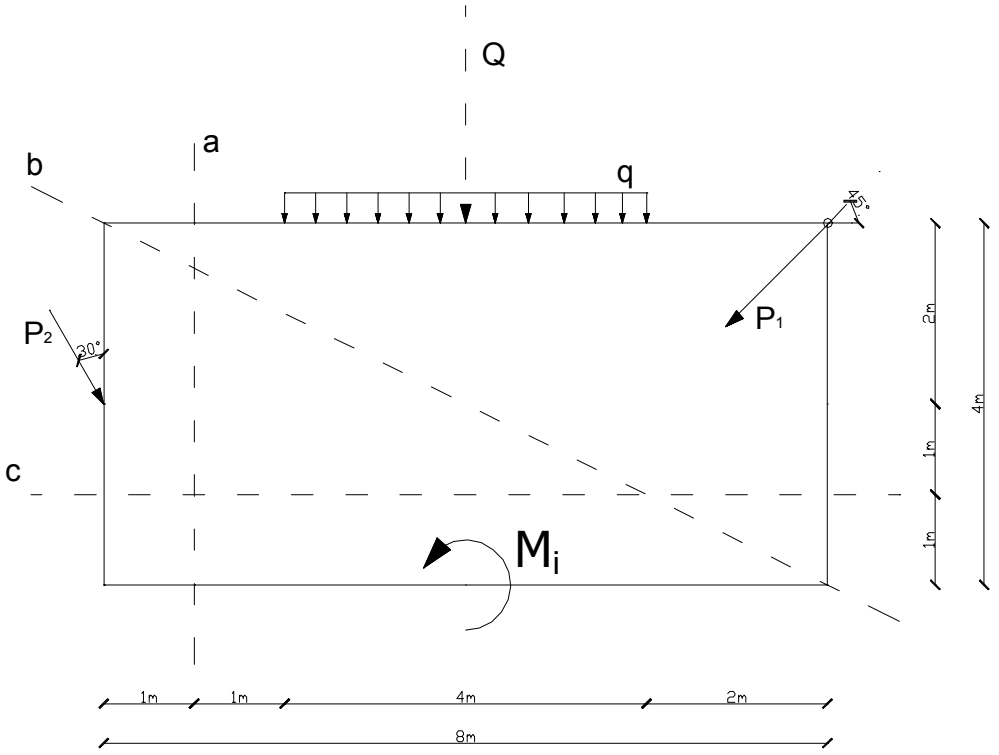


Nije potrebno raditi pojedinačno svaku sliku, već se sve može napraviti na jednoj.



Zadatak 15: (Culmannova metoda)

Isto opterećenje kao u prethodnom zadatku samo imamo **koncentrirani moment M** koji djeluje na sustav :

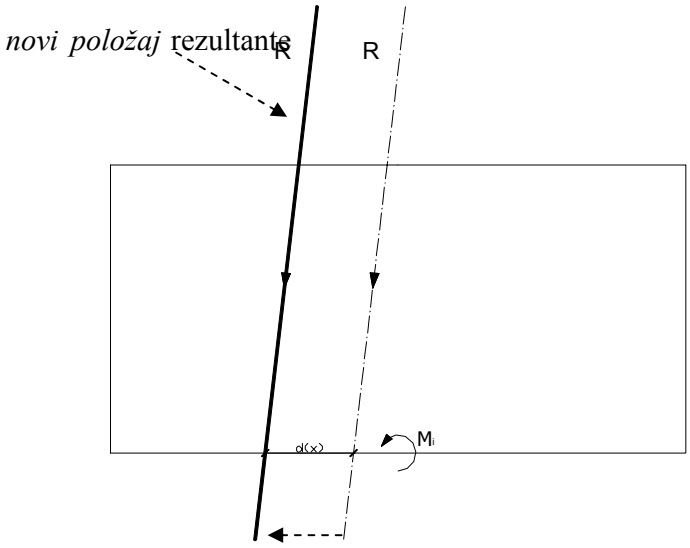


Rješenje:

- postupak je isti samo moramo u položaj rezultante unesti i promjenu od momenta.

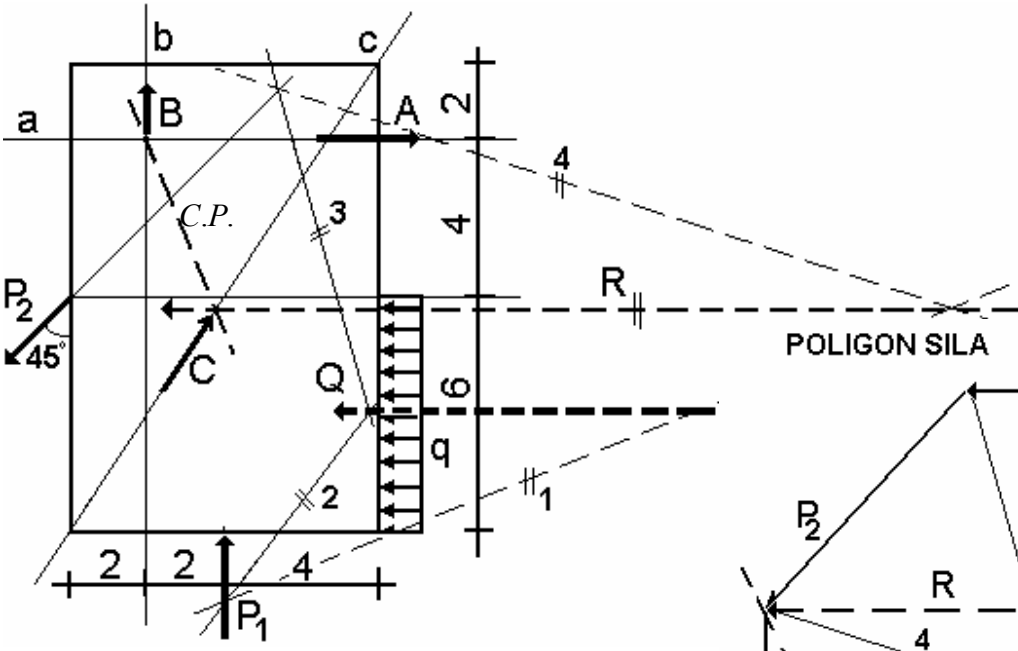
$M = R \times d \rightarrow \quad d = \frac{M}{R}$ ili $M = R_y \times \mathbf{x} \rightarrow \quad x = \frac{M}{R_y}$

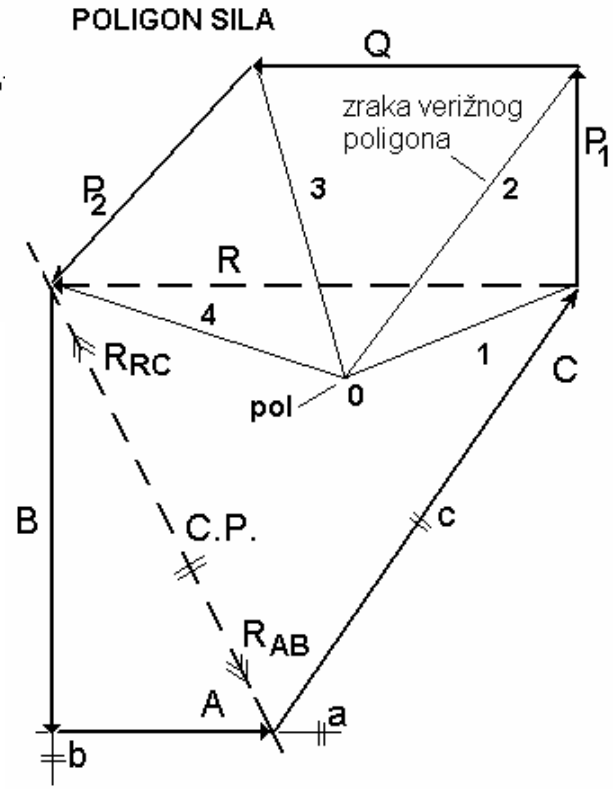
- rezultanta se pomakne za vrijednost d ili x



Zadatak 16 : (Culmannova metoda)

Na pravcima a , b i c odredite sile A , B i C tako da sustav bude u ravnoteži. Zadatak riješiti grafičkim postupkom





g = 15 kN/m => Q = 15 × 6 = 90 kN
P_1 = 60 kN
P_2 = 90k N

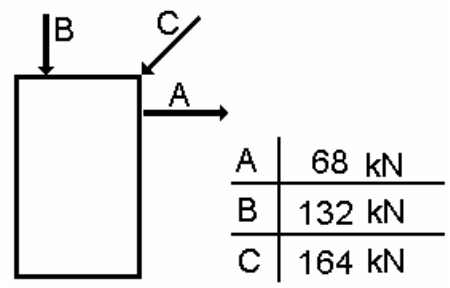
Rješenje:

1. Nađemo rezultantu R
2. Položaj rezultante R na tijelu
3. Culmannov pravac

Očitamo :

R = 7,7 cm ⇒ 154 kN
A = 3,4 cm ⇒ 68 kN
B = 6,6 cm ⇒ -132 kN
C = 8,2 cm ⇒ 164 kN

Skica stvarnog djelovanja

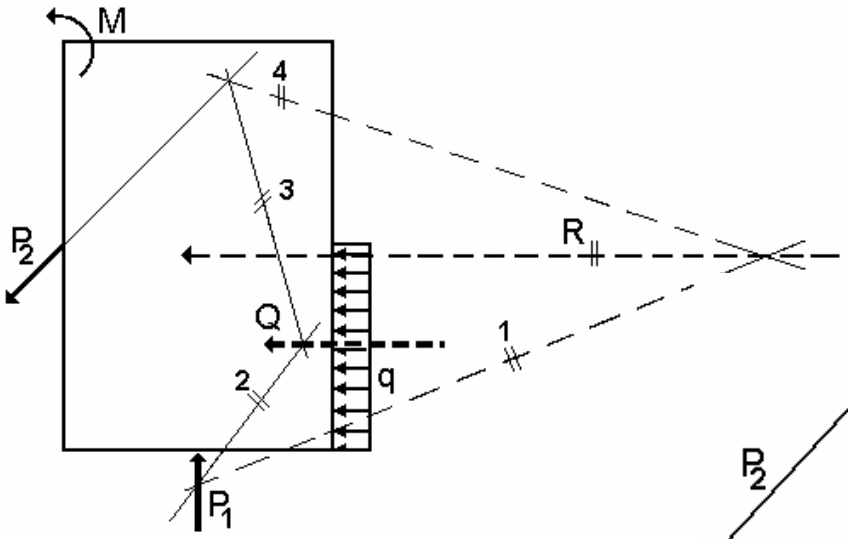


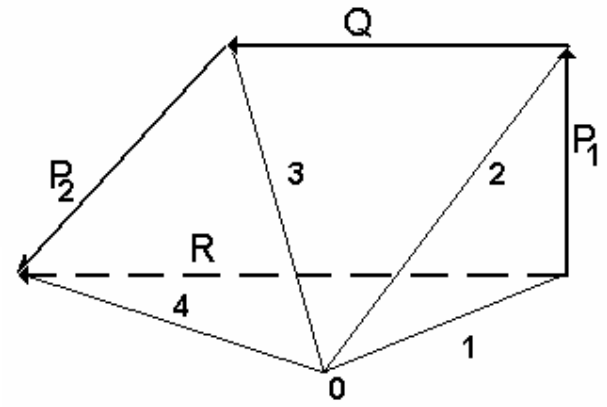
A	68 kN
B	132 kN
C	164 kN

Zadatak 17: (Culmannova metoda)

Isto opterećenje kao u prethodnom zadatku samo je dodan još **koncentrirani moment M.**

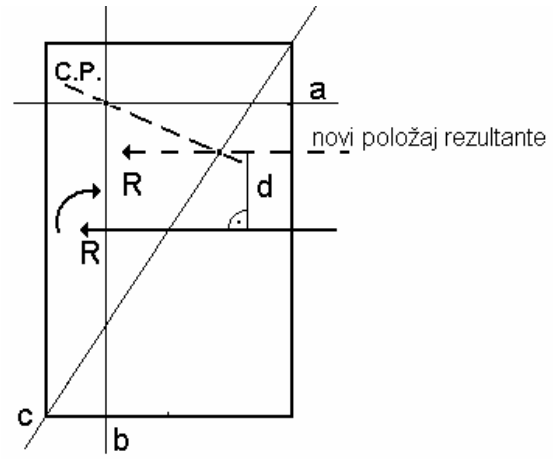
g =15 kN/m => Q = 15 $\times$ 6 = 90 kN
P_1 = 60 kN
P_2 = 90 kN
M = 330 kNm





Rješenje:▪ Prvo nađemo R pomoću poligona sila

Mj. sila 1 cm = 20 kN



▪ Nakon što smo našli R trebamo ju translatirati tako da poništimo moment M = 330 kNm .*(translacija prema gore da dobijemo negativan moment)*

(redukcija sile na točku)

M = R · d

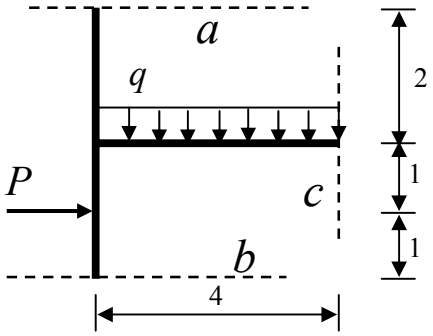
$d = \frac{M}{R}$ = $\frac{330}{148}$ = 2,30 m

▪ Kada smo našli novi položaj R radimo Culmannov postupak kao u prethodnom zadatku .

$\vec{R} + \vec{A} + \vec{B} + \vec{C}$ = 0

ZADATAK 18:

Grafički i računski naći rezultantu R te sile A, B i C koje se nalaze na pravcima a, b i c i sa zadanim opterećenjem čine ravnotežu .

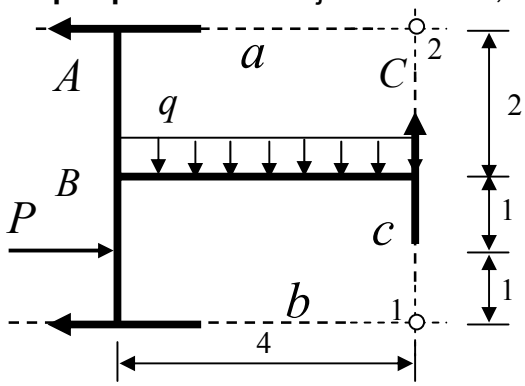


g = 25 kN/m $\Rightarrow$ Q = 4 $\times$ 25 = 100 kN
P = 120 kN

1.) Analitičko rješenje

▪ **Rezultanta R** $R_x = P = 120$ kN
$R_y = -q \cdot 4 = -100$ kN
R = $\sqrt{120^2 + 100^2}$ = 156,2 kN

▪ **pretpostavimo** smjerove sila A , B i C



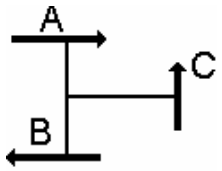
$\sum$ **M$_{(1)}$ = 0**
- P · 1 + A · 4 + Q · 2 = 0 $\Rightarrow$ A = - 20 kN

$\sum$ **M$_{(2)}$ = 0**
P · 3 - B · 4 + Q · 2 = 0 $\Rightarrow$ **B = 140 kN**

$\sum$ **y = 0**
C – Q = 0 $\Rightarrow$ **C = 100 kN**

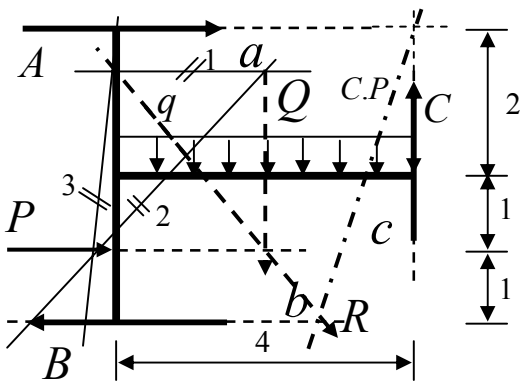
KONTROLA ! $\sum$ x = 0 - A – B + P = 0
-(-20) – 140 +120 = 0

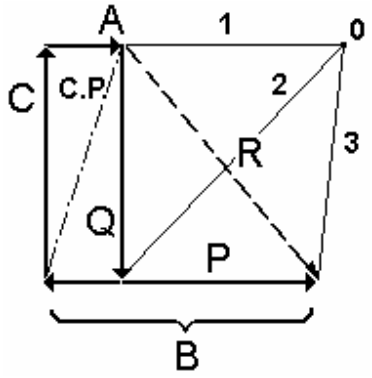
Skica stvarnog djelovanja:



A	20 kN
B	140 kN
C	100 kN

2.) Grafičko rješenje Mj. sila 1cm :: 40 kN



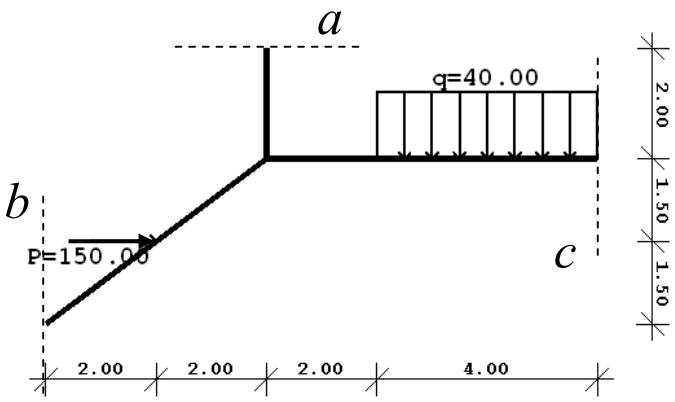


Očitano :

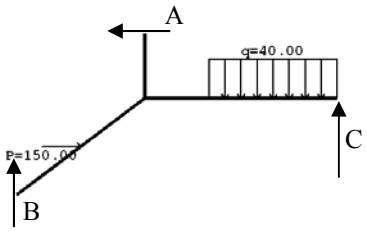
R = 3,9 cm $\Rightarrow$ **156 kN**
A = 0,5 cm $\Rightarrow$ **-20 kN**
B = 3,5 cm $\Rightarrow$ **140 kN**
C = 2,5 cm $\Rightarrow$ **100 kN**

ZADATAK 19:

Za zadano opterećenje P i q potrebno je odrediti: rezultantu R te sile A, B i C na pravcima a, b i c koje će sa zadanim opterećenjem (P i q) činiti ravnotežu zadanog sustava! Zadatak riješiti analitičkim i grafičkim postupkom.

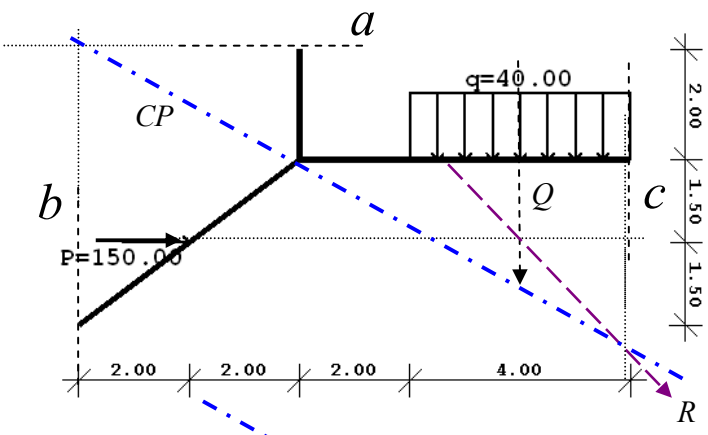


Rješenje:

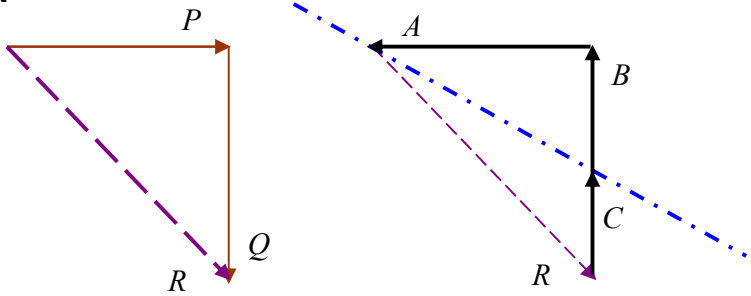


R=219.3 kN;
A=150kN; B=84,5kN; C=75,5kN

Grafičko rješenje



Mj. sila 1cm::50 kN

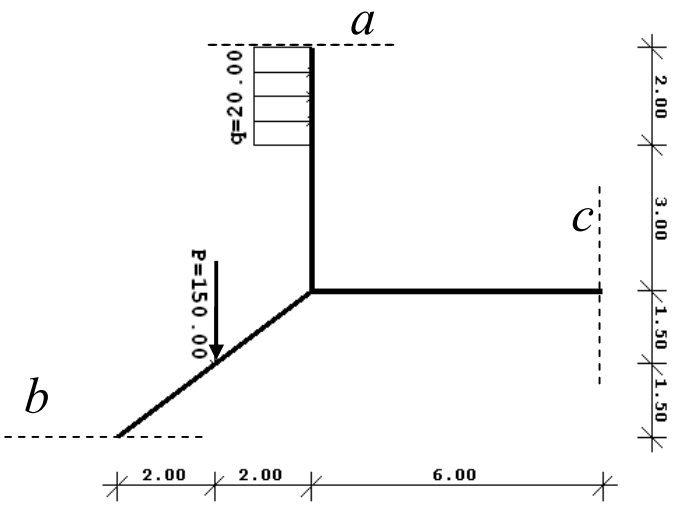


Očitano:

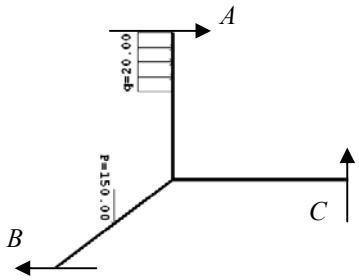
R= 4,4 cm ⇒R= 220 kN; A=3 cm ⇒150 kN; B=1,7 cm ⇒85 kN; C=1,5 cm ⇒75kN

ZADATAK 20:

Za zadano opterećenje P i q potrebno je odrediti: rezultantu R te sile A, B i C na pravcima a, b i c koje će sa zadanim opterećenjem (P i q) činiti ravnotežu zadanog sustava! Zadatak riješiti analitičkim i grafičkim postupkom.

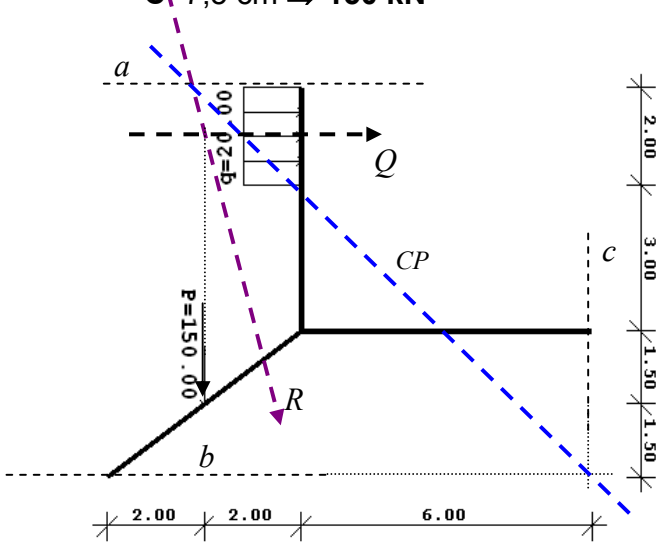


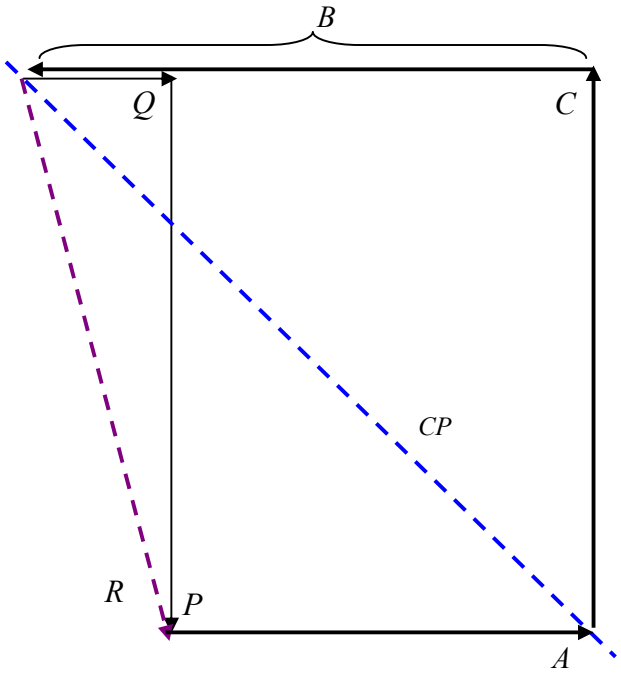
Rješenje:
R=155,2 kN
A=115kN; B=155 kN; C=150 kN



Grafičko rješenje:
Mjerilo sila 1cm ::20kN ⇒ očitano:

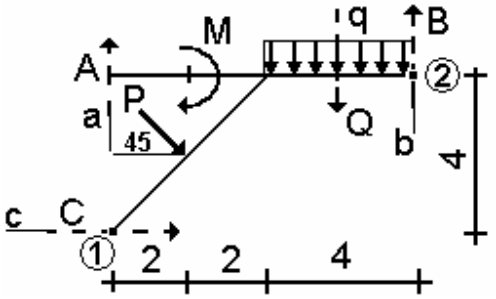
R=7,8 cm ⇒**156 kN**
A=5,8 cm ⇒**116 kN**
B=7,8 cm ⇒**156 kN**
C=7,5 cm ⇒ **150 kN**





Zadatak 21:

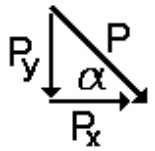
Za zadano opterećenje potrebno je na pravcima a , b i c odrediti sile A, B i C koje će tijelo držati u ravnoteži .



g = 20 kN/m $\Rightarrow$ Q = 20 $\times$ 4 = 80 kN
P_1 = 100 kN
M = 320 kNm

Rješenje:

1. Pretpostavimo smjerove sila A, B i C
2. Projiciramo silu P



$P_x = P \cdot \cos 45° = 70{,}71$ kN
$P_y = P \cdot \sin 45° = 70{,}71$ kN

3. Nađemo rezultantu **R**

$\sum P_x = R_x \Rightarrow R_x = 70{,}71$ kN ; $\sum P_y = R_y \Rightarrow -Q - P_y = R_y \Rightarrow R_y = 150{,}71$ kN

$R = \sqrt{R_x^2 + R_y^2} = 166{,}47$ kN

4. Pišemo jednadžbe ravnoteže $\sum$ **x = 0 ;** $\sum$ **y = 0 ;** $\sum M_{(T)} = 0$

$\Sigma M_{(1)} = 0$

$-P_y \cdot 2 - P_x \cdot 2 - M - Q \cdot 6 + B \cdot 8 = 0 \Rightarrow B = 135{,}355$ kN

$\sum$ y = 0

$C + P_x = 0 \Rightarrow$ **C =** $-P_x$ = **-70,71 kN**

$\sum$ x = 0

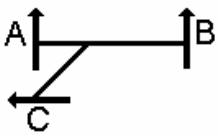
$A + B - P_y - Q = 0 \Rightarrow$ **A = 15,355 kN**

KONTROLA!

$\Sigma M_{(2)} = 0$

$-A \cdot 8 - M + C \cdot 4 + P_x \cdot 2 + P_y \cdot 6 + Q \cdot 2 = 0$

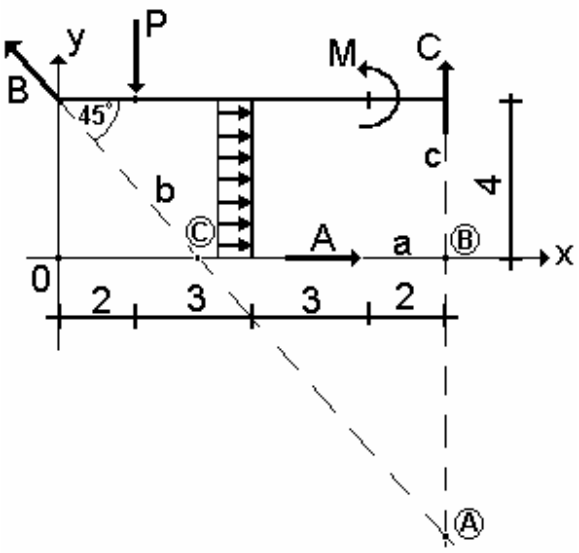
Skica



A	15,355 kN
B	135,355 kN
C	70,71 kN

Zadatak 22:

Za zadano opterećenje potrebno je na pravcima a , b i c odrediti sile A, B i C koje će tijelo držati u ravnoteži .



g = 25 kN/m $\Rightarrow$ Q = 25 $\times$ 4 = 100 kN
P = 100 kN
M = 400 kNm

Rješenje:

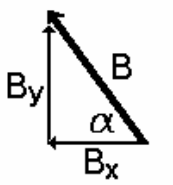
1. Nađemo rezultantu R

R_x = Q = 100 kN
R_y = -P = -100 kN
R = $\sqrt{100^2 + 100^2}$ = 141,4 kN

2. Pretpostavimo smjerove sila A, B i C

3. Projiciramo silu B

B_x = 0,7071 B
B_y = 0,7071 B



4. Pišemo jednadžbe ravnoteže $\sum$ **x = 0 ;** $\sum$ **y = 0 ;** $\sum$ $\mathbf{M_{(T)} = 0}$

$\sum$ **x = 0** Q + A -B_x = 0 $\Rightarrow$ 2 nepoznanice

$\sum$ **y = 0** -P + B_y + C = 0 $\Rightarrow$ 2 nepoznanice

- Tražimo točke u kojima će se pojaviti samo jedna nepoznanica → tražimo Ritterove točke koordinate točaka

A $\Rightarrow$ x = 10 m , y = -6 m
B $\Rightarrow$ x = 10 m , y = 0 m
C $\Rightarrow$ x = 4 m , y = 0 m

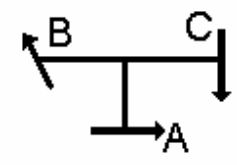
$\sum$ $\mathbf{M_{(A)} = 0}$ P · 8 - A · 6 - Q · (2+6) + M = 0 $\Rightarrow$ **A = 66,67 kN**

$\sum$ $\mathbf{M_{(B)} = 0}$ -B_y · 10 + B_x · 4 - Q · 2 + P · 8 + M = 0 $\Rightarrow$ **B = 235,70 kN**
B_x = B_y = 166,67 kN

$\sum$ $\mathbf{M_{©} = 0}$ P · 2 - Q · 2 + M + C · 6 = 0 $\Rightarrow$ **C = -66,67 kN**

KONTROLA ! $\sum$ x = 0 -B_x + Q + A = 0; -166,67 + 100 + 66,67 = 0
$\sum$ y = 0 B_y - P + C = 0; 166,67 - 100 - 66,67 = 0

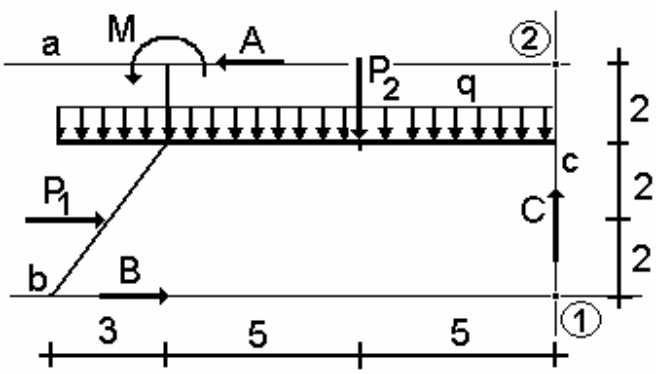
Skica



A	66,67 kN
B	235,70 kN
C	66,67 kN

Zadatak 23:

Zadano tijelo treba uravnotežiti silama A, B i C na pravcima a , b i c.



g = 10 kN/m $\Rightarrow$ Q = 10 $\times$ 13 = 130 kN
P_1 = 50 kN
P_2 = 50 kN
M = 20 kNm

Rješenje:

Jednadžbe ravnoteže :

$\sum$ **y = 0** $\quad C - q \cdot 13 - P_2 = 0 \Rightarrow$ **C = 180 kN**

$\sum$ **$M_{(1)}$ = 0**

$-P_1 \cdot 2 + A \cdot 6 + M + P_2 \cdot 5 + q \cdot 13 \cdot 6{,}5 = 0 \quad \Rightarrow \quad A = -169{,}2$ kN

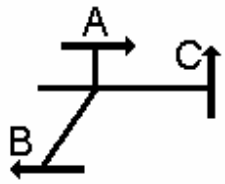
$\sum$ **x = 0** $\quad -A + B + P_1 = 0 \Rightarrow$ **B = -219,2 kN**

KONTROLA !

$\sum$ **$M_{(2)}$ = 0**

$20 + P_1 \cdot 4 + P_2 \cdot 5 + Q \cdot 6{,}5 + B \cdot 6 = 0$ zadovoljava

Skica

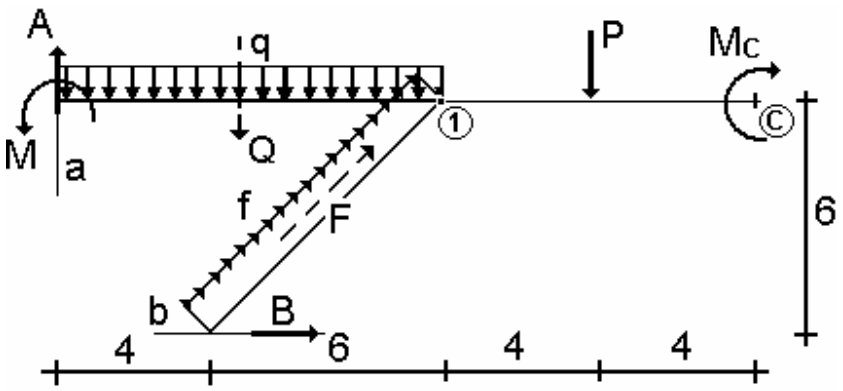


A	169,2 kN
B	219,2 kN
C	180 kN

Zadatak 24:

(*često dolazi na ispitu ili kolokviju, a ako se dobro ne savlada izazove tužne uzdahe*) **!!!!!!!**

Zadano tijelo u ravnini uravnotežiti silama A i B na pravcima a i b te momentom M_C u točki C.

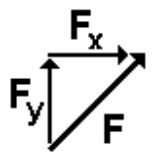


Rješenje:

$f = 20\ kN/m \Rightarrow F = f \cdot l = 20 \cdot 8{,}485 = 169{,}7\ kN$

$F_x = F \cdot \cos\alpha = 169{,}7 \cdot 0{,}707 = 120\ kN$ ili $\mathbf{F_x = f \cdot l_x} = 20 \cdot 6 = 120\ kN$ **!!!!!**

$F_y = F \cdot \sin\alpha = 169{,}7 \cdot 0{,}707 = 120\ kN$ ili $\mathbf{F_y = f \cdot l_y} = 20 \cdot 6 = 120\ kN$ **!!!!!**



$Q = 10 \times 5 = 50\ kN$
$P = 100\ kN$
$M = 200\ kNm$

$\sum \mathbf{x = 0}$ $\quad B + F_x = 0 \Rightarrow \mathbf{B = -120\ kN}$

$\sum \mathbf{y = 0}$ $\quad -Q + F_y - P + A = 0 \Rightarrow \mathbf{A = 30\ kN}$

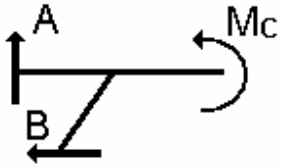
$\sum \mathbf{M_{©} = 0}$ $\quad -M_C + P \cdot 4 + Q \cdot 13 + M + B \cdot 6 - F_y \cdot 8 - A \cdot 18 = 0 \Rightarrow \mathbf{M_C = -970\ kNm}$

KONTROLA !

$\sum \mathbf{M_{(1)} = 0}$

$-A \cdot 10 + M + Q \cdot 5 - P \cdot 4 - M_C + B \cdot 6 = 0$ zadovoljava

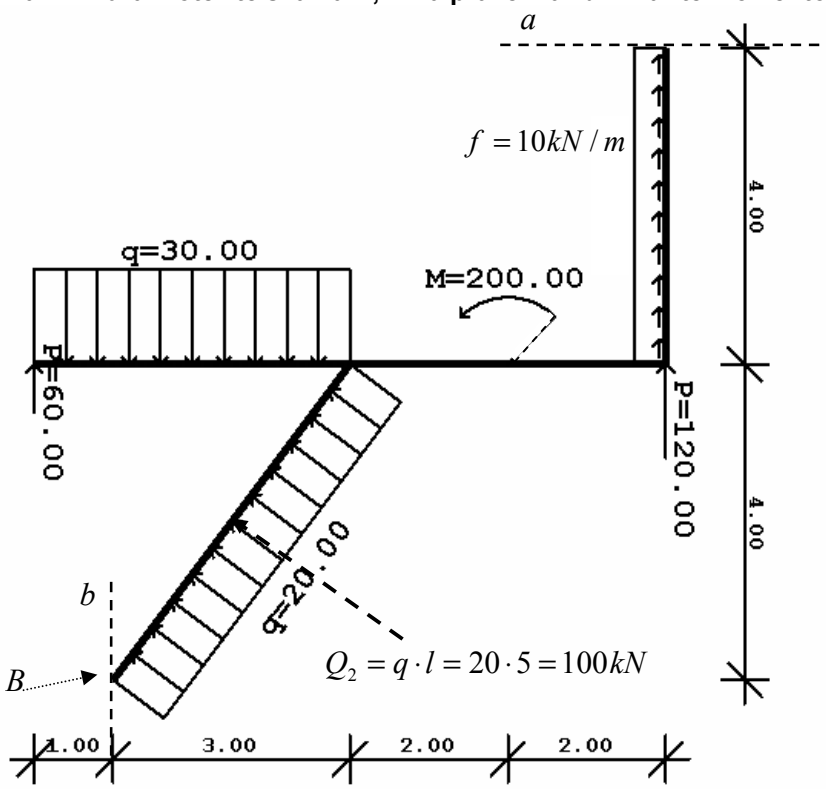
Skica



A	30 kN
B	120 kN
M_C	970 kNm

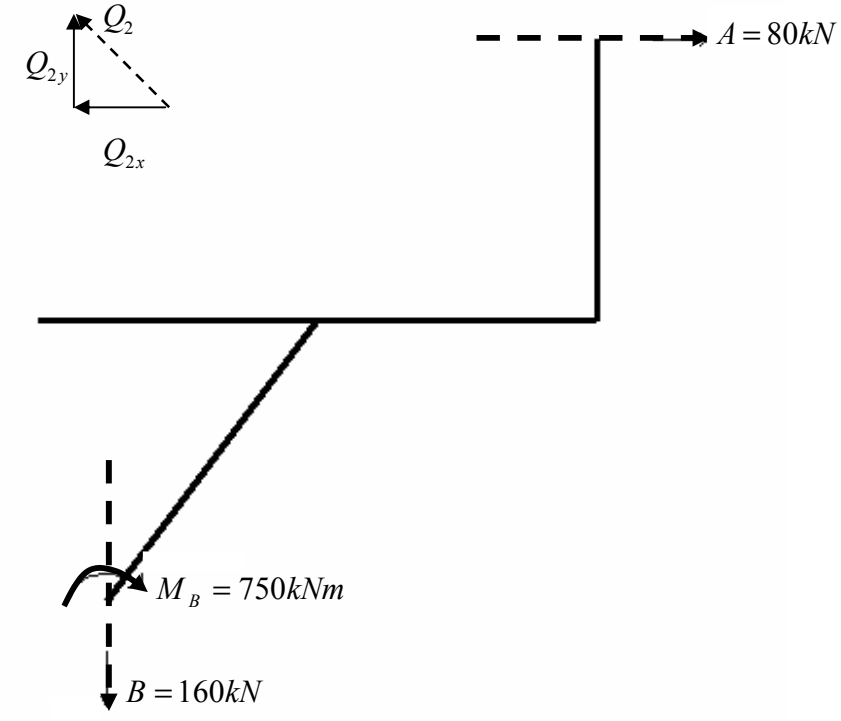
Zadatak 25: *(primjer ispitnog zadatka 31. 1. 2006.)*

Zadano tijelo u ravnini uravnotežite silama A, B na pravcima *"a"* i *"b"* te momentom M_B u točki B.



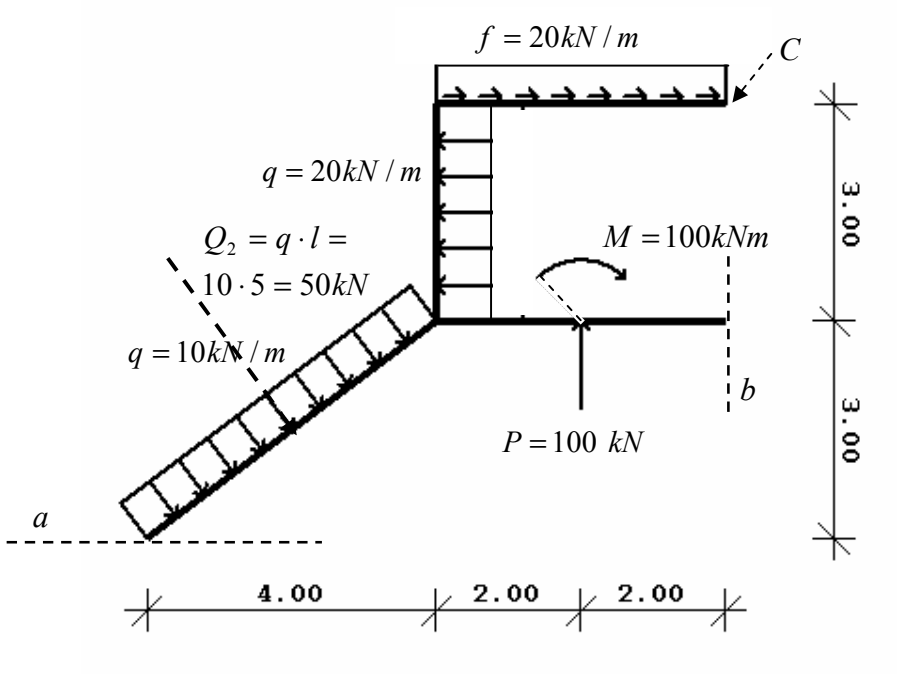
(dimenzije u metrima)

Rješenje: ***($Q_{2x}=q_2 \cdot l_y=20 \cdot 4=80$ kN; $Q_{2y}=q_2 \cdot l_x=20 \cdot 3=60$ kN !!!!!!)***



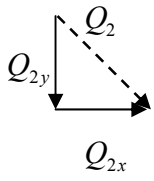
Zadatak 26: *(primjer ispitnog zadatka 14. 11. 2005.)*

Zadano tijelo u ravnini uravnotežiti silama na pravcu *"a"* i na pravcu *"b"* te momentom M_C u točki *"C"*.

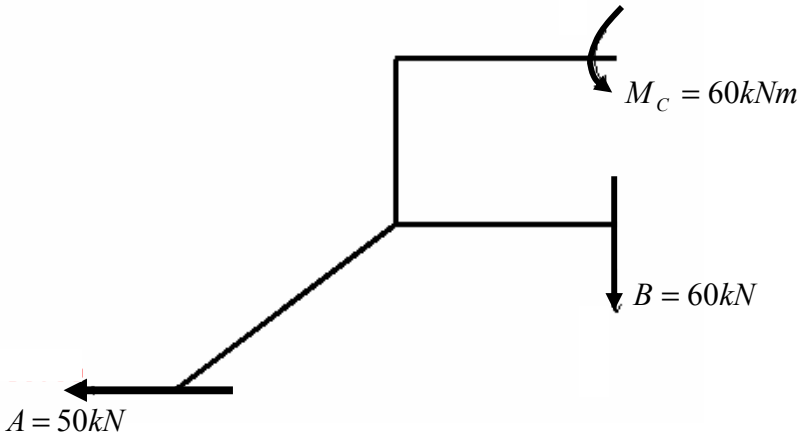


(dimenzije u metrima)

Rješenje:



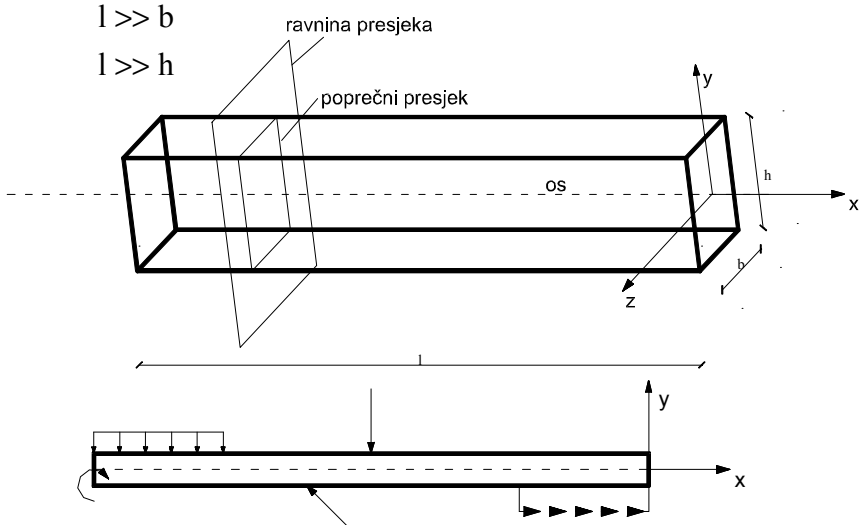
($Q_{2x}=q_2 \cdot l_y=10 \cdot 3=30$ kN;
$Q_{2y}=q_2 \cdot l_x=10 \cdot 4=40$ kN !!!!!!)



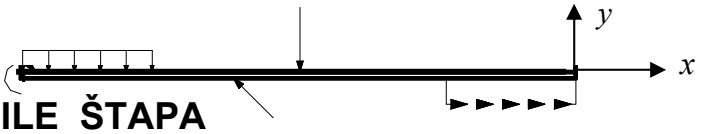
2. DIO – primjeri i zadaci za 2. kolokvij

STATIKA ŠTAPNIH SUSTAVA

Štap je tijelo s jednom istaknutom dimenzijom *(jedna dimenzija je znatno veća od druge dvije) :*

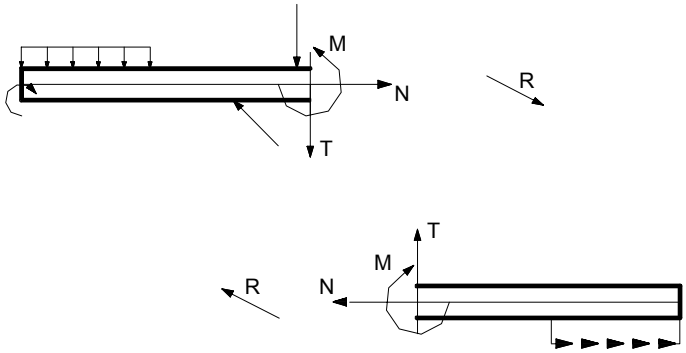


- mi štap prikazujemo samo jednom dimenzijom (l) tj. crtamo samo njegovu os, a sve opterećenje se nalazi u ravnini x-y.



UNUTARNJE SILE ŠTAPA

U analizi unutarnjih sila pretpostavljamo štap u ravnoteži i rastavljamo ga na dva dijela. Tako rastavljeni štap više nije u ravnoteži te mi tražimo sile koje će uravnotežiti štap, tj. tražimo **unutarnje sile.**



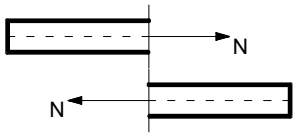
M – moment savijanja
T – poprečna sila(transverzalna)
N – uzdužna sila(normalna)

(smjerovi u kojim su definirane karakteristike materijala)

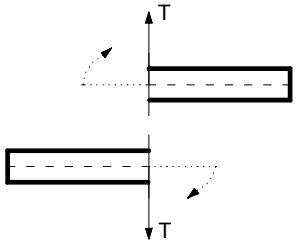
Prema **III. Newtonovom zakonu** akcije i reakcije djelovanja unutarnjih sila na oba dijela moraju biti jednaka.

Unutarnje sile zamjenjuju odbačeni dio štapa tj. uravnotežuju dio štapa koji promatramo.

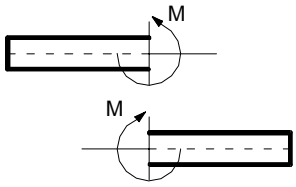
KONVENCIJE PREDZNAKA :



N – je pozitivan ako izlazi iz štapa tj. sila je vlačna



T – je pozitivan ako N zarotiramo u smjeru kazaljke na satu



M – je pozitivan na vlačnoj strani *(vlačna strana će biti objašnjena u zadacima)*

Postavljamo jednadžbe ravnoteže iz kojih računamo unutarnje sile:

$\sum x = 0$ ➔ **N**

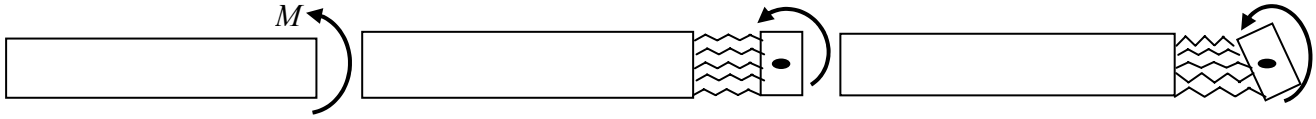
$\sum y = 0$ ➔ **T**

$\sum M = 0$ ➔ **M**

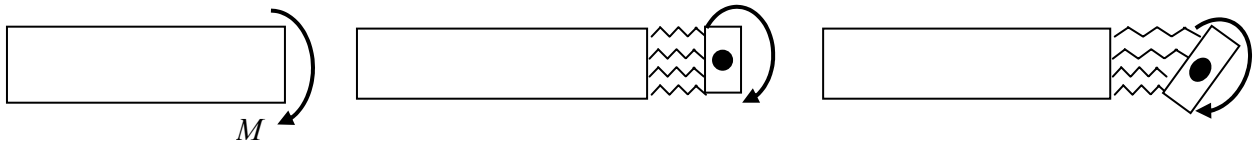
Objašnjenje predznaka unutarnjih sila:

Zamislimo na kraju našeg poprečnog presjeka krutu pločicu oprugama vezanu za promatrani presjek štapa.

Primjer** djelovanja **momenta savijanja

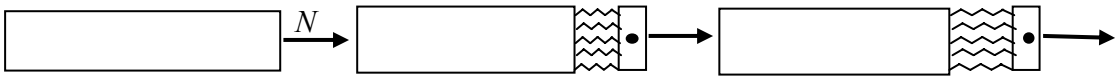


Ako pločicu rotiramo u smjeru pretpostavljenog momenta *(u prikazanom primjeru suprotno od smjera kazaljke na satu),* vidjet ćemo da se donja opruga isteže – znači u **donjoj zoni** našeg štapa djeluje **vlačno** naprezanje, dok se gornja opruga skraćuje – znači u gornjoj zoni štapa djeluje tlačno naprezanje.

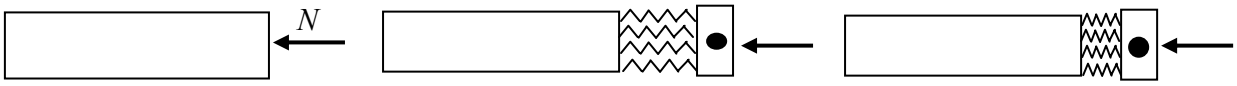


Ako pločicu rotiramo u smjeru pretpostavljenog momenta *(u smjeru kazaljke na satu),* vidjet ćemo da se sada gornja opruga isteže – znači u **gornjoj** zoni našeg štapa djeluje **vlačno** naprezanje, dok se donja opruga skraćuje – znači u donjoj zoni štapa djeluje tlačno naprezanje.

Primjer** djelovanja **uzdužne sile



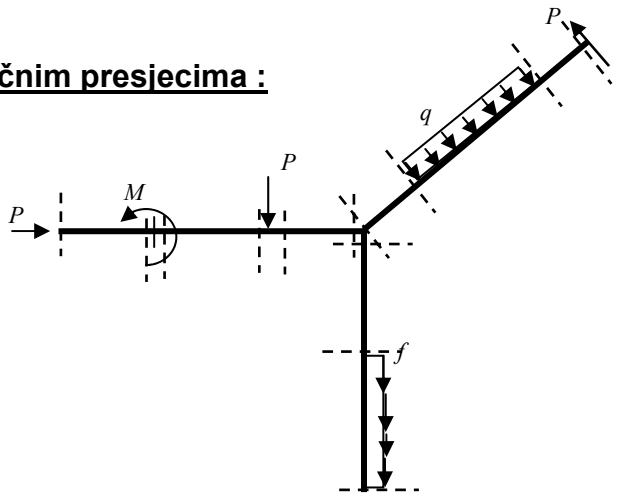
Ako uzdužna sila djeluje iz presjeka (štapa), imamo djelovanje pozitivne uzdužne sile. Vidimo da će se opruge rastezati, što znači da u našem presjeku (štapu) djeluje **vlačno** naprezanje.



Ako uzdužna sila djeluje prema presjeku (štapu), imamo djelovanje negativne uzdužne sile. Vidimo da će se opruge stisnuti, što znači da u našem presjeku (štapu) djeluje **tlačno** naprezanje.

<u>Unutarnje sile računamo u karakterističnim presjecima :</u>

- sustav možemo presjeći bilo gdje tj. gdje god nam je potrebno
- unutarnje sile možemo računati na oba dijela i pokazati da rješenja oba sustava moraju biti ista.
- kada izračunamo sve potrebne sile možemo crtati dijagrame.



- **karakteristični presjeci su**:
  - početak i kraj štapa, promjena smjera štapa
  - početak i kraj kontinuiranog opterećenja
  - ispred i iza djelovanja koncentrirane sile i koncentriranog momenta

Između karakterističnih točaka koristimo diferencijalne odnose.

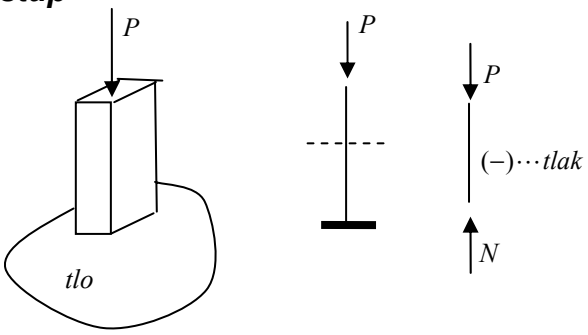
<u>Diferencijalni odnosi za crtanje dijagrama :</u>

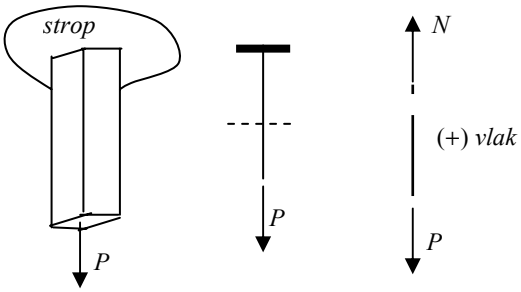
$$\frac{dM_{(x)}}{dx} = T_x \qquad \frac{dT_{(x)}}{dx} = -q_{(x)}$$

$$\frac{d^2M_{(x)}}{dx^2} = -q_{(x)} \qquad \frac{dN_{(x)}}{dx} = -f_{(x)}$$

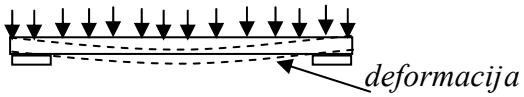
primjeri opterećenja i unutarnjih sila:

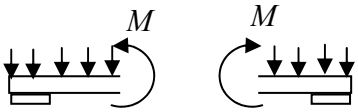
stup





greda

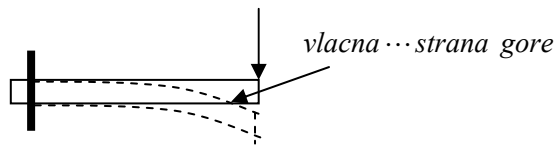


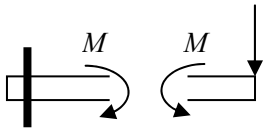


donja strana je vlacna

momentni dijagram ⋯ NEMA predznak
ovdje ga crtamo ga na DONJOJ strani

konzola – balkon



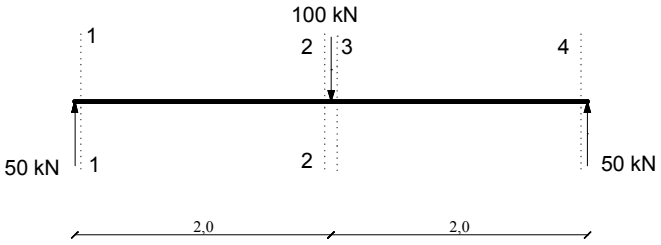


momentni dijagramcrtamo
na vlacnoj, GORNJOJstrani

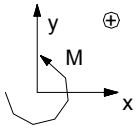
ODREĐIVANJE I CRTANJE DIJAGRAMA UNUTARNJIH SILA

Primjer 1: koncentrirana sila

Za uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



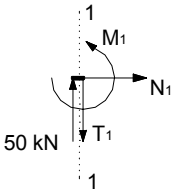
Pretpostavljeni smjerovi :

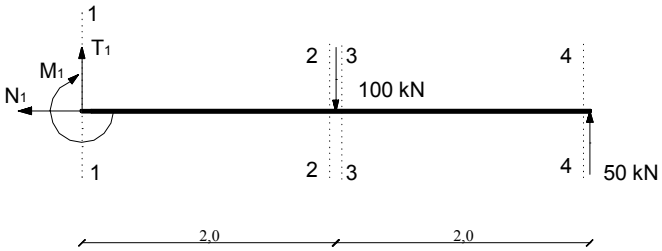


- sile koje djeluju na štap su u ravnoteži.

Proračun unutarnjih sila:

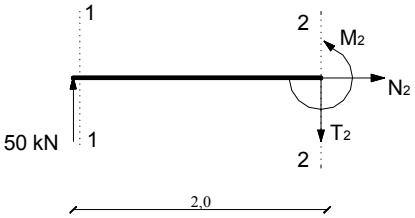
Presjek 1-1

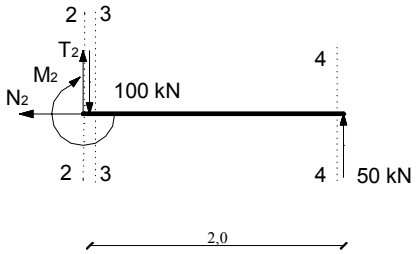




	lijevi dio		*desni dio*	
$\sum x = 0$	$N_1+0=0$	**$N_1=0$**	$-N_1+0=0$	**$N_1=0$**
$\sum y = 0$	$50-T_1=0$;	**$T_1=50$kN**	$50-100+T_1=0$;	**$T_1=50$kN**
$\sum M = 0$	$-50\times0+M_1=0$;	**$M_1=0$**	$50\times4-100\times2-M_1=0$;	**$M_1=0$**

Presjek 2-2

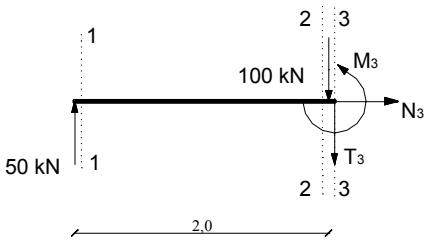




	lijevi dio		*desni dio*	
$\sum x = 0$	$N_2+0=0$	**$N_2=0$**	$N_2+0=0$	**$N_2=0$**
$\sum y = 0$	$50-T_2=0$;	**$T_2=50$ kN**	$50-100+T_2=0$;	**$T_2=50$ kN**
$\sum M = 0$	$-50\times2+M_2=0$;	**$M_2=100$ kNm**	$50\times2-100\times0-M_2=0$;	**$M_2=100$ kNm**

- pokazano je da računajući s obje strane unutarnje sile su iste za isti poprečni presjek

Presjek 3-3

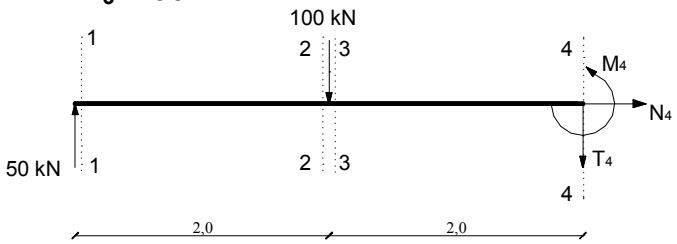


$\sum x = 0$ $N_3+0=0$ **$N_3=0$**

$\sum y = 0$ $50-100-T_3=0$; **$T_3= -50$ kN**

$\sum M = 0$ $-50\times2+M_3=0$; **$M_3=100$ kNm**

Presjek 4-4



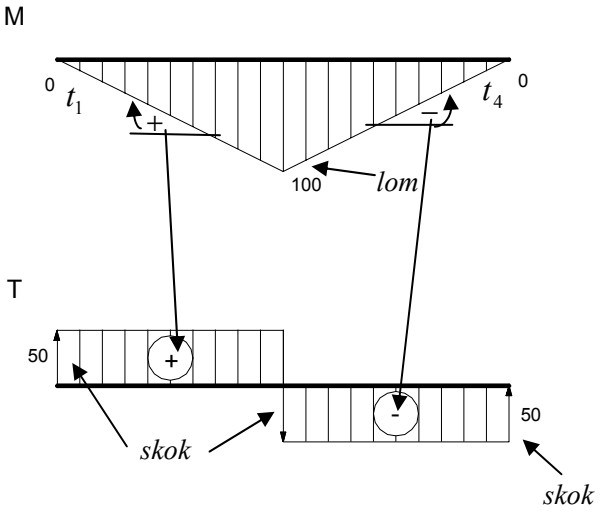
$\sum x = 0$ $N_4+0=0$ **$N_4=0$**

$\sum y = 0$ $50-100-T_4=0$; **$T_4=-50$ kN**

$\sum M = 0$ $-50\times4+100\times2+M_4=0$; **$M_4=0$**

- *lakše bi bilo da smo presjeke 3-3, 4-4 računali s lijeve strane (manje posla)*

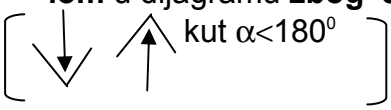
Dijagrami :



$\frac{dM_x}{dx} = T_1 \Rightarrow \frac{100-0}{2} = 50\ kN$ *nagib* t_1 (+)

$\frac{dM_x}{dx} = T_4 = \frac{0-100}{2} = -50\ kN$ *nagib* t_4 (−)

M** dijagram **- linearan
- crta se na **''vlačnoj strani''**
- crta se **bez predznaka**
- **lom** u dijagramu **zbog sile**



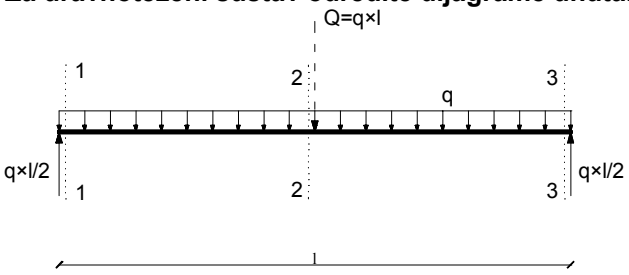
T** dijagram **- konstantan
- crta se **+ ➔ gore**
- ➔ dolje
- crta se **s predznacima**
- **skok** u dijagramu za **iznos i smjer** koncentrirane **sile**

***N** dijagram*
- nemamo uzdužnih sila
- crta se + ➔ gore
- ➔ dolje

(*T i N kako dobijemo rezultat samo crtamo + gore – dolje)*

Primjer 2: kontinuirano opterećenje

Za uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



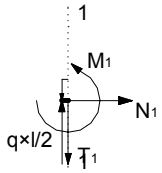
- sile koje djeluju na štap su u ravnoteži.

q = 10kN/m; l=5m

→ Q=50kN q×l/2=25kN

Proračun unutarnjih sila:

Presjek 1-1

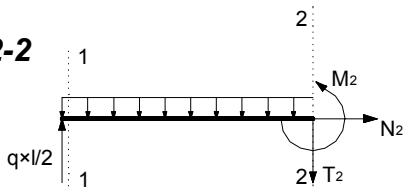


- *kod rješavanja paralelno se rješavalo*

općim brojevima i konkretnim brojevima

$\sum x = 0$	$N_1+0=0$	**$N_1=0$**	;	$N_1+0=0$	**$N_1=0$**
$\sum y = 0$	$q\times l/2-T_1=0$;	**$T_1=q\times l/2$**	;	$10\times 5/2-T_1=0$;	**$T_1=25$kN**
$\sum M = 0$	$-q\times l/2\times 0+M_1=0$;	**$M_1=0$**	;	$-q\times l/2\times 0+M_1=0$;	**$M_1=0$ kNm**

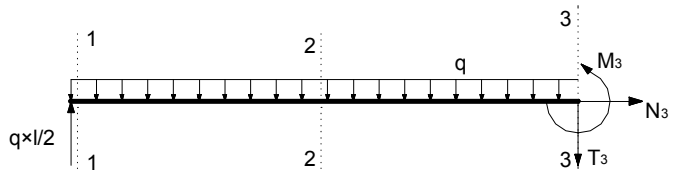
Presjek 2-2



(osim u karakterističnim točkama maksimume možemo tražiti u točkama gdje očekujemo najveće vrijednosti sila, a to je u našem slučaju presjek 2-2)

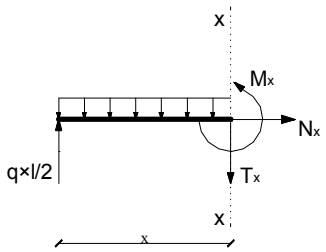
$\sum x = 0$	$N_2+0=0$	**$N_2=0$**	;	$N_2+0=0$	**$N_2=0$ kN**
$\sum y = 0$	$q\times l/2-q\times l/2-T_2=0$;	**$T_2=0$**	;	$25-25-T_2=0$;	**$T_2=0$ kN**
$\sum M = 0$	$-q\times l/2\times l/2+q\times l/2\times l/4+M_2=0$;	**$M_2=q\times l^2/8$;**		$25\times 2.5+25\times 1.25+M_2=0$;	**$M_2=31.2$kNm**

Presjek 3-3



$\sum x = 0$	N_3+0=0	**N_3=0**	;N_3+0=0	**N_3=0**
$\sum y = 0$	q×l/2-q×l-T_3=0 ;	**T_3=-q×l/2**	;10×5/2-10×5-T_3=0 ;	**T_3=-25kN**
$\sum M = 0$	-q×l/2×l+q×l×l/2+M_3=0;	**M_3=0**	;-10×5/2×5+10×5×5/2+M_3=0	**M_3=0**

Presjek x-x



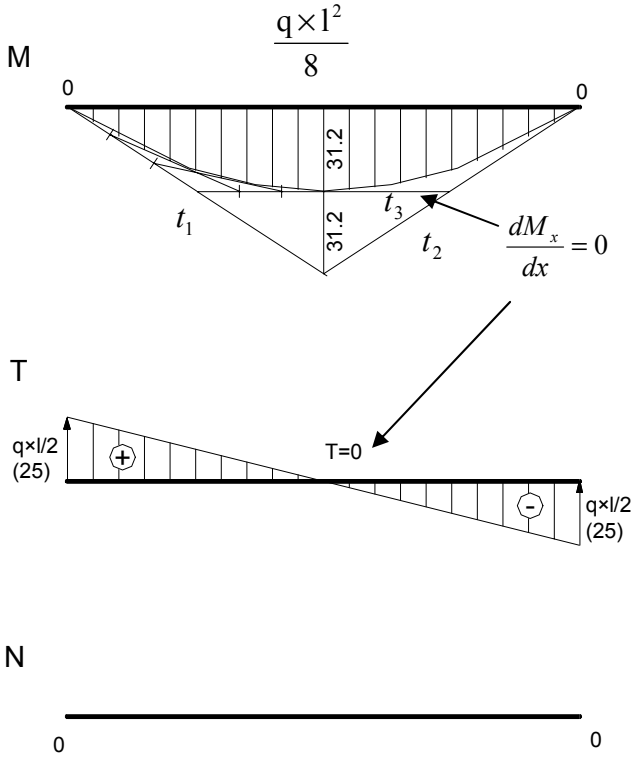
- možemo tražiti vrijednosti unutarnjih sila na bilo kojoj udaljenosti x.

$\sum x = 0$	N_x+0=0	**N_x=0**	
$\sum y = 0$	q×l/2-q×x-T_x=0 ;	**T_x=** q×l/2-q×x	*(jednadžba pravca)*
$\sum M = 0$	-q×l/2×x+q×x×x/2+M_x=0;	**M_x=**- q×x^2/2 + q×l×x/2	*(jednadžba parabole)*

Dijagrami :

- **kontinurirano opterećenje** daje **momentni dijagram u** obliku **parabole**
- **nultočka** poprečnih sila - gdje je **tangenta** u momentnom dijagramu **horizontalna**

poprečna sila = 0 ($\frac{dM_x}{dx} = 0$)



<u>Konstrukcija dijagrama:</u>

- nanosimo dvije vrijednosti momenta **m_0** u sredini raspona okomito na os nosača (kao da imamo Q silu **m_0 = qxl^2/8**)
- povlačimo prve dvije tangente t_1 i t_2
- povlačimo tangentu t_3 paralelnu sa zaključnom linijom (ovdje s osi štapa)
- konstruiramo pomoćne tangente za parabolu
- *parabola ima "trbuh" u smjeru djelovanja opterećenja*

- **kontinurirano opterećenje** daje **dijagram poprečnih** u obliku **pravca**

- nemamo uzdužnih sila

ODREĐIVANJE MAKSIMALNE VRIJEDNOSTI MOMENTA
(molim ne preskočiti, poneki puta se zbog toga pada na ispitu)

Vrijednost **maksimalnog momenta** određujemo *iz uvjeta* da je **poprečna sila** na tom mjestu **jednaka nuli**, tj T_x=0.

U našem slučaju to znači da nam je na mjestu t_3 (horizontalna tangenta) poprečna sila nula, a istovremeno vrijednost momenta maksimalna.

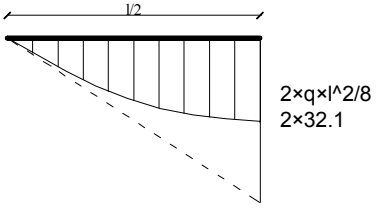
$$T_x = -q \cdot x + \frac{ql}{2} = 0 \cdots \Rightarrow x = \frac{l}{2} \Rightarrow M_{\max} = \frac{ql^2}{8}$$

Diferencijalni odnosi:

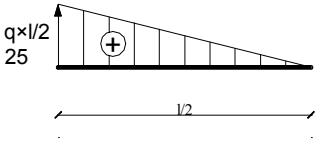
$$\frac{dM_{(x)}}{dx} = T_x \qquad \frac{2 \times \frac{q \times l^2}{8}}{\frac{l}{2}} = \frac{q \times l}{2} = T_1$$

$$\frac{dT_{(x)}}{dx} = -q_{(x)} \qquad \frac{0 - \frac{q \times l}{2}}{\frac{l}{2}} = -q_{(x)}$$

M

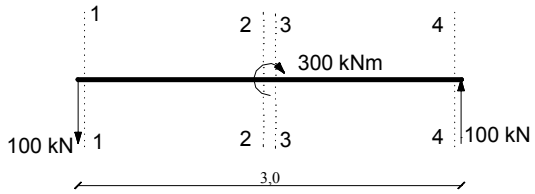


T



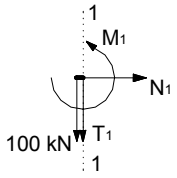
Primjer 3: koncentrirani moment

Za uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



Proračun unutarnjih sila:

Presjek 1-1

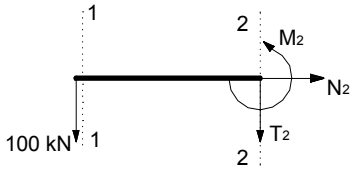


$\sum x = 0$ $N_1+0=0$ **$N_1=0$**

$\sum y = 0$ $-100-T_1=0$; **$T_1=-100$kN**

$\sum M = 0$ $100\times0+M_1=0$; **$M_1=0$**

Presjek 2-2

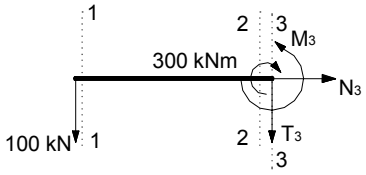


$\sum x = 0$ $N_2+0=0$ **$N_2=0$**

$\sum y = 0$ $-100-T_2=0$; **$T_2=-100$ kN**

$\sum M = 0$ $100\times1.5+M_2=0$; **$M_2=-150$ kNm**

Presjek 3-3

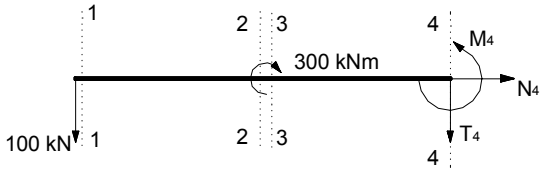


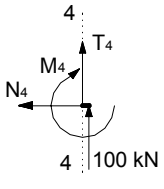
$\sum x = 0$ $N_3+0=0$ **$N_3=0$**

$\sum y = 0$ $-100-T_3=0$; **$T_3=-100$ kN**

$\sum M = 0$ $100\times1.5-300+M_3=0$; **$M_3=150$ kNm**

Presjek 4-4



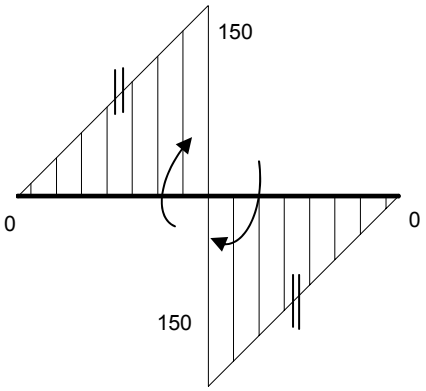


	lijevo		*desno*	
$\sum x = 0$	$N_4+0=0$	**$N_4=0$**	$N_4+0=0$	**$N_4=0$**
$\sum y = 0$	$-100-T_4=0$;	**$T_4=-100$ kN**	$100+T_4=0$;	**$T_4=-100$ kN**
$\sum M = 0$	$100\times3-300+M_4=0$;	**$M_4=0$**	$100\times0-M_4=0$;	**$M_4=0$**

računamo kako nam je lakše

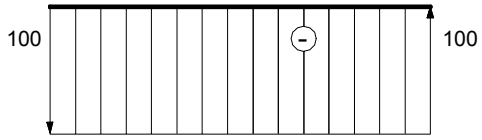
Dijagrami:

M



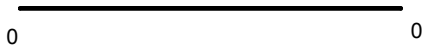
- **skok** u **momentnom** dijagramu **zbog koncetriranog momenta**
- **skok** u smjeru strijelice
- **nagibi** tangente isti

T



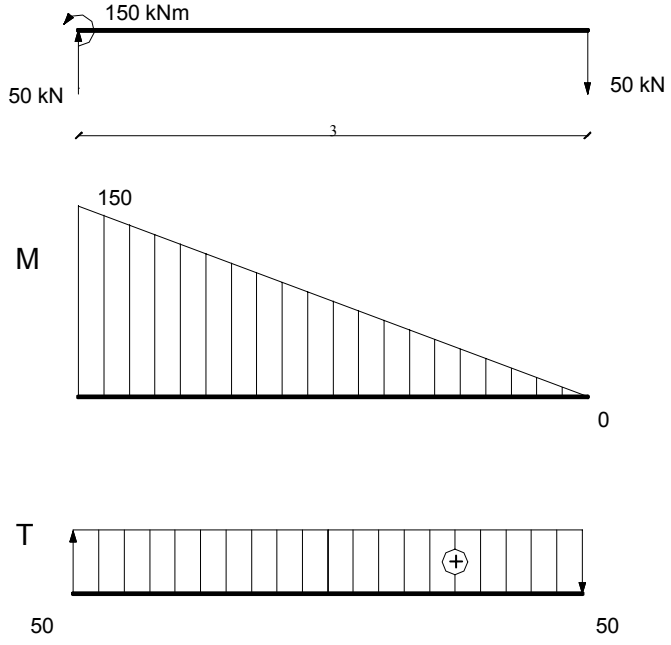
- **poprečna** sila je negativna na cijelom štapu (kako gledaju sile)

N



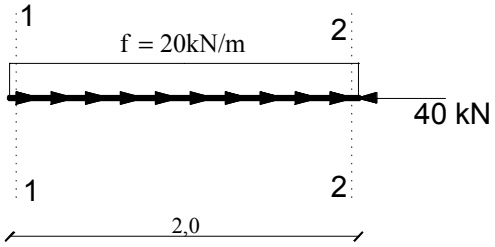
- nemamo uzdužnih sila

dodatni primjer:



Primjer 4: kontinuirano opterećenje u smjeru uzdužne osi štapa

Za uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



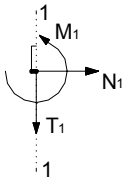
Proračun unutarnjih sila:

Presjek 1-1

$\sum x = 0$ $N_1+0=0$ **$N_1=0$**

$\sum y = 0$ $0-T_1=0$; **$T_1=0$**

$\sum M = 0$ $0+M_1=0$; **$M_1=0$**

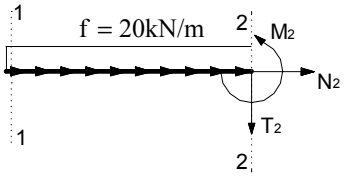


Presjek 2-2

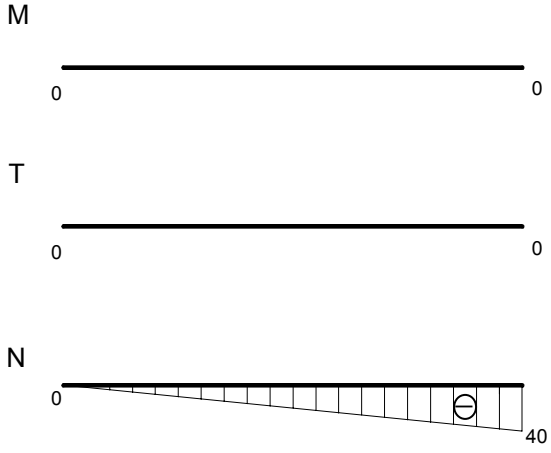
$\sum x = 0$ $N_2+f\times 2=0$ **$N_2=-40$ kN**

$\sum y = 0$ $0-T_2=0$; **$T_2=0$**

$\sum M = 0$ $0+M_2=0$; **$M_2=0$**



Dijagrami :

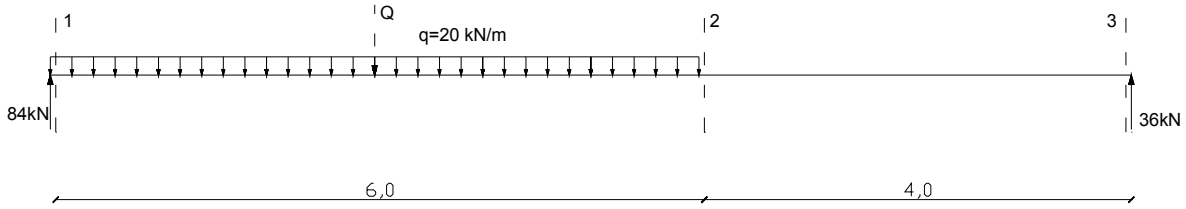


- **nemamo momenata** (opterećenje u smjeru osi ne doprinosi momentnom dijagramu)
- **nemamo poprečnih sila** (opterećenje u smjeru osi ne doprinosi momentnom dijagramu)
- **kontinuirano** opterećenje **u smjeru osi štapa** daje **linearan** dijagram **uzdužnih** sila

$$\frac{dN_x}{dx} = -f_x \qquad -\frac{-40}{2} = +20\,kN/m$$

Zadatak 27:

Za zadani uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N te vrijednost maksimalnog momenta savijanja M_{max}.

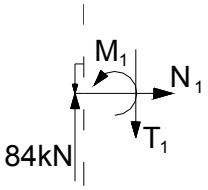


Postupak:

3 karakteristična presjeka

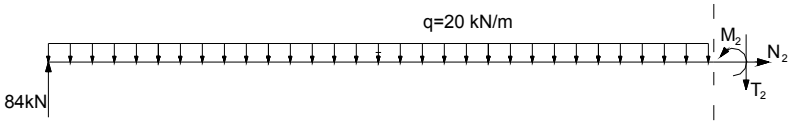
$$q = 20kN/m \rightarrow Q = 20 \cdot 6 = 120kN$$

Pr. 1.



$$\Sigma x = \phi$$
$$\Sigma y = \phi$$
$$84 - T_1 = \phi \qquad T_1 = 84kN$$
$$\Sigma M_{(1)} = \phi \rightarrow M_1 = 0$$

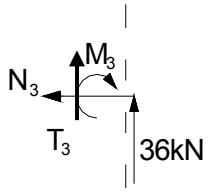
Pr. 2.



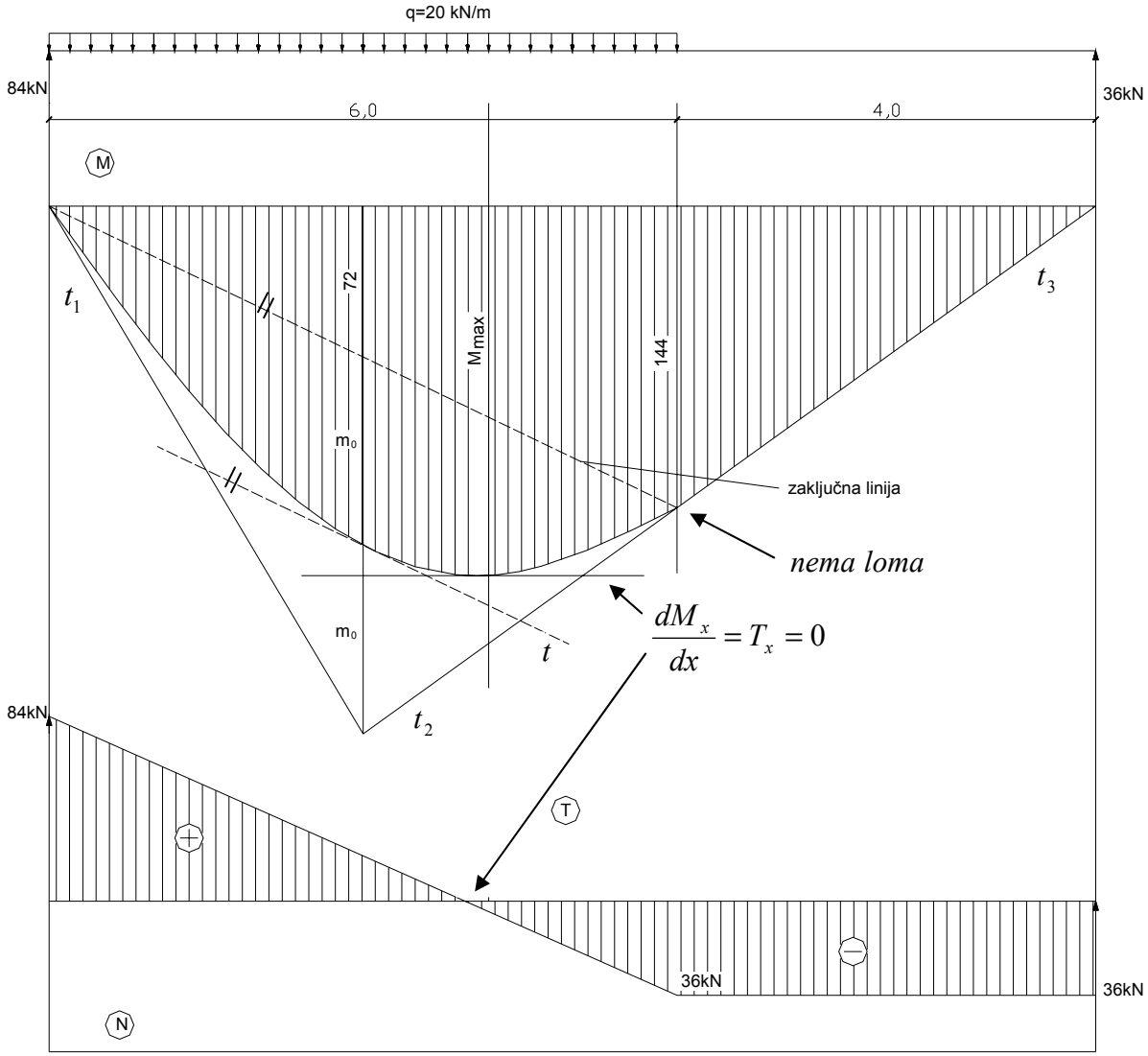
$$\Sigma x = \phi \rightarrow N_2 = 0$$
$$\Sigma y = \phi \rightarrow 84 - q \cdot 6 = \phi$$
$$T_2 = 84 - 120 = -36kN$$

$$\Sigma M_{(2)} = \phi$$
$$M_2 - 84 \cdot 6 + q \cdot 6 \cdot 3 = \phi$$
$$M_2 = 84 \cdot 6 - 120 \cdot 3 = 144kNm$$

Pr.3.



$$\Sigma x = \phi \rightarrow N_3 = 0$$
$$\Sigma y = \phi \rightarrow T_3 + 36 = \phi \rightarrow T_3 = -36kN$$
$$\Sigma M_3 = \phi \rightarrow M_3 = 0$$



$$m_0 = \frac{q \cdot l^2}{8} = \frac{20 \cdot 6^2}{8} = 90kNm$$

***t** II sa zaključnom linijom*

$$T_1 = \frac{72 + 90 + 90 - 0}{3} = 84kN$$

$$\frac{dMx}{dx} = Tx$$

$$T_2 = \frac{144 - (72 + 90 + 90)}{3} = -36kN$$

$$T_3 = \frac{0 - 144}{4} = -36kN$$

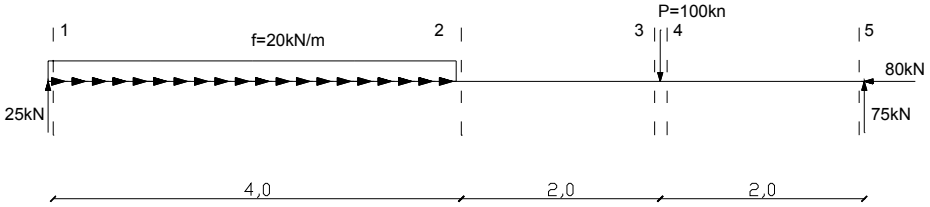
- **na mjestu gdje je T_x=0 → imamo M_{max} !!!!!**
-
- **točan** položaj odredimo iz T_x= 0 → T_x=84-q·x=0 → x = 4.2m

$$M_{\max} = 84 \cdot 4{,}2 - \frac{20 \cdot 4{,}2^2}{2} = 176{,}4kNm$$

$$\frac{dTx}{dx} = -q_x \qquad \frac{-36 - 84}{6} = -20kN/m$$

Zadatak 28:

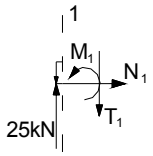
Za zadani uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



Postupak:

5 karakterističnih presjeka

Pr. 1.



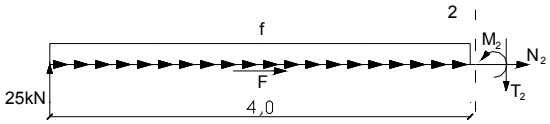
$\Sigma x = 0 \quad \mathbf{N_1} = 0$

$\Sigma y = 0 \quad 25 - T_1 = 0$

$\mathbf{T_1} = 25kN$

$\Sigma M_1 = 0 \quad \mathbf{M_1} = 0$

Pr. 2.

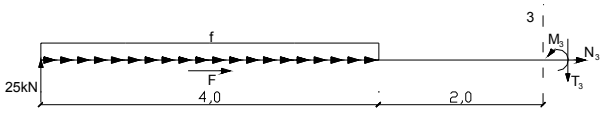


$\Sigma x = 0 \quad N_2 + f \cdot 4 = 0 \rightarrow \mathbf{N_2} = -80kN$

$\Sigma y = 0 \quad 25 - T_2 = 0 \rightarrow \mathbf{T_2} = 25kN$

$\Sigma M_2 = 0 \quad M_2 - 25 \cdot 4 = 0 \rightarrow \mathbf{M_2} = 100kNm$

Pr. 3.

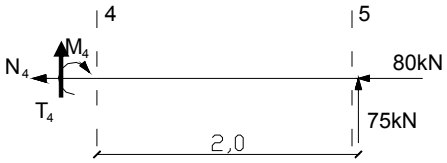


$\Sigma x = 0 \quad N_3 + f \cdot 4 = 0 \rightarrow \mathbf{N_3} = -80kN$

$\Sigma y = 0 \quad 25 - T_3 = 0 \rightarrow \mathbf{T_3} = 25kN$

$\Sigma M_3 = 0 \quad M_3 - 25 \cdot 6 = 0 \rightarrow \mathbf{M_3} = 150kNm$

Pr. 4.



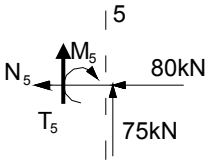
$\Sigma x = 0 \quad -N_4 - 80 = 0 \rightarrow \mathbf{N_4} = -80kN$

$\Sigma y = 0 \quad T_4 + 75 = 0 \rightarrow \mathbf{T_4} = -75kN$

$\Sigma M_4 = 0 \quad -M_4 + 75 \cdot 2 = 0 \rightarrow \mathbf{M_4} = 150kNm$

- ***kontrola***! $M_4 = M_3$

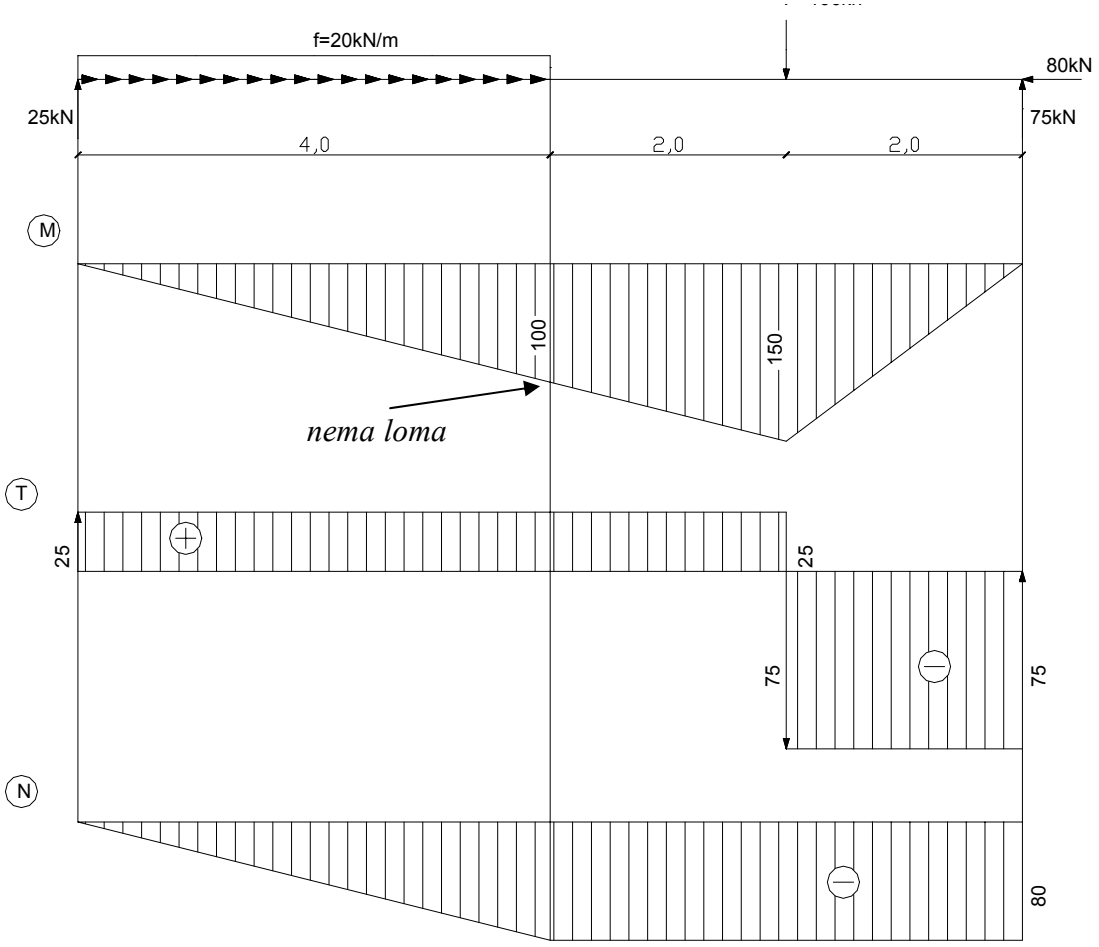
Pr.5.



$\Sigma x = 0 \quad -N_5 - 80 = 0 \rightarrow \mathbf{N_5} = -80kN$

$\Sigma y = 0 \quad T_5 + 75 = 0 \rightarrow \mathbf{T_5} = -75kN$

$\Sigma M_5 = 0 \quad \rightarrow \quad \mathbf{M_5} = 0$



- **f** $\rightarrow$ **nema** utjecaja na **momentni** dijagram

$$\frac{dMx}{dx} = Tx$$

$$T_3 = T_2 = T_1 = \frac{150}{6} = 25kN$$

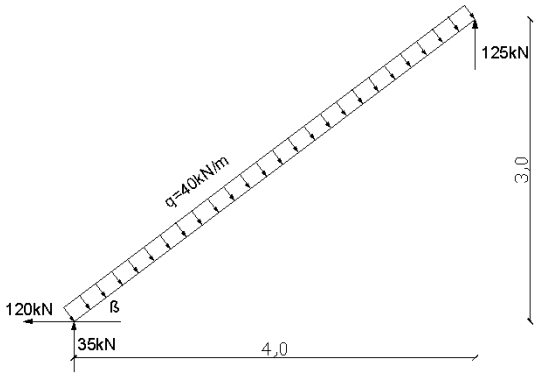
$$T_4 = T_5 = -\frac{150}{2} = -75kN$$

$$\frac{dNx}{dx} = -f(x)$$

$$-\left(\frac{-80}{4}\right) = 20kN/m$$

Zadatak 29:

Za zadani uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.

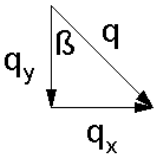


$$\text{tg}\beta = \frac{l_y}{l_x} = \frac{3}{4} = 0{,}75$$

$$\cos\beta = \frac{l_x}{l}$$

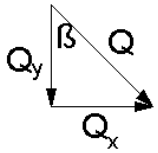
$$\sin\beta = \frac{l_y}{l}$$

Postupak:



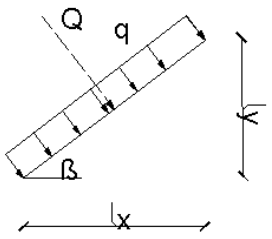
$$Q = q \cdot l$$

$$Q_y = Q \cdot \cos\beta = q \cdot l \cdot \frac{l_x}{l} = q \cdot l_x$$!!!!!!!



$$Q_x = Q \cdot \sin\beta = q \cdot l \cdot \frac{l_y}{l} = q \cdot l_y$$

- kod **kontinuiranog opterećenja** koje djeluje **okomito na os** koso položenog tijela dobro je projicirati opterećenje u smjeru globalnih osi x i y, tj. Q na $\rightarrow$ Q_x i Q_y. Zamjenu izvodimo u težištu djelovanja opterećenja.
-



$$Q = q \cdot l = q \cdot \sqrt{l_x^2 + l_y^2} = 40 \cdot \sqrt{4^2 + 3^2} = 40 \cdot 5 = 200kN$$

$$Q_y = q \cdot l_x = 40 \cdot 4 = 160kN$$

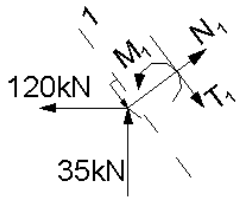
$$Q_x = q \cdot l_y = 40 \cdot 3 = 120kN$$

ili

$$Q_y = Q \cdot \cos\beta = 200 \cdot 0.8 = 160kN$$

$$Q_x = Q \cdot \sin\beta = 200 \cdot 0.6 = 120kN$$

Presjek 1.

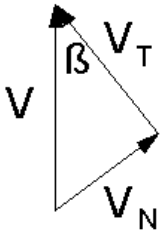


$\Sigma x = 0 \rightarrow N_{1x} + T_{1x} - 120 = 0$

$\Sigma y = 0 \rightarrow N_{1y} - T_{1y} + 35 = 0$

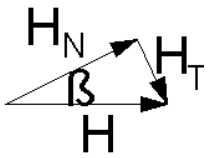
$\rangle$ N_1 i T_1 nepoznate

ili **projiciramo sile V i H u smjeru sila N i T !!!!!**



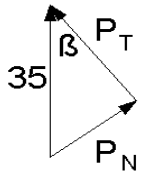
$V_N = V \cdot \sin ß$

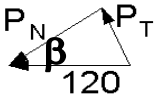
$V_T = V \cdot \cos ß$



$H_N = H \cdot \cos ß$

$H_T = H \cdot \sin ß$





ΣT= 0

$-T_1 + 35 \cdot \cos ß + 120 \cdot \sin ß = 0$

T_1 $=35 \cdot 0{,}8 + 120 \cdot 0{,}6$ **=100kN**

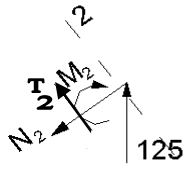
ΣN= 0

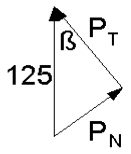
$N_1 + 35 \cdot \sin ß - 120 \cdot \cos ß = 0$

N_1 $=-35 \cdot 0{,}6 + 120 \cdot 0{,}8$**=75kN**

$\Sigma M_{(1)} = 0 \rightarrow M_1 = 0$

Presjek 2.





$P_T = P \cdot \cos ß = 125 \cdot 0{,}8 = 100kN$

$P_N = P \cdot \sin ß = 125 \cdot 0{,}6 = 75kN$

ΣT= 0

$T_2 + P_T = 0$

T_2= -100kN

ΣN= 0

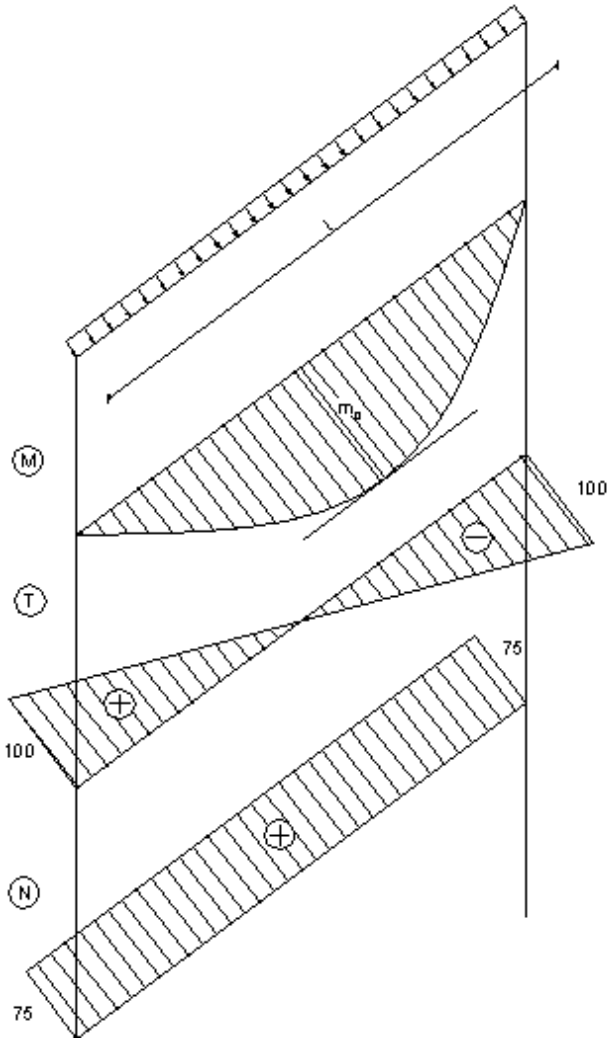
$N_2 - P_N = 0$

N_2 = 75kN

$\Sigma M_{(2)} = 0 \rightarrow M_2 = 0$

Pogledajte zadatke za vježbu *(na kraju 2. dijela)* ***kako vas ne bi iznenadili kosi štapovi s horizontalnim ili vertikalnim opterećenjima na svojim krajevima!!!!***

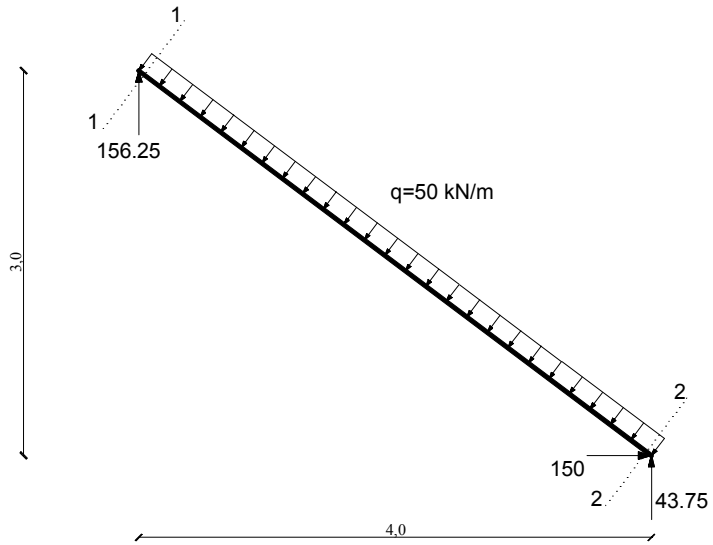
- znamo da je **M** $\rightarrow$ **parabola**, a kod kosog štapa parabolu konstruiramo isto kao i kod horizontalnog
- dijagram poprečnih sila je linearan, ali se pojavljuje i dijagram uzdužnih sila koji je konstantan



$$m_0 = \frac{ql^2}{8} = \frac{40 \cdot 5^2}{8} = 125\ kNm$$

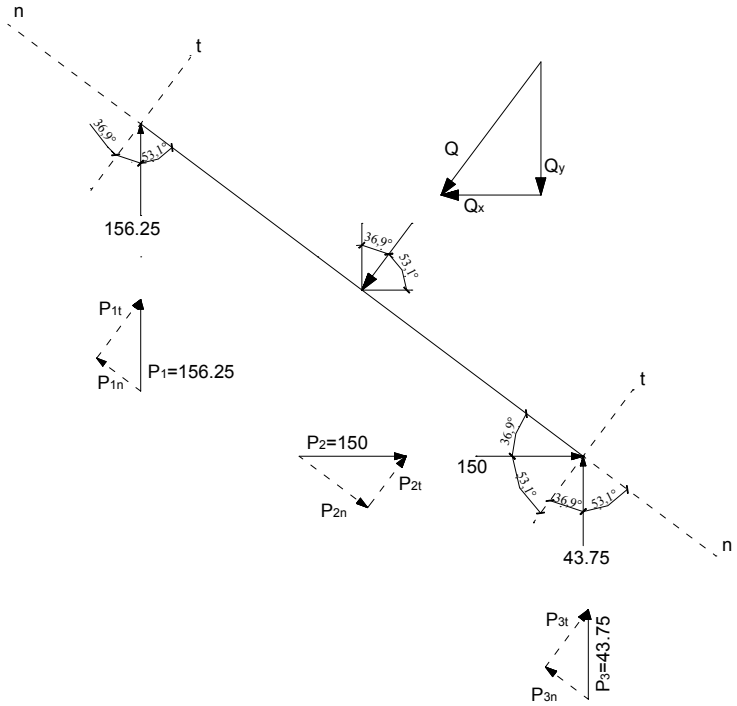
Zadatak 30:

Za zadani uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



Postupak:

•*projiciramo sile na smjerove pogodne za crtanje dijagrama:* ***u smjeru štapa i okomito na štap***



$$\text{tg}\alpha = \frac{3}{4} = 0.75 \Rightarrow \alpha = 36.9\,°$$

$\cos\alpha = 0.8$

$\sin\alpha = 0.6$

Q

$Q_x = Q \times \sin\alpha$ = 250×0.6 = **150 kN**

$Q_y = Q \times \cos\alpha$ = 250×0.8 = **200 kN**

P_1

$P_{1n} = P_1 \times \sin\alpha$ = 156.25×0.6 = **93.75 kN**

$P_{1t} = P_1 \times \cos\alpha$ = 156.25×0.8 = **125,0 kN**

P_2

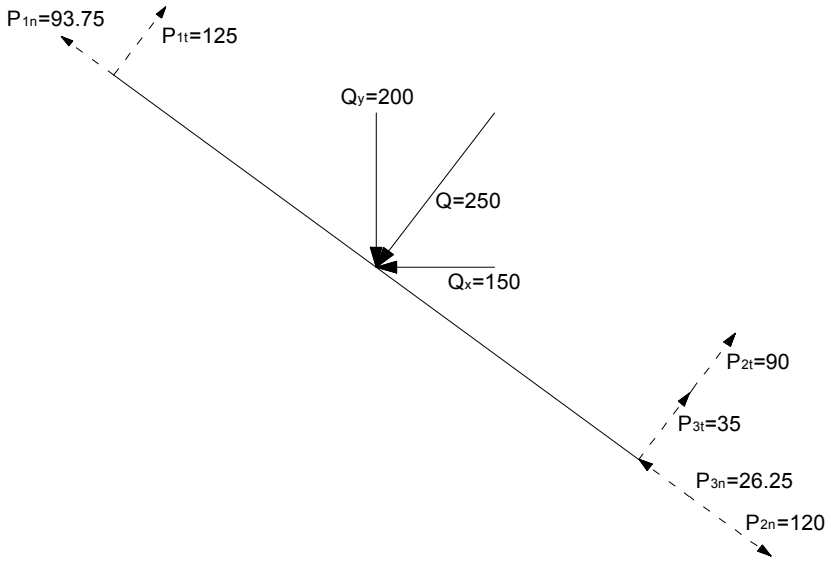
$P_{2n} = P_2 \times \cos\alpha$ = 150×0.8 = **120,0 kN**

$P_{2t} = P_2 \times \sin\alpha$ = 150×0.6 = **90,0 kN**

P_3

$P_{3n} = P_3 \times \sin\alpha$ = 43.75×0.6 = **26,25 kN**

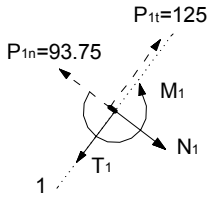
$P_{3t} = P_3 \times \cos\alpha$ = 43.75×0.8 = **35,0 kN**



- sile i štapovi mogu biti zadani pod bilo kojim kutom, ali sve se opet svodi na projiciranje sila na prikladne osi (najčešće u smjeru štapa i okomito na štap)

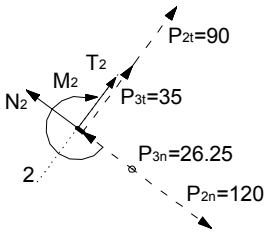
-

Presjek 1-1



$\sum x = 0$	N_1-93.75=0	**N_1=93.75 kN**
$\sum y = 0$	125-T_1=0 ;	**T_1=125 kN**
$\sum M = 0$	0+M_1=0;	**M_1=0**

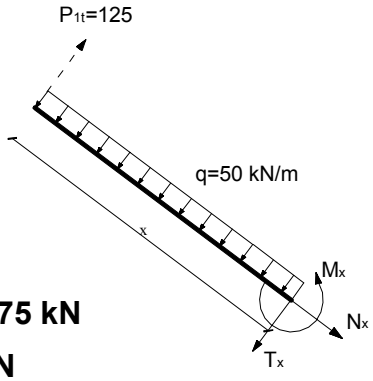
Presjek 2-2



$\sum x = 0$	-N_2-26.25+120=0	**N_2=93.75**
$\sum y = 0$	35+90+T_2=0 ;	**T_2=-125 kN**
$\sum M = 0$	0+M_2=0;	**M_2=0**

Presjek x-x

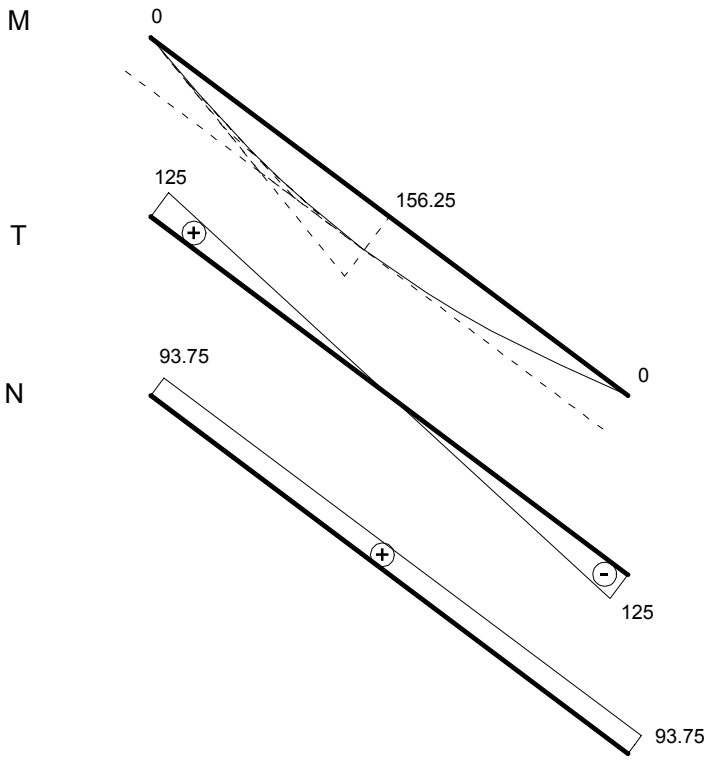
- za ovaj slučaj mjesto maksimalnog momenta je gdje je poprečna sila jednaka 0, u l/2



uvjet → **T_x=0**

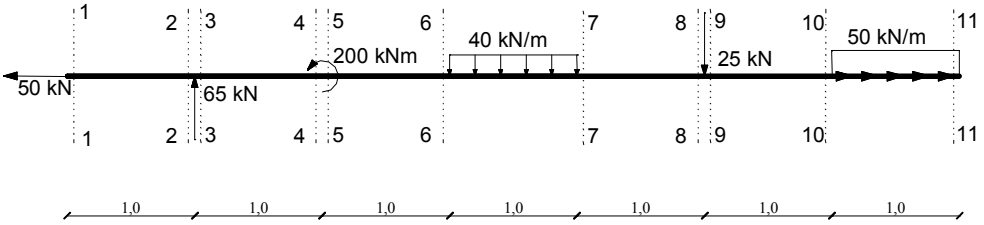
125-50×x-T_x=0 → x = 2.5m = [l/2]

$\sum x = 0$	N_x-93.75=0	**N_x=93.75 kN**
$\sum y = 0$	125-T_x-50×2.5=0 ;	**T_x=0 kN**
$\sum M = 0$	-125×2.5+50×2.5×1.25+M_x=0;	**M_x=156.25 kNm**



Zadatak 31:

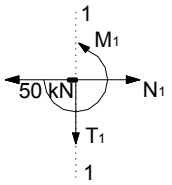
Za zadani uravnoteženi sustav odredite dijagrame unutarnjih sila M, T i N.



Postupak:
Proračun unutarnjih sila:

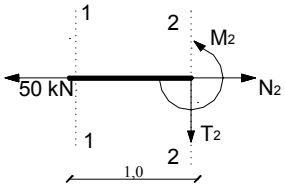
11 karakterističnih presjeka

Presjek 1-1



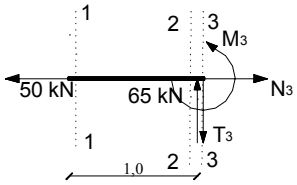
$\sum x = 0$	N_1-50=0	**N_1=50kN**
$\sum y = 0$	-T_1+0=0 ;	**T_1=0**
$\sum M = 0$	M_1+0=0;	**M_1=0**

Presjek 2-2



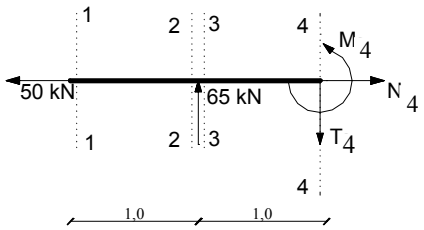
$\sum x = 0$	N_2-50=0	**N_2=50kN**
$\sum y = 0$	-T_2+0=0 ;	**T_2=0**
$\sum M = 0$	M_2+0=0;	**M_2=0**

Presjek 3-3



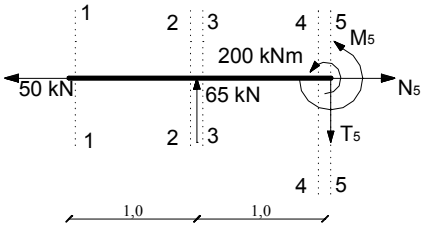
$\sum x = 0$	N_3-50=0	**N_3=50kN**
$\sum y = 0$	-T_3+65=0 ;	**T_3=65kN**
$\sum M = 0$	M_3+0=0;	**M_3=0**

Presjek 4-4



$\sum x = 0$	N_4-50=0	**N_4=50kN**
$\sum y = 0$	-T_4+65=0 ;	**T_4=65kN**
$\sum M = 0$	M_4-65×1,0=0;	**M_4=65kNm**

Presjek 5-5



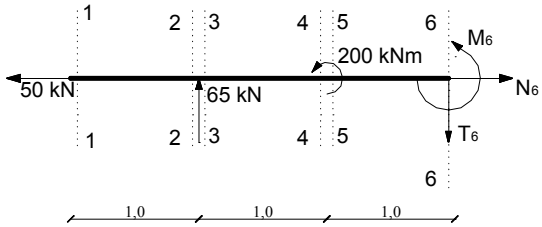
$\sum x = 0$	N_5-50=0	**N_5=50kN**
$\sum y = 0$	-T_5+65=0 ;	**T_5=65kN**
$\sum M = 0$	M_5-65×1,0+ 200=0;	**M_5=-135kNm**

Presjek 6-6

$\sum x = 0$ N_6-50=0 **N_6=50kN**

$\sum y = 0$ -T_6+65=0 ; **T_6=65kN**

$\sum M = 0$ M_6-65×2,0+ 200=0; **M_6=-70kNm**

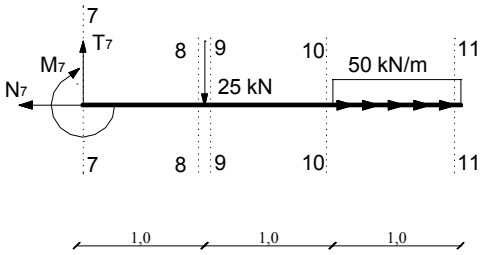


Presjek 7-7

$\sum x = 0$ -N_7+50×1.0=0 **N_7=50kN**

$\sum y = 0$ T_7-25=0 ; **T_7=25kN**

$\sum M = 0$ -M_7-25×1,0=0; **M_7=-25kNm**

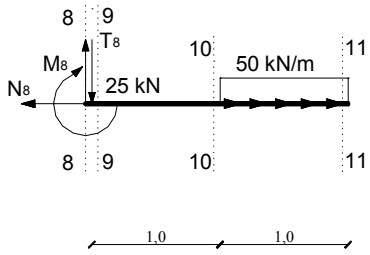


Presjek 8-8

$\sum x = 0$ -N_8+50×1.0=0 **N_8=50kN**

$\sum y = 0$ T_8-25=0 ; **T_8=25kN**

$\sum M = 0$ -M_8+0=0; **M_8=0**

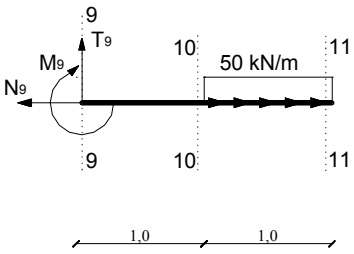


Presjek 9-9

$\sum x = 0$ -N_9+50×1.0=0 **N_9=50kN**

$\sum y = 0$ T_9+0=0 ; **T_9=0**

$\sum M = 0$ -M_9+0=0; **M_9=0**

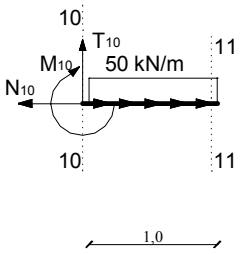


Presjek 10-10

$\sum x = 0$ -N_{10}+50×1.0=0 **N_{10}=50kN**

$\sum y = 0$ T_{10}+0=0 ; **T_{10}=0**

$\sum M = 0$ -M_{10}+0=0; **M_{10}=0**

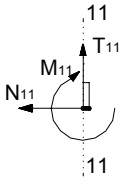


Presjek 11-11

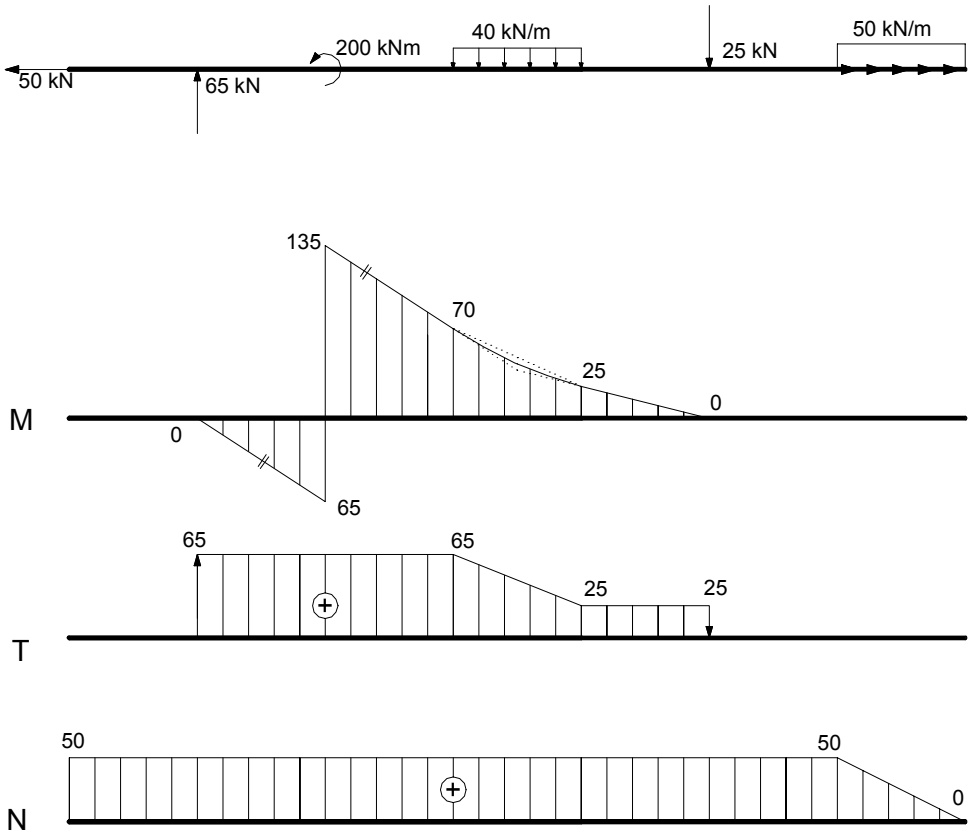
$\sum x = 0$ -N_{11}+50×0=0 **N_{11}=0**

$\sum y = 0$ T_{11}+0=0 ; **T_{11}=0**

$\sum M = 0$ -M_{11}+0=0; **M_{11}=0**



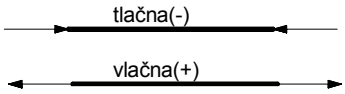
Dijagrami :



objašnjenje dijagrama :

N dijagram

- uzdužna sila može biti tlačna ili vlačna.
- po konvenciji: vlačna sila je pozitivna(+), a tlačna negativna(-)



- pozitivne vrijednosti(+) crtamo gore, a negativne(-) dolje
- na dijagram **N** utječu samo uzdužna opterećenja **(P_N i f** *(u smjeru osi štapa*)**)**, **N** dijagram je potpuno neovisan o dijagramima M i T

koncetrirana sila u smjeru osi štapa P_N – uzrokuje skok **N** u dijagramu, a između sila dijagram je konstantan

kontinuirano opterećenje u smjeru osi štapa – uzrokuje **linearan** pad ili rast uzdužne sile ovisno o smjeru opterećenja.

T dijagram

- poprečna sila može biti pozitivna(+) ili negativna(-)
- pozitivne vrijednosti(+) crtamo gore, a negativne(-) dolje
- na dijagram **T** utječu samo poprečna opterećanja **(P_T i q** *(okomito na os štapa)***)**.

koncetrirana sila – uzrokuje skok u **T** dijagramu, između je dijagram konstantan.

kontinuirano opterećenje – uzrokuje linearan pad ili rast poprečne sile ovisno o smjeru opterećenja.

M dijagram

- pozitivne vrijednosti(+) **M** crtamo na **vlačnoj** strani (**ne upisujemo predznak**)
- na dijagram **M** utječu poprečna opterećenja i koncetrirani momenti(**P_T i q i M**).

koncetrirani moment – uzrokuje skok u **M** dijagramu, a između sila dijagram je linearan (*nagib lijevo i desno od presjeka je jednak*)

koncetrirana sila – uzrokuje **''lom''** dijagrama, linearan pad ili rast dijagrama, a u ovisnosti o veličini sile mijenja se nagib pravca *(veća sila - strmiji pravac)*

kontinuirano opterećenje okomito na os štapa – daje **parabolu** u **M** dijagramu, ovisno o veličini opterećenja mijenjaju se karakteristike parabole.

opaska :

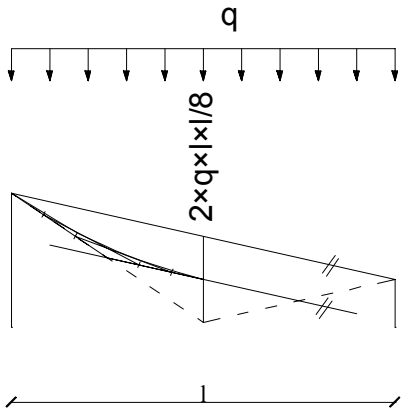
dijagrami

M	***T***
parabola	linearan
linearan	konstantan
konstantan	nema

- diferencijalni odnosi

$$\frac{dM_{(x)}}{dx} = T_x \qquad \frac{dT_{(x)}}{dx} = -q_{(x)}$$

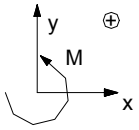
konstrukcija:



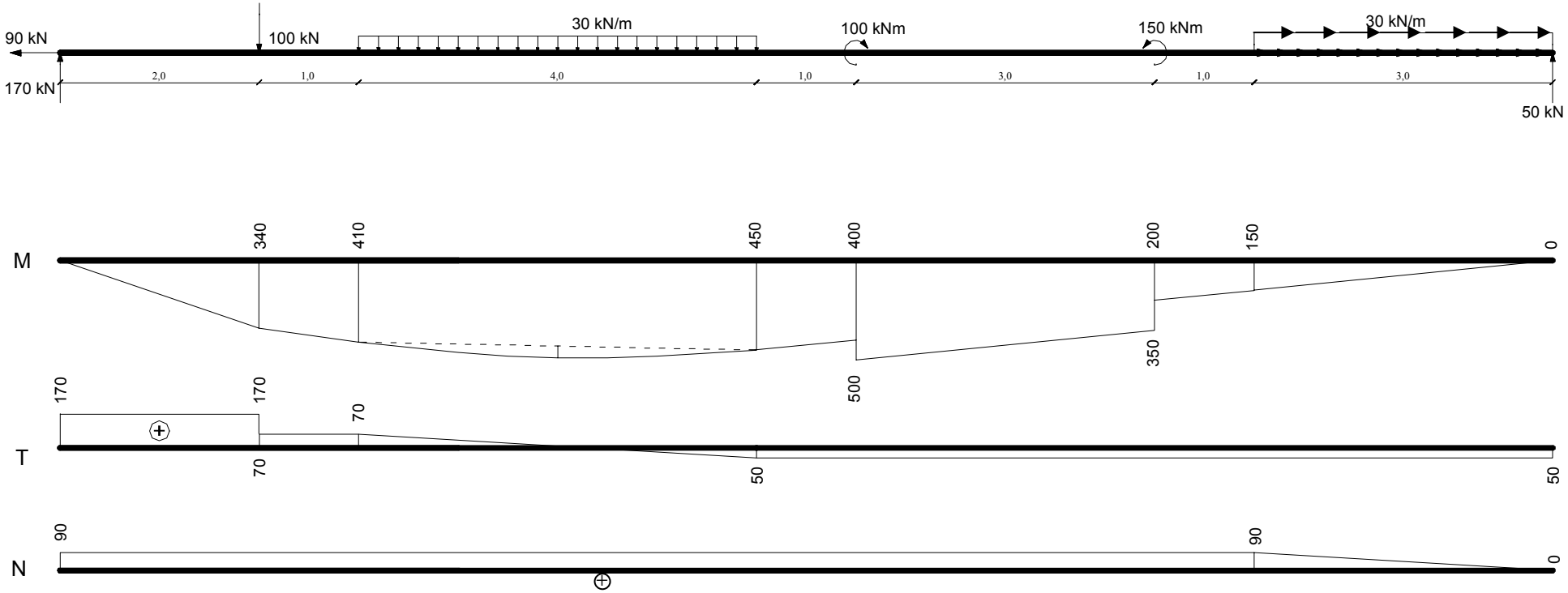
OPASKA!

Smjerovi unutarnjih sila su dogovorena konvencija, ali pri postavljanju jednadžbi ravnoteže mi sami biramo **''pozitivne smjerove'' !!!**

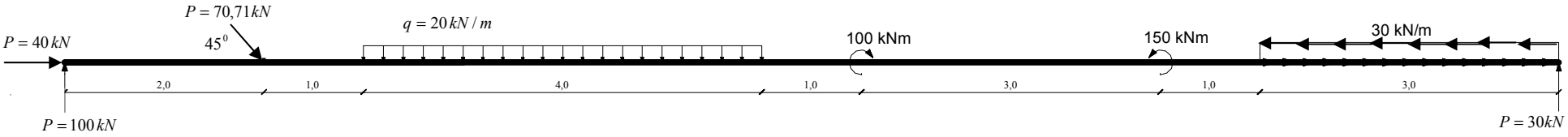
u svim primjerima izabrani pozitivni smjerovi su :

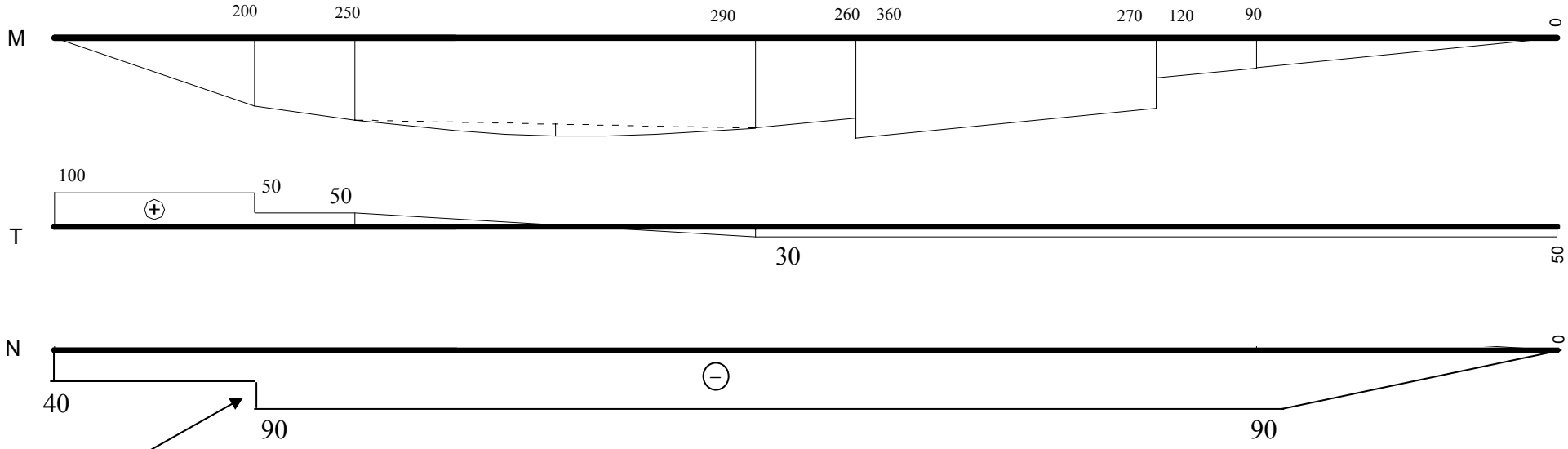


Rekapitulacija 1:



Rekapitulacija 2:

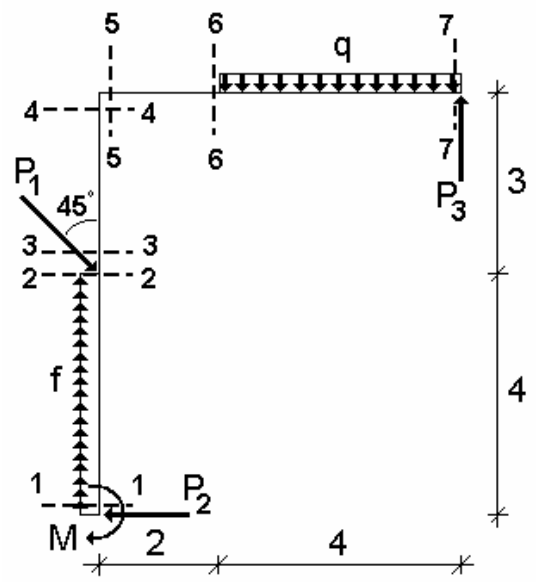




ODREĐIVANJE UNUTARNJIH SILA – složeniji sustavi

Zadatak 32:

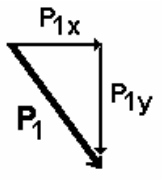
Za prikazani uravnoteženi sustav odrediti nacrtati dijagrame unutarnjih sila M,T i N



$P_1 = 70$ kN ,
$P_2 = 50$ kN ,
$P_3 = 90$ kN,
M = 20 kNm ,
q = 20 kN/m i
f = 10 kN/m

Postupak:

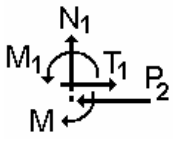
$P_{1x} = P_1 \cdot 0{,}707 = 50$ kN
$P_{1y} = P_1 \cdot 0{,}707 = 50$ kN



7 karakterističnih presjeka

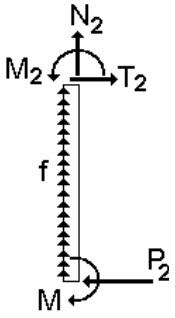
Presjek 1.

Σ x = 0 $T_1 - P_2 = 0$ **T_1** = 50 kN
Σ y = 0 **N_1** = 0
$\sum M_{(1)} = 0$ $M_1 - M = 0$ **M_1**= 20 kNm
(vlak s desne strane)



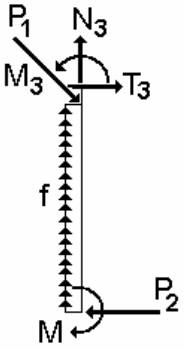
Presjek 2.

Σ x = 0 **T_2** = 50 kN
Σ y = 0 $N_2 + f \cdot 4 = 0$ **N_2** = - 40 kN
$\sum M_{(2)} = 0$ $M_2 - M - P_2 \cdot 4 = 0$ **M_2**= 220 kNm
(vlak s desne strane)



Presjek 3.

Σ y = 0 $N_3 - P_{1y} + f \cdot 4 = 0$ **N_3** = 10 kN
Σ x = 0 $T_3 - P_2 + P_{1x} = 0$ **T_3** = 0
$\sum M_{(3)} = 0$ **M_3** = M_2 = 220 kNm
(vlak s desne strane)



Presjek 4.

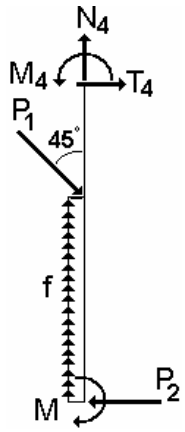
Σ x = 0 $\quad \mathbf{T_4} = 0$

Σ y = 0 $\quad \mathbf{N_4} = N_3 = 10$ kN

$\sum \mathbf{M_{(4)} = 0} \quad M_4 - M - P_2 \cdot 7 + P_{1x} \cdot 3 = 0$

$\mathbf{M_4} = 220$ kNm

(vlak s desne strane)



Presjek 5.

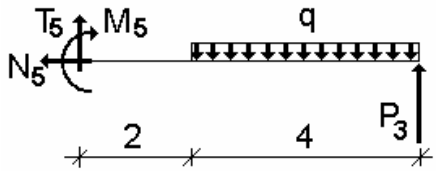
Σ x = 0 $\quad \mathbf{N_5} = 0$

Σ y = 0 $\quad T_5 + P_3 - q \cdot 4 = 0 \quad \mathbf{T_5} = -10$ kN

$\sum \mathbf{M_{(5)} = 0} \quad - M_5 - q \cdot 4 \cdot 4 + P_3 \cdot 6 = 0$

$\mathbf{M_5} = 220$ kNm

(vlak s donje strane)



Presjek 6.

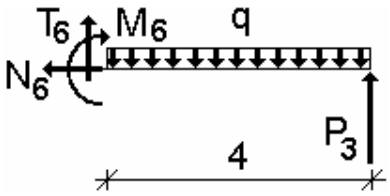
Σ x = 0 $\quad \mathbf{N_6} = 0$

Σ y = 0 $\quad T_6 - q \cdot 4 + P_3 = 0 \quad \mathbf{T_6} = -10$ kN

$\sum \mathbf{M_{(6)} = 0} \quad - M_6 - q \cdot 4 \cdot 2 + P_3 \cdot 4 = 0$

$\mathbf{M_6} = 200$ kNm

(vlak s donje strane)

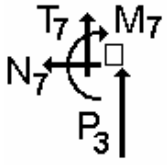


Presjek 7.

Σ x = 0 $\quad \mathbf{N_7} = 0$

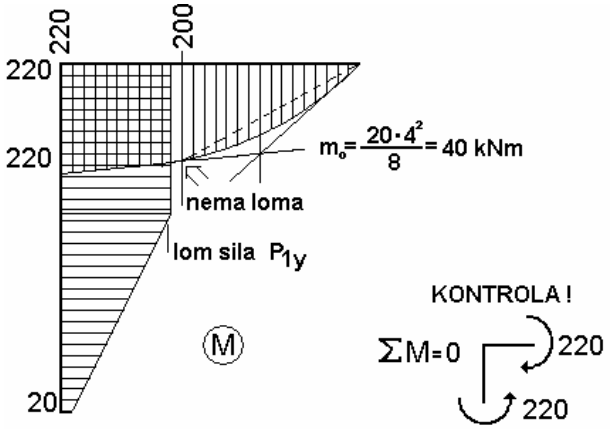
Σ y = 0 $\quad \mathbf{T_7} = -90$ kN

$\sum \mathbf{M_{(7)} = 0} \quad \mathbf{M_7} = 0$

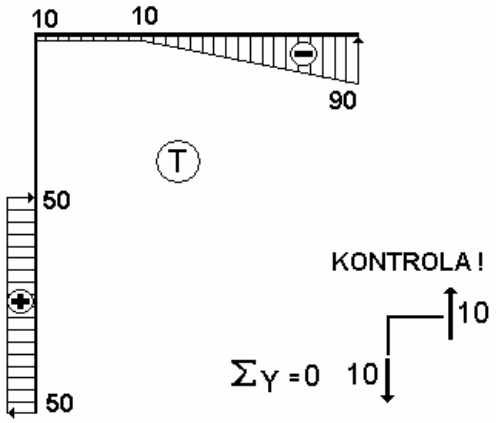


Dijagrami unutarnjih sila M,T i N

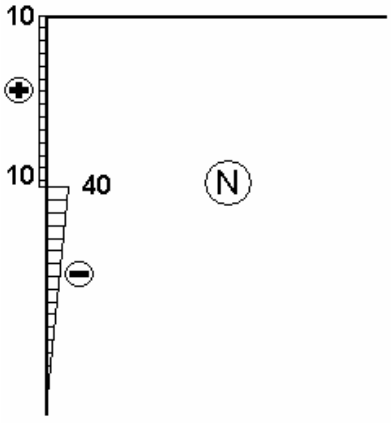
M - dijagram :



T – dijagram :

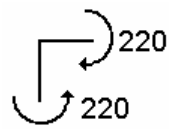


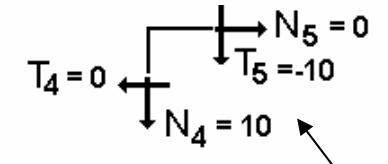
N – dijagram :



KONTROLA ! (u čvoru)

Σ M = 0





Σ x = 0

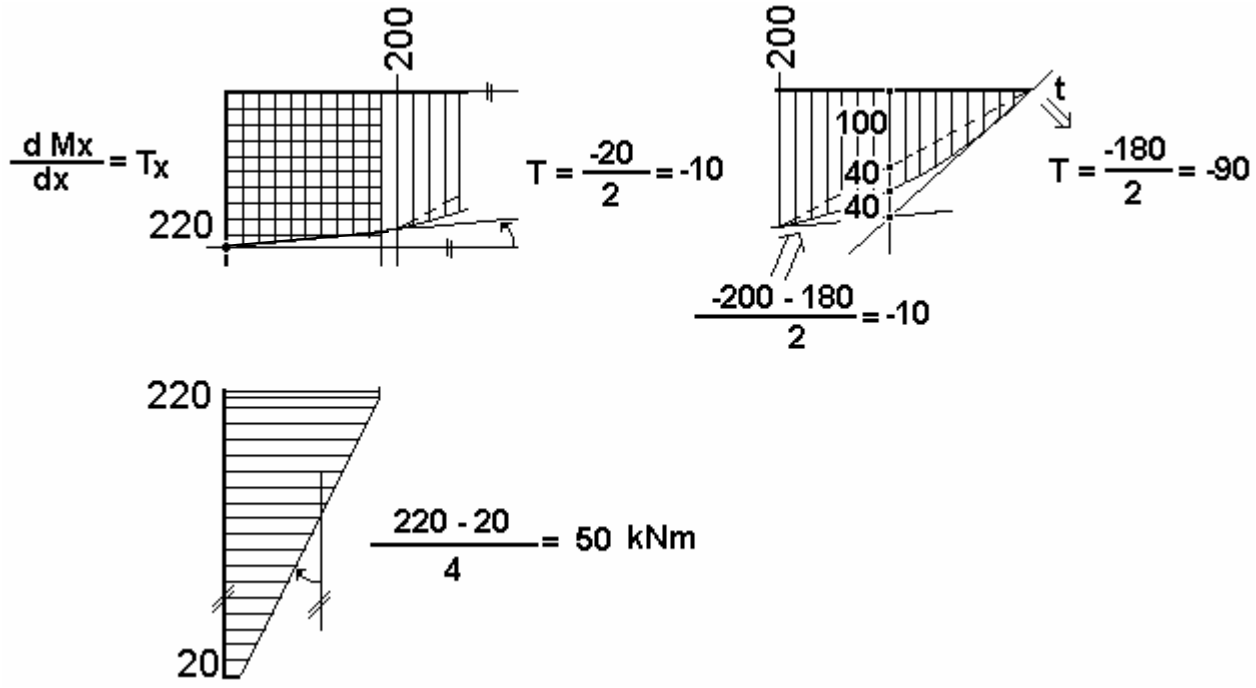
Σ y = 0

ovdje su nacrtani pozitivi smjerovi unutarnjih sila

Diferencijalni odnosi !

⇒ **momentni** dijagram ⇒ **parabole** ispod **q**
⇒ **skok** kod koncentriranog **M**
⇒ linearan na mjestu **f** te između koncentriranih sila

⇒ dijagram **poprečnih sila** ⇒ **linearan** ispod **q**
⇒ **nula** na konstantnom dijelu M dijagrama
⇒ **skok** kod sile **P**

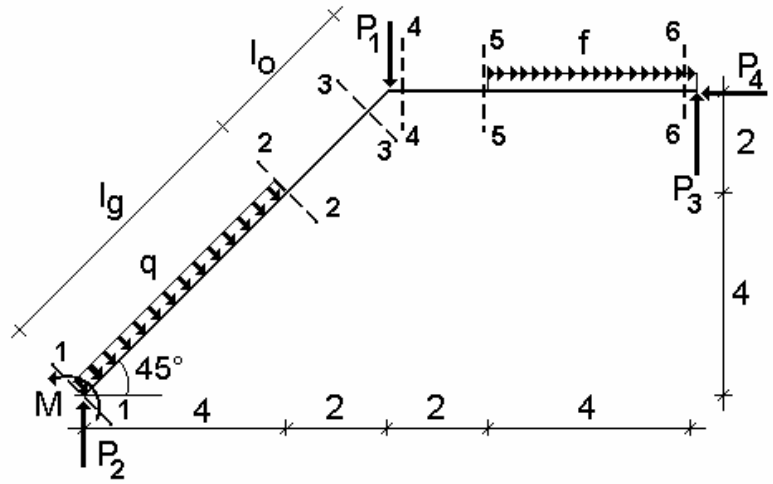


⇒ dijagram **uzdužnih sila** ⟶ **linearan** na dijelu **f**.

Zadatak 33:

Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N

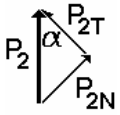
P_1 = 120 kN, P_2 = 220 kN, P_3 = 20 kN, P_4 = 160 kN , M = 240 kNm ,
q = 30 kN/m i f =10 kN/m



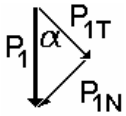
Postupak:

▪ 6 karakterističnih presjeka

$P_{2N} = P_2 \cdot \sin\alpha = 220 \cdot \sin 45° = 155{,}56$ kN
$P_{2T} = P_2 \cdot \cos\alpha = 220 \cdot \cos 45° = 155{,}56$ kN



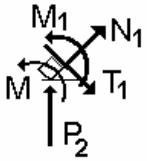
$P_{1T} = 120 \cdot 0{,}707 = 84{,}8$ kN
$P_{1N} = 120 \cdot 0{,}707 = 84{,}8$ kN



$l = l_g + l_o$ $\quad l = \sqrt{6^2 + 6^2} = 8{,}49$ m

$l_g = \sqrt{4^2 + 4^2} = 5{,}66$ m $\quad l_o = 2{,}83$ m

Presjek 1.



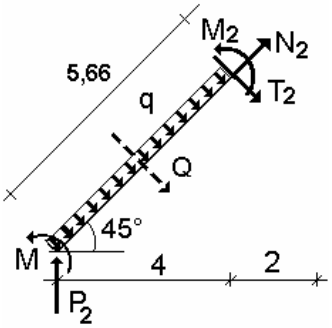
Σ N = 0 $\quad N_1 + P_{2N} = 0 \quad$ **N_1** = -155,56 kN
Σ T = 0 $\quad T_1 - P_{2T} = 0 \quad$ **T_1** = 155,56 kN
∑ $M_{(1)}$ = 0 $\quad M_1 + M = 0 \quad$ **M_1**= - M = - 240 kNm (vlak gore)

Presjek 2.

Σ N = 0 $\quad N_2 + P_{2N} = 0 \quad$ **N_2** = -155,56 kN
Σ T = 0 $\quad T_2 - P_{2T} + q \cdot l_g = 0$
T_2 = - 30 · 5,66 + 155,56 = -14,15 kN

∑ $M_{(2)}$ = 0 $\quad M_2 + M - P_2 \cdot 4 + q \cdot \frac{5{,}66^2}{2} = 0$

M_2= 160 kNm (vlak dolje)



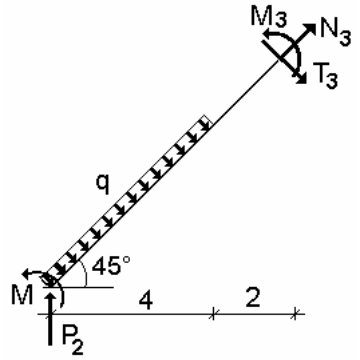
Presjek 3.

Σ N = 0 $N_3 + P_{2N} = 0$ **N_3** = - 155,56 kN

Σ T = 0 $T_3 - P_{2T} + q \cdot l_g = 0$ **T_3** = - 14,15 kN

$\sum$ **$M_{(3)}$ = 0** $M_3 + M - P_2 \cdot 6 + q \cdot 5{,}66 \cdot \left(\frac{5{,}66}{2} + l_o \right) = 0$

M_3 = 120 kNm (vlak dolje)

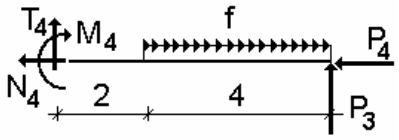


Presjek 4.

Σ x = 0 $-N_4 - P_4 + f \cdot 4 = 0$ **N_4** = -160 + 10 · 4 = -120 kN

Σ y = 0 $T_4 + P_3 = 0$ **T_4** = - 20 kN

$\sum$ **$M_{(4)}$ = 0** $-M_4 + P_3 \cdot 6 = 0$ **M_4** = 120 kNm (vlak dolje)

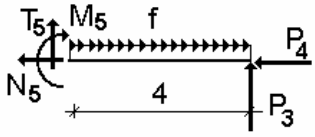


Presjek 5.

Σ x = 0 $-N_5 - P_4 + f \cdot 4 = 0$ **N_5** = -120 kN

Σ y = 0 $P_3 + T_5 + = 0$ **T_5** = -20 kN

$\sum$ **$M_{(5)}$ = 0** $-M_5 + P_3 \cdot 4 = 0$ **M_5** = 80 kNm (vlak dolje)

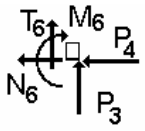


Presjek 6.

Σ x = 0 $-N_6 - P_4 = 0$ **N_6** = -160 kN

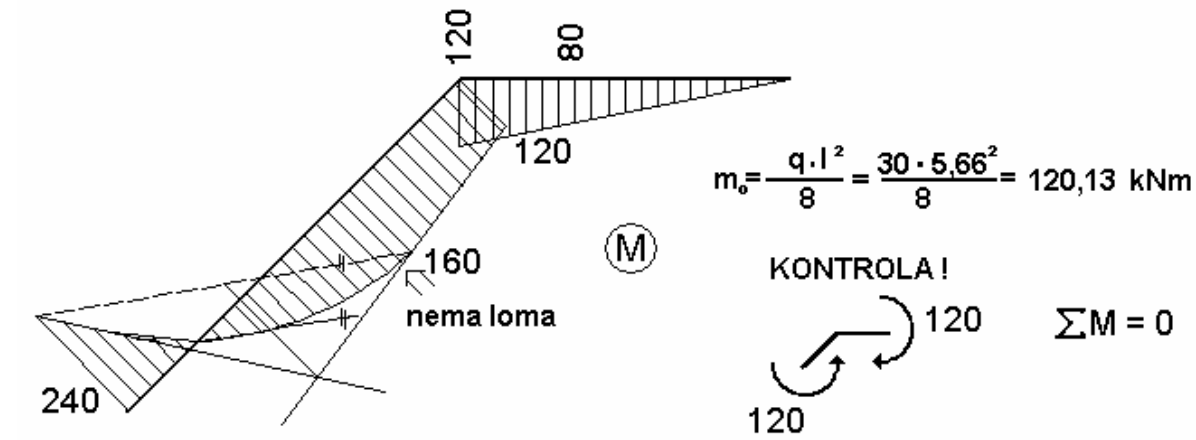
Σ y = 0 $T_6 + P_3 = 0$ **T_6** = -20 kN

$\sum$ **$M_{(6)}$ = 0** **M_6** = 0

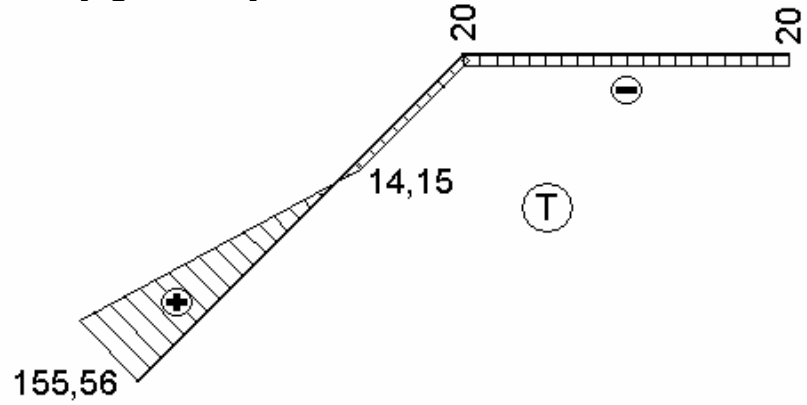


Dijagrami unutarnjih sila M,T i N

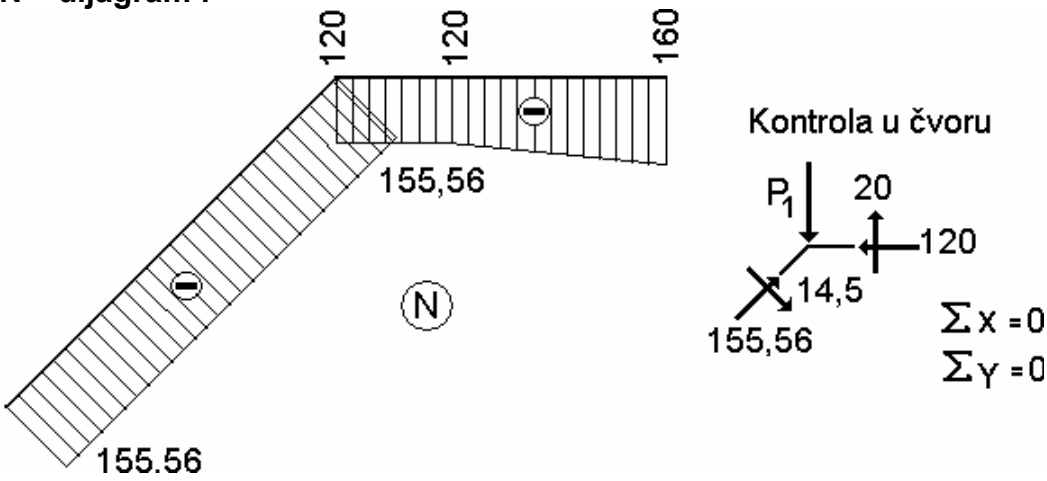
M - dijagram : Mj. sila 1 cm = 100 kNm



T – dijagram : Mj.sila 1cm = 100 kN



N – dijagram :

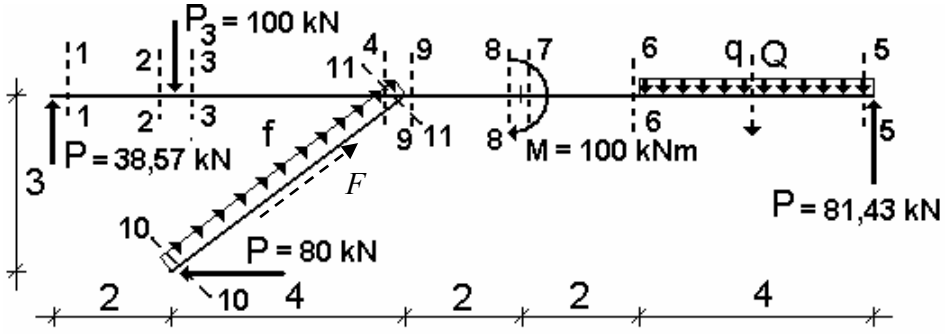


Zadatak 34: (kontinuirano opterećenje f u smjeru kosog štapa !!!!)

Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N

f = 20 kN/m→ **F** = 100 kN $\Rightarrow$ $\mathbf{F_x = f \cdot l_x}$=20 ·4 = 80 kN; $\mathbf{F_y = f \cdot l_y}$=20 ·3 = 60 kN; !!!!!

q = 20 kN/m→ **Q**= 80 kN



Postupak:

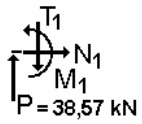
- **11 karakterističnih presjeka**

Presjek 1.

Σ x = 0 $\mathbf{N_1}$ = 0

Σ y = 0 38,57 - T_1 = 0 $\mathbf{T_1}$ = 38,57 kN

$\sum \mathbf{M_{(1)} = 0}$ $\mathbf{M_1}$= 0

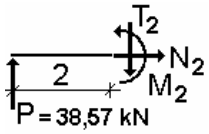


Presjek 2.

Σ x = 0 $\mathbf{N_2}$ = 0

Σ y = 0 38,57 - T_2 = 0 $\mathbf{T_2}$ = 38,57 kN

$\sum \mathbf{M_{(2)} = 0}$ $M_2 - 38,57 \cdot 2 = 0$ $\mathbf{M_2}$ = 77,14 kNm (vlak dolje)

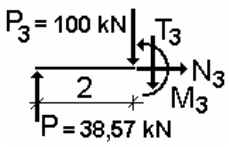


Presjek 3.

Σ x = 0 $\mathbf{N_3}$ = 0 kN

Σ y = 0 38,57 – 100 - T_3 = 0 $\mathbf{T_3}$ = - 61,43 kN

$\sum \mathbf{M_{(3)} = 0}$ $M_3 - 38,57 \cdot 2 = 0$ $\mathbf{M_3}$ = 77,14 kNm



Presjek 4.

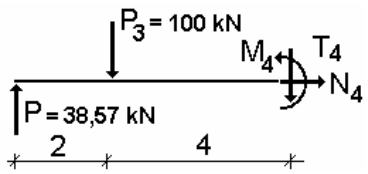
Σ x = 0 $\mathbf{N_4}$ = 0 kN

Σ y = 0 38,57 – 100 - T_4 = 0

$\mathbf{T_4}$ = - 61,43 kN

$\sum \mathbf{M_{(4)} = 0}$ $M_4 - 38,57 \cdot 6 + 100 \cdot 4 = 0$

$\mathbf{M_4}$ = - 168,58 kNm (vlak gore)

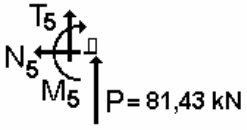


Presjek 5.

Σ x = 0 $\mathbf{N_5}$ = 0

Σ y = 0 T_5 + 81,43 = 0 $\mathbf{T_5}$ = - 81,43 kN

$\sum \mathbf{M_{(5)} = 0}$ $\mathbf{M_5}$ = 0



Presjek 6.

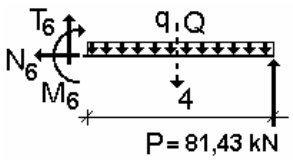
Σ x = 0 $\mathbf{N_6}$ = 0

Σ y = 0 T_6 + 81,43 – 20 · 4 = 0

$\mathbf{T_6}$ = -1,43 kN

$\sum \mathbf{M_{(6)} = 0}$ $-M_6 - q \cdot 4 \cdot 2 + 81,43 \cdot 4 = 0$

$\mathbf{M_6}$ = 165,72 kNm (vlak dolje)



Presjek 7.

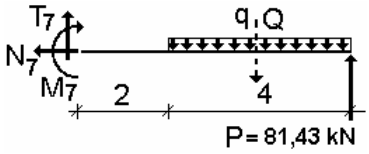
Σ x = 0 $\quad N_7 = 0$

Σ y = 0 $\quad T_7 + 81{,}43 - q \cdot 4 = 0$

$\quad T_7 = -1{,}43$ kN

$\sum M_{(7)} = 0 \quad -M_7 - q \cdot 4 \cdot 4 + 81{,}43 \cdot 6 = 0$

$\quad M_7 = 168{,}58$ kNm (vlak dolje)



Presjek 8.

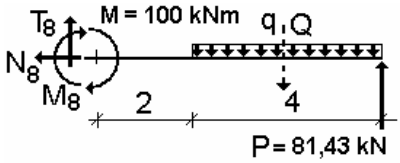
Σ x = 0 $\quad N_8 = 0$ kN

Σ y = 0 $\quad T_8 - q \cdot 4 + 81{,}43 = 0$

$\quad T_8 = -1{,}43$ kN

$\sum M_{(8)} = 0 \quad -M_8 - q \cdot 4 \cdot 4 + 81{,}43 \cdot 6 - 100 = 0$

$\quad M_8 = 68{,}58$ kNm (vlak dolje)



Presjek 9.

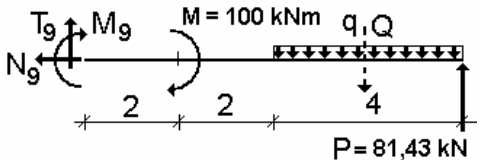
Σ x = 0 $\quad N_9 = 0$ kN

Σ y = 0 $\quad T_9 + 81{,}43 - q \cdot 4 = 0$

$\quad T_9 = -1{,}43$ kN

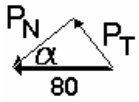
$\sum M_{(9)} = 0 \quad -M_9 - 100 - q \cdot 4 \cdot 6 + 81{,}43 \cdot 8 = 0$

$\quad M_9 = 71{,}44$ kNm (vlak dolje)



Presjek 10.

$tg\alpha = \frac{3}{4} \Rightarrow \alpha = 36{,}87^\circ$

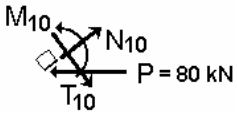


$P_T = P \cdot \sin\alpha = 80 \cdot 0{,}6 = 48$ kN

$P_N = P \cdot \cos\alpha = 80 \cdot 0{,}8 = 64$ kN

Σ N = 0 $\quad N_{10} - P_N = 0 \quad$ **N_{10}** = 64 kN

Σ T = 0 $\quad T_{10} - P_T = 0 \quad$ **T_{10}** = 48 kN



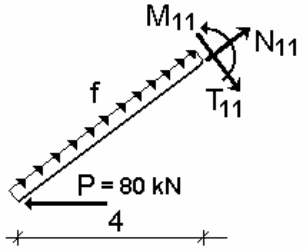
Presjek 11.

Σ N = 0 $\quad N_{11} + f \cdot l - P_N = 0 \quad$ **N_{11}**$= - 20 \cdot 5 + 64 = - 36$ kN

Σ T = 0 $\quad T_{11} - P_T = 0 \quad$ **T_{11}** = 48 kN

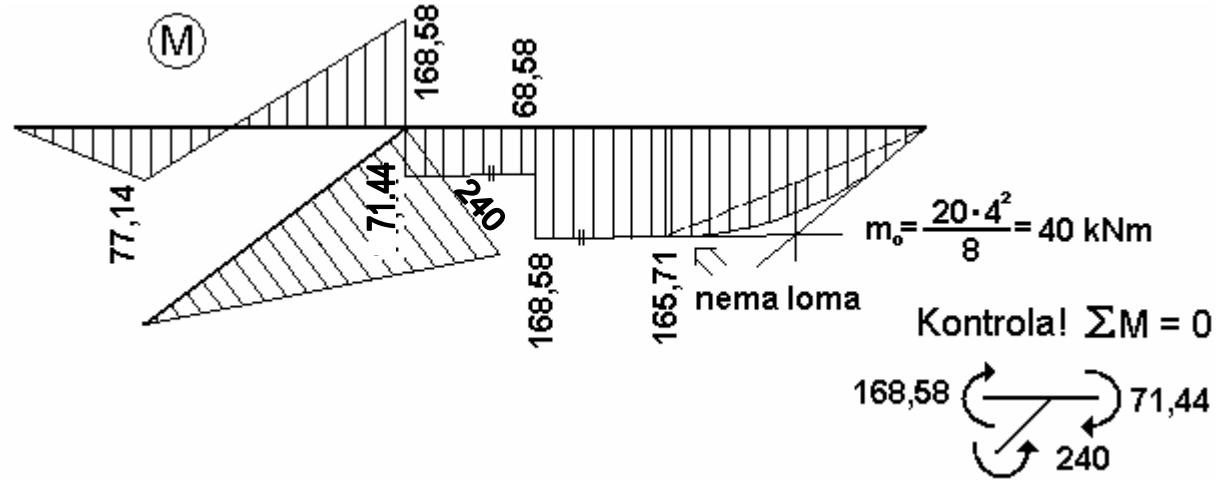
$\sum M_{(11)} = 0 \quad M_{11} - 80 \cdot 3 = 0 \quad$ **M_{11}** = 240 kNm

(vlak dolje desno)

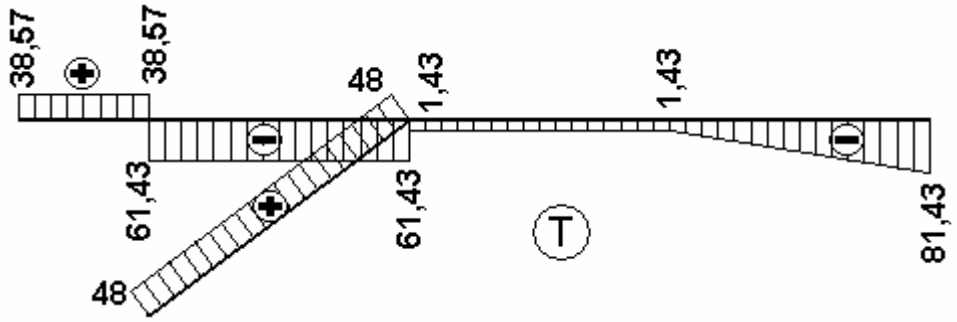


Dijagrami unutarnjih sila M,T i N

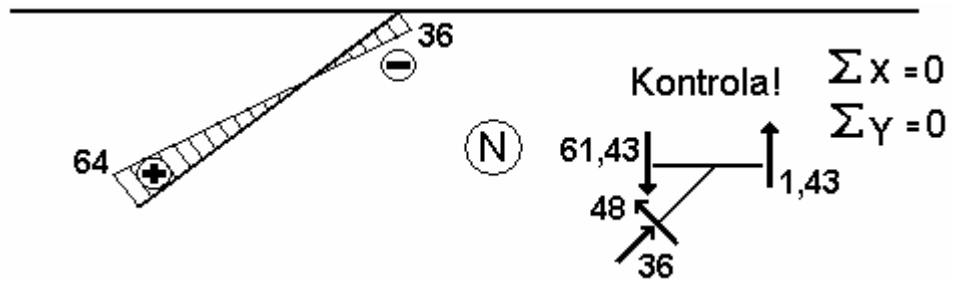
M – dijagram : Mj.sila 1cm = 100 kNm



T – dijagram : Mj.sila 1cm = 100 kNm

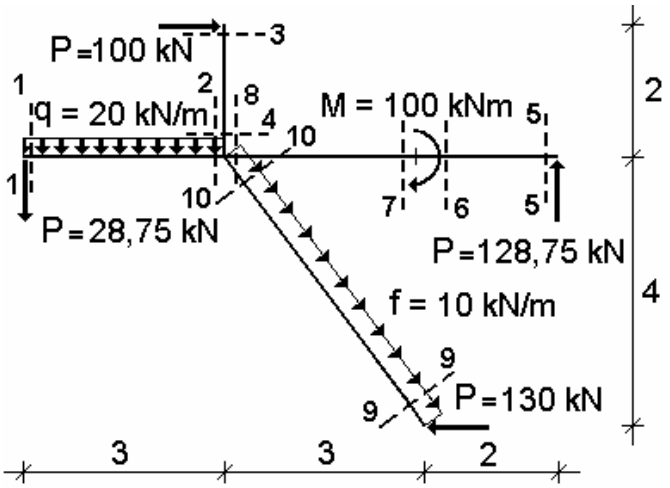


N – dijagram :



Zadatak 35: (kontinuirano opterećenje f u smjeru kosog štapa !!!!)

Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N



Q = 20 · 3 = 60 kN; **F** = 10 · 5 = 50 kN; $\Rightarrow$ $\mathbf{F_x = f \cdot l_x}$=10 ·3 = 30 kN; !!!!!!!!!!!!!!

$\mathbf{F_y = f \cdot l_y}$=10 ·4 = 40 kN;!!!!!!!!!!!!!!!

Postupak:

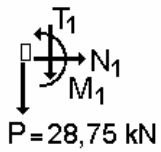
- **10 karakterističnih presjeka**

Presjek 1.

Σ x = 0 $\mathbf{N_1}$ = 0

Σ y = 0 $\mathbf{T_1}$ = - 28,75 kN

$\sum \mathbf{M_{(1)} = 0}$ $\mathbf{M_1}$= 0



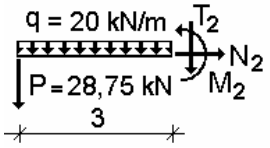
Presjek 2.

Σ x = 0 $\mathbf{N_2}$ = 0

Σ y = 0 $-T_2 - q \cdot 3 - 28{,}75 = 0$ $\mathbf{T_2}$ = - 88,75 kN

$\sum \mathbf{M_{(2)} = 0}$ $M_2 + 28{,}75 \cdot 3 + q \cdot 3 \cdot 1{,}5 = 0$

$\mathbf{M_2}$ = - 176,25 kNm (vlak gore)

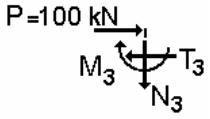


Presjek 3.

Σ x = 0 $100 - T_3 = 0$ $\mathbf{T_3}$ = 100 kN

Σ y = 0 $\mathbf{N_3}$ = 0

$\sum \mathbf{M_{(3)} = 0}$ $\mathbf{M_3}$ = 0

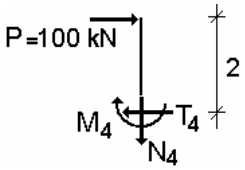


Presjek 4.

Σ x = 0 $\mathbf{T_4}$ = 100 kN

Σ y = 0 $\mathbf{N_4}$ = 0

$\sum \mathbf{M_{(4)} = 0}$ $-M_4 - 100 \cdot 2 = 0$ $\mathbf{M_4}$ = -200 kNm (vlak lijevo)

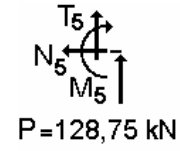


Presjek 5.

Σ x = 0 $\mathbf{N_5}$ = 0

Σ y = 0 $\mathbf{T_5}$ = - 128,75 kN

$\sum \mathbf{M_{(5)} = 0}$ $\mathbf{M_5}$ = 0



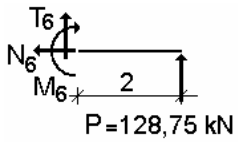
Presjek 6.

Σ x = 0 $\mathbf{N_6} = 0$

Σ y = 0 $\mathbf{T_6} = -128{,}75$ kN

$\sum \mathbf{M_{(6)} = 0}$ $-M_6 + 128{,}75 \cdot 5 = 0$

$\mathbf{M_6} = 257{,}50$ kNm (vlak dolje)



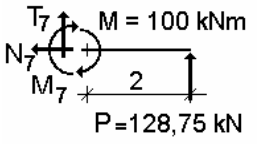
Presjek 7.

Σ x = 0 $\mathbf{N_7} = 0$

Σ y = 0 $\mathbf{T_7} = -128{,}75$ kN

$\sum \mathbf{M_{(7)} = 0}$ $-M_7 - M + 128{,}75 \cdot 2 = 0$

$\mathbf{M_7} = 157{,}50$ kNm



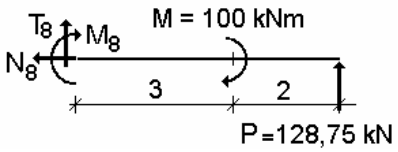
Presjek 8.

Σ x = 0 $\mathbf{N_8} = 0$ kN

Σ y = 0 $\mathbf{T_8} = -128{,}75$ kN

$\sum \mathbf{M_{(8)} = 0}$ $-M_8 - M + 128{,}75 \cdot 5 = 0$

$\mathbf{M_8} = 543{,}75$ kNm

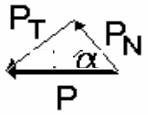


Presjek 9.

$$tg\alpha = \frac{4}{3} \Rightarrow \alpha = 53{,}13^\circ$$

$P_T = P \cdot \sin\alpha = 130 \cdot 0{,}8 = 104$ kN

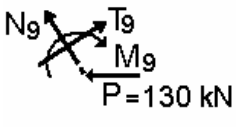
$P_N = P \cdot \cos\alpha = 130 \cdot 0{,}6 = 78$ kN



$\sum \mathbf{N = 0}$ $N_9 + P_N = 0$ $\mathbf{N_9} = -78$ kN

$\sum \mathbf{T = 0}$ $T_9 - P_T = 0$ $\mathbf{T_9} = 104$ kN

$\sum \mathbf{M_{(9)} = 0}$ $\mathbf{M_9} = 0$



Presjek 10.

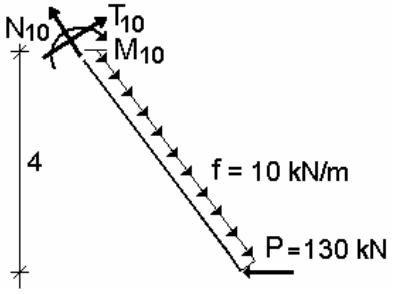
$\sum \mathbf{N = 0}$ $N_{10} - f \cdot L + P_N = 0$

$\mathbf{N_{10}} = 10 \cdot 5 - 78 = -28$ kN

$\sum \mathbf{T = 0}$ $\mathbf{T_{10}} = 104$ kN

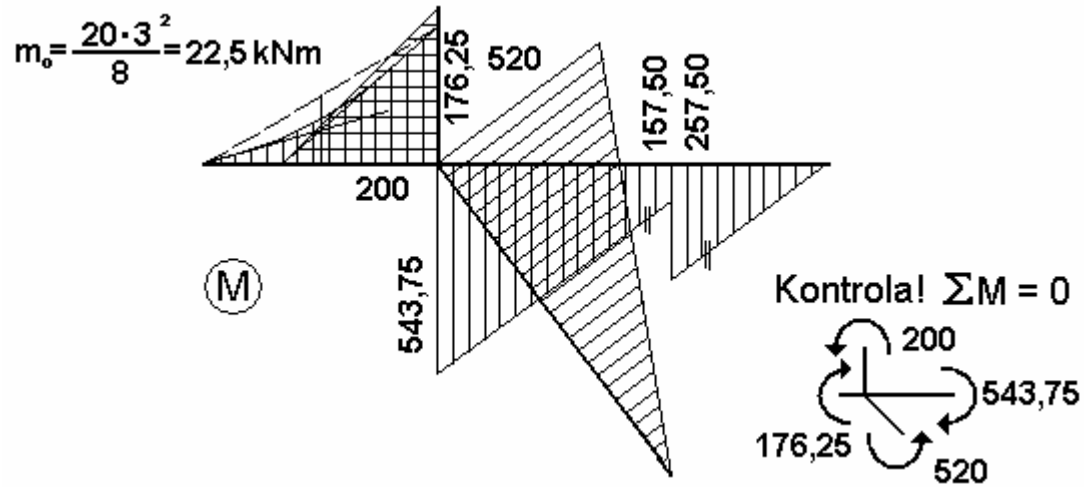
$\sum \mathbf{M_{(10)} = 0}$ $-M_{10} - 130 \cdot 4 = 0$

$\mathbf{M_{10}} = -520$ kNm (vlak desno)

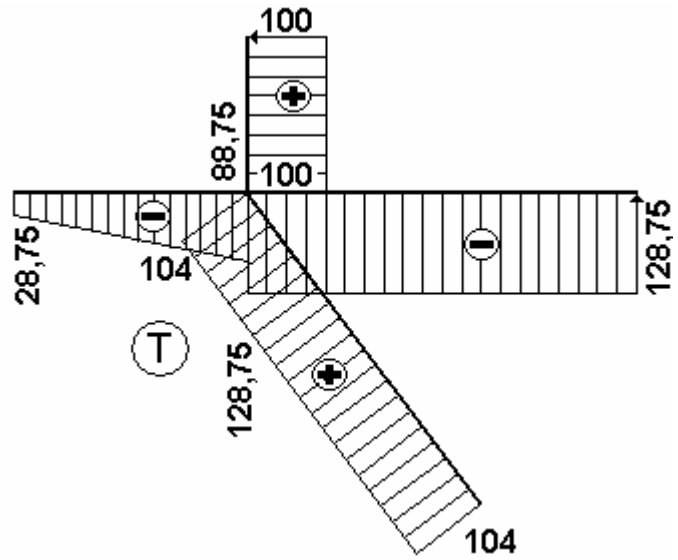


Dijagrami unutarnjih sila M,T i N

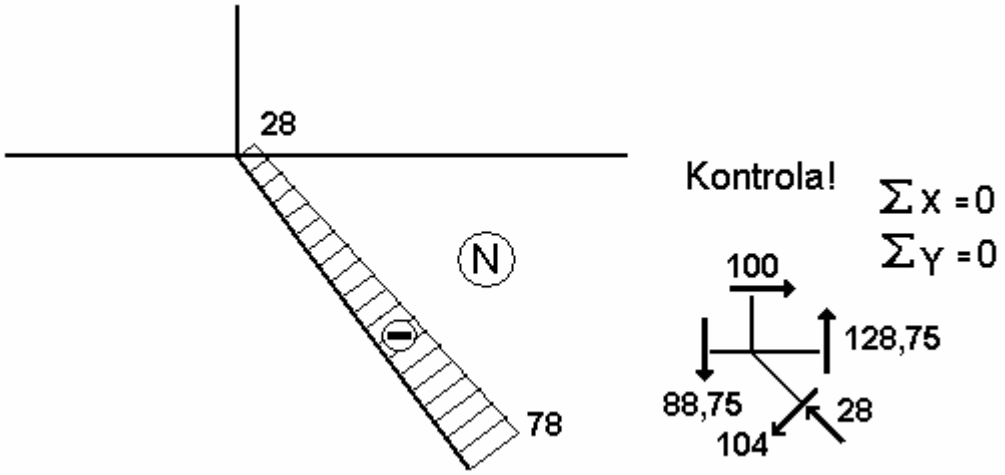
M – dijagram : Mj.sila 1cm = 100 kNm



T – dijagram : Mj.sila 1cm = 100 kNm

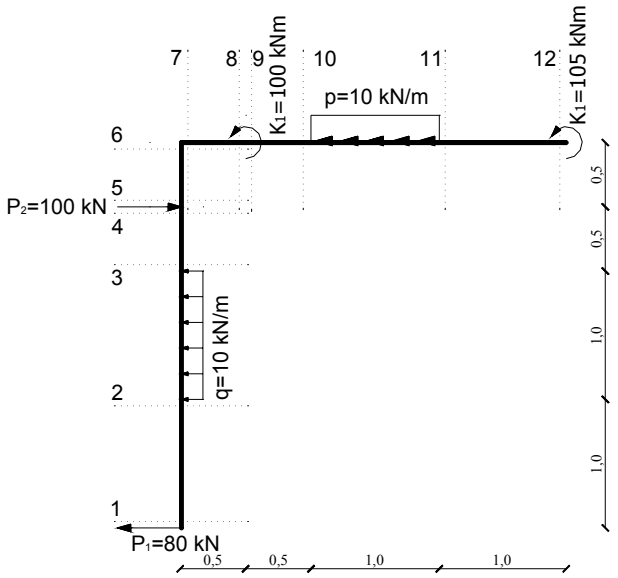


N – dijagram :

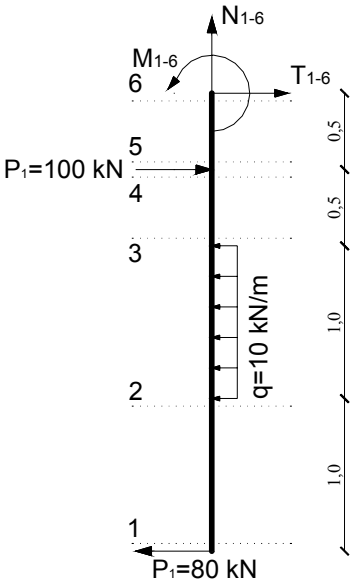


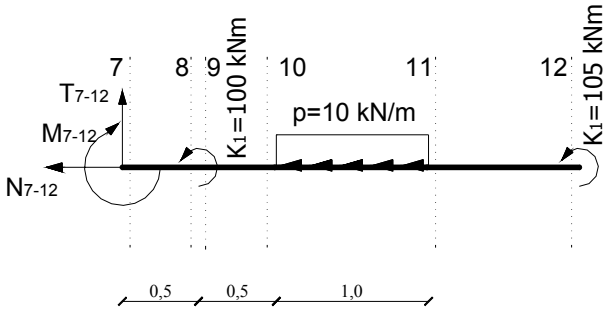
Zadatak 36:

Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N



Postupak*:*
smjerovi unutarnjih sila za pojedine presjeke :





Proračun unutarnjih sila:

Presjek 1-1

$\sum x = 0$	$-T_1+80=0$;	**$T_1=80$ kN**
$\sum y = 0$	$N_1+0=0$	**$N_1=0$**
$\sum M = 0$	$M_1+0=0$;	**$M_1=0$**

Presjek 2-2

$\sum x = 0$	$-T_2+80=0$;	**$T_2=80$ kN**
$\sum y = 0$	$N_2+0=0$	**$N_2=0$**
$\sum M = 0$	$M_2-80\times1=0$;	**$M_2=80$ kNm**

Presjek 3-3

$\sum x = 0$	$-T_3+80+10\times1=0$;	**$T_2=90$ kN**
$\sum y = 0$	$N_3+0=0$	**$N_3=0$**
$\sum M = 0$	$M_3-80\times2-10\times1\times0.5=0$;	**$M_3=165$ kNm**

Presjek 4-4

$\sum x = 0$	$-T_4+80+10\times1=0$;	**$T_4=90$ kN**
$\sum y = 0$	$N_4+0=0$	**$N_4=0$**
$\sum M = 0$	$M_4-80\times2.5-10\times1\times1.0=0$;	**$M_4=210$ kNm**

Presjek 5-5

$\sum x = 0$	$T_5+80+10\times1-100=0$;	**$T_5=-10$ kN**
$\sum y = 0$	$-N_5+0=0$	**$N_5=0$**
$\sum M = 0$	$M_5-80\times2.5-10\times1\times1.0+100\times0=0$;	**$M_5=210$ kNm**

Presjek 6-6

$\sum x = 0$	$T_6+80+10\times1-100=0$;	**$T_6=-10$ kN**
$\sum y = 0$	$-N_6+0=0$	**$N_6=0$**
$\sum M = 0$	$-M_6-80\times3-10\times1\times1.5+100\times0.5=0$;	**$M_6=205$ kNm**

Presjek 12-12

$\sum x = 0$	$-N_{12}+0=0$	**$N_{12}=0$**
$\sum y = 0$	$T_{12}+0=0$;	**$T_{12}=0$**
$\sum M = 0$	$-M_{12}+105=0$;	**$M_{12}=105$ kNm**

Presjek 11-11

$\sum x = 0$	$-N_{11}+0=0$	**$N_{11}=0$**
$\sum y = 0$	$T_{11}+0=0$;	**$T_{11}=0$**
$\sum M = 0$	$-M_{11}+105=0$;	**$M_{11}=105$ kNm**

Presjek 10-10

$\sum x = 0$	$-N_{10}-10\times1.0=0$	**$N_{10}=-10$kN**
$\sum y = 0$	$T_{10}+0=0$;	**$T_{10}=0$**
$\sum M = 0$	$-M_{10}+105=0$;	**$M_{10}=105$ kNm**

Presjek 9-9

$\sum x = 0$	$-N_{9}-10\times1.0=0$	**$N_{9}=-10$kN**
$\sum y = 0$	$T_{9}+0=0$;	**$T_{9}=0$**
$\sum M = 0$	$-M_{9}+105=0$;	**$M_{9}=105$ kNm**

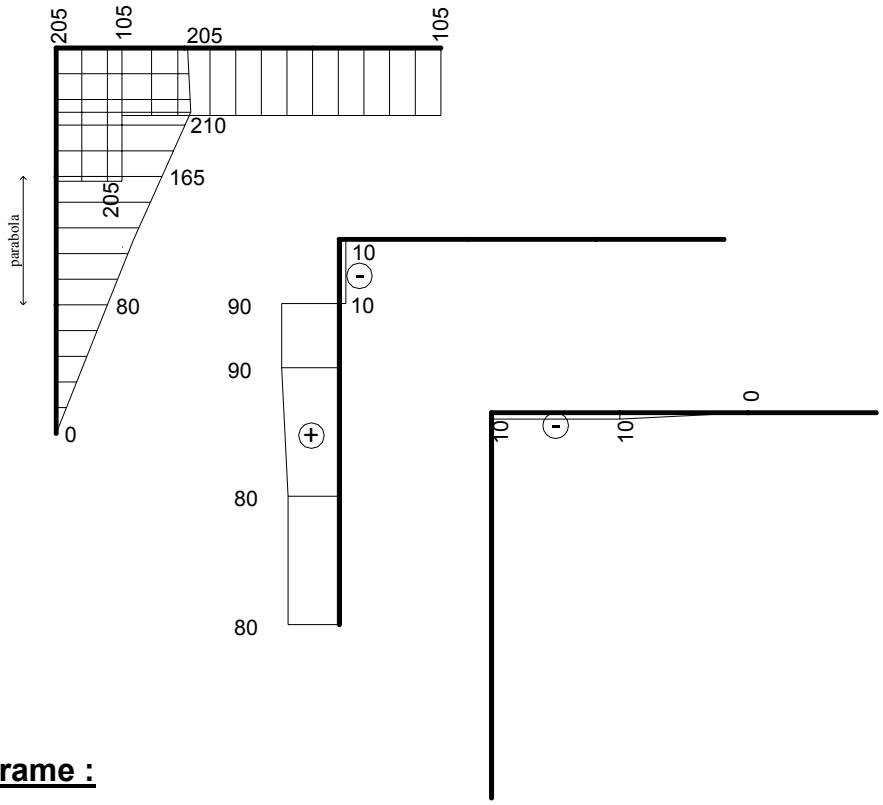
Presjek 8-8

$\sum x = 0$	$-N_{8}-10\times1.0=0$	**$N_{8}=-10$kN**
$\sum y = 0$	$T_{8}+0=0$;	**$T_{8}=0$**
$\sum M = 0$	$-M_{8}+105+100=0$;	**$M_{8}=205$ kNm**

Presjek 7-7

$\sum x = 0$	$-N_{7}-10\times1.0=0$	**$N_{7}=-10$kN**
$\sum y = 0$	$T_{7}+0=0$	**$T_{7}=0$**
$\sum M = 0$	$-M_{7}+105=0$;	**$M_{7}=205$ kNm**

Dijagrami :



<u>komentari uz dijagrame :</u>

M

ravnoteža momenata – primjećujemo da u spoju grede i stupa imamo momente jednakog iznosa, suprotnog smjera tj. imamo ravnotežu čvora

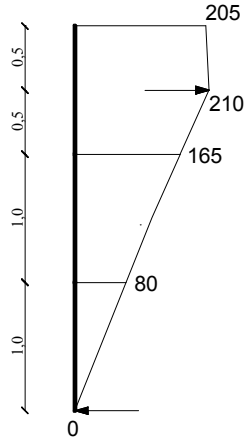
N i T

ravnoteža sila u čvoru :

$$\sum x = 0 \qquad \sum y = 0$$

<u>Opaska</u> : *<u>poprečne sile stupa postaju uzdužne sile grede</u>*
<u>poprečne sile grede postaju uzdužne sile stupa</u>(u ovom slučaju su 0)

<u>kako možemo iz dijagrama momenata odrediti sile koje djeluju:</u>

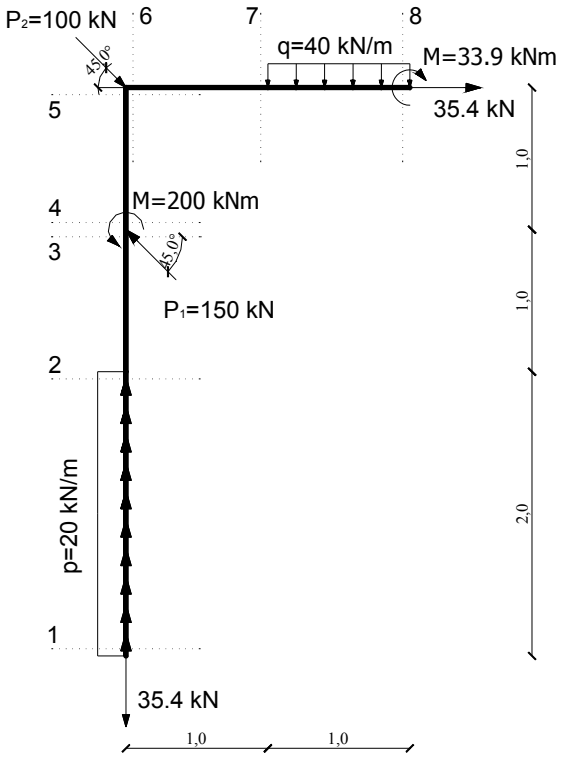


(210-205)/0.5 + (210-165)/0.5 = **<u>P_2 =100 kN</u>**

(80-0)/1.0 = **<u>P_1 = 80 kN</u>**

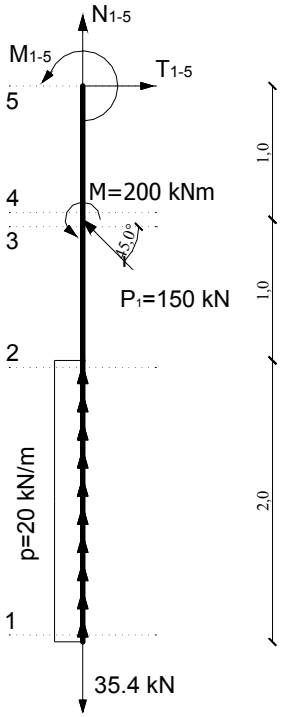
Zadatak 37:

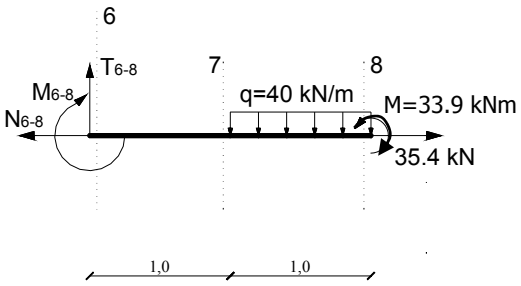
Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N



Postupak:

smjerovi unutarnjih sila za pojedine presjeke :





rastavljamo sile :

P_1

$P_{1x} = P_1 \times \cos 45° =$ 150.0 × 0.707 = **106.1 kN**

$P_{1y} = P_1 \times \sin 45° =$ 150.0 × 0.707 = **106.1 kN**

P_2

$P_{2x} = P_2 \times \cos 45° =$ 100.0 × 0.707 = **70.7 kN**

$P_{2y} = P_2 \times \sin 45° =$ 100.0 × 0.707 = **70.7 kN**

<u>Proračun unutarnjih sila:</u>

Presjek 1-1

$\sum x = 0$	$-T_1+0=0$;	**T_1=0**
$\sum y = 0$	$N_1-35.4=0$	**N_1=35.4kN**
$\sum M = 0$	$M_1+0=0$;	**M_1=0**

Presjek 2-2

$\sum x = 0$	$-T_2+0=0$;	**T_2=0**
$\sum y = 0$	$N_2-35.4+20\times2=0$	**N_2=-4.6kN**
$\sum M = 0$	$M_2+0=0$;	**M_2=0**

Presjek 3-3

$\sum x = 0$	$-T_3+0=0$;	**T_3=0**
$\sum y = 0$	$N_3-35.4+20\times2=0$	**N_3=-4.6kN**
$\sum M = 0$	$M_3+0=0$;	**M_3=0**

Presjek 4-4

$\sum x = 0$	$-T_4+106.1=0$;	**T_4=106.1 kN**
$\sum y = 0$	$N_4-35.4+20\times2+106.1=0$	**N_4=-110.7 kN**
$\sum M = 0$	$M_4+200=0$;	**M_4=-200 kNm**

Presjek 5-5

$\sum x = 0$	$-T_5+106.1=0$;	**T_5=106.1 kN**
$\sum y = 0$	$N_5-35.4+20\times2+106.1=0$	**N_5=-110.7 kN**
$\sum M = 0$	$M_5+200-106.1\times1=0$;	**M_5=-93.9 kNm**

Presjek 8-8

$\sum x = 0$	$-N_8+35.4=0$	**N_8=35.4 kN**
$\sum y = 0$	$T_8+0=0$;	**T_8=0**
$\sum M = 0$	$-M_8-33.9=0$;	**M_8=-33.9 kNm**

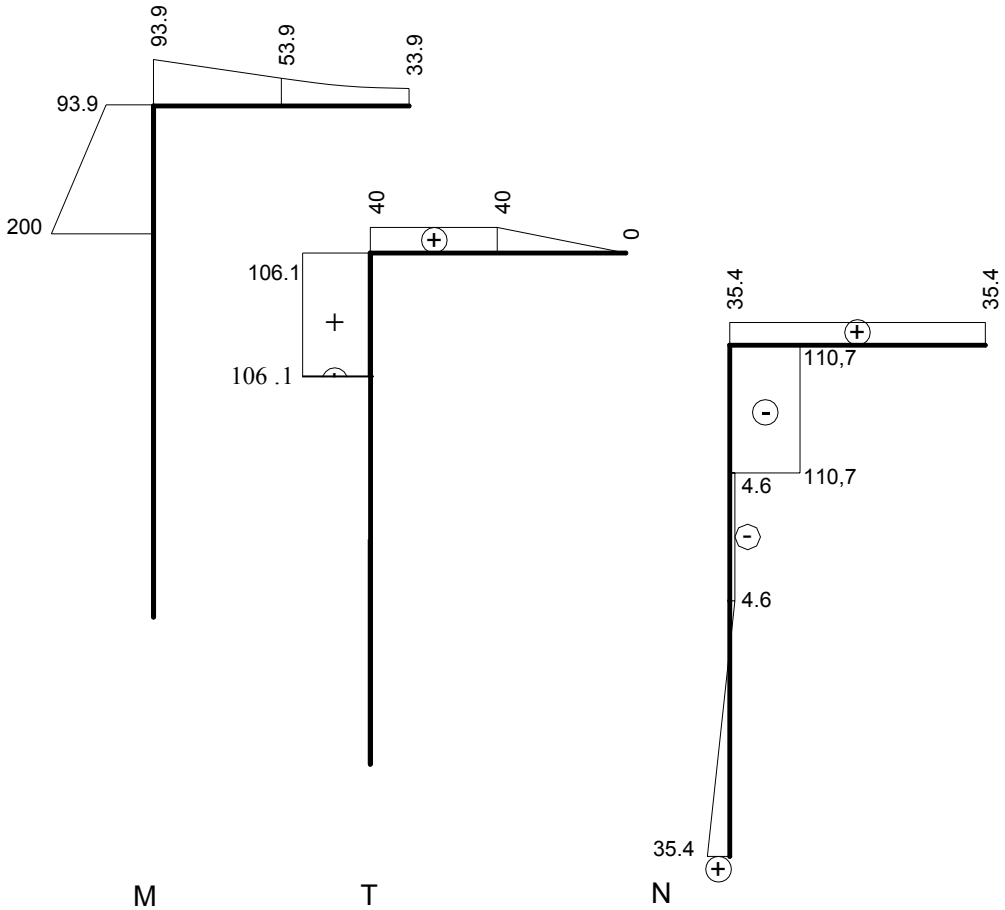
Presjek 7-7

$\sum x = 0$	$-N_7+35.4=0$	**N_7=35.4 kN**
$\sum y = 0$	$T_7-40\times1=0$;	**T_7=40 kN**
$\sum M = 0$	$-M_7-33.9-40\times1\times0.5=0$;	**M_7=-53.9 kNm**

Presjek 6-6

$\sum x = 0$	$-N_6+35.4=0$	**N_6=35.4 kN**
$\sum y = 0$	$T_6-40\times1=0$;	**T_6=40 kN**
$\sum M = 0$	$-M_6-33.9-40\times1\times1.5=0$;	**M_6=-93.9 kNm**

Dijagrami :

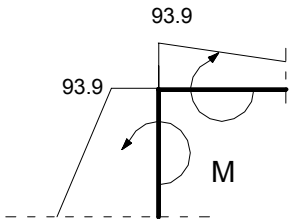


komentari uz dijagrame:

- na spoju stupa i grede moramo imati zadovoljene uvjete ravnoteže.

M

ravnoteža momenata *(ako računamo momente sa dvije strane možemo se kontrolirati postavljanjem ravnoteže čvora)*



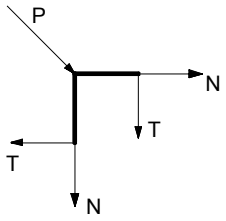
- momenti su suprotnog smjera i poništavaju se znači ravnoteža je zadovoljena.

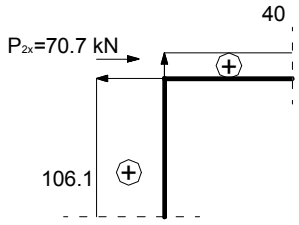
N i T

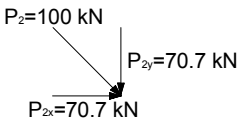
ravnoteža sila u čvoru :

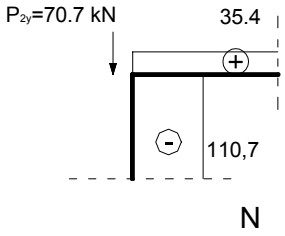
$$\sum x = 0$$

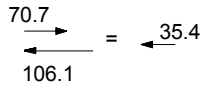
$$\sum y = 0$$

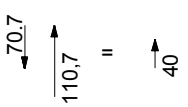












- *poprečna sila iz stupa + vanjsko djelovanje = uzdužna sile grede*

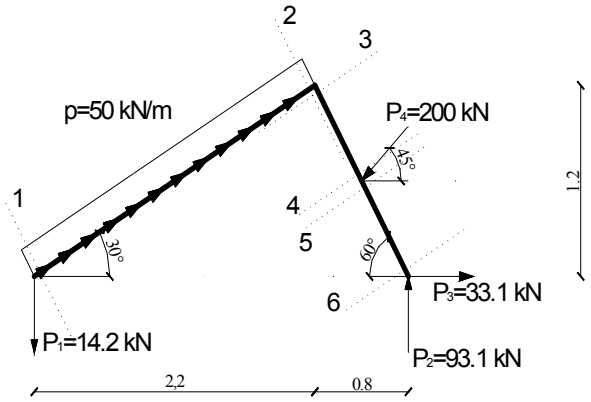
- *uzdužna sila iz stupa + vanjsko djelovanje = poprečna sila grede*

***OPASKA** :*

prilikom proračuna unutarnjih sila nismo upotrebljavali silu P_2, samo je poslužila kao kontrola ravnoteže sila u čvoru.

Zadatak 38:

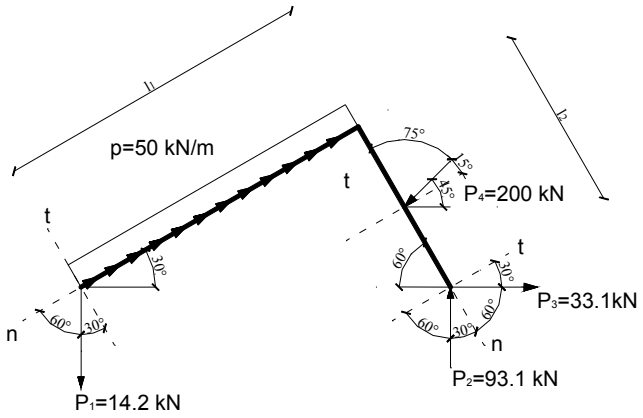
Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N



Postupak:

$$l_1 = \sqrt{2.2^2 + 1.2^2} = 2.5\text{m}$$
$$l_2 = \sqrt{0.8^2 + 1.2^2} = 1.44\text{m}$$

cos 60° = 0.5
cos 30° = 0.87
cos 15° = 0.97
cos 75° = 0.26



P1

$$P_{1n} = P_1 \times \cos 60° = 14.2 \times 0.5 \quad \textbf{= 7.1 kN}$$
$$P_{1t} = P_1 \times \cos 30° = 14.2 \times 0.87 \quad \textbf{= 12.4 kN}$$

P2

$$P_{2n} = P_2 \times \cos 30° = 93.1 \times 0.87 \quad \textbf{= 80.1 kN}$$
$$P_{2t} = P_2 \times \cos 60° = 93.1 \times 0.5 \quad \textbf{= 46.6 kN}$$

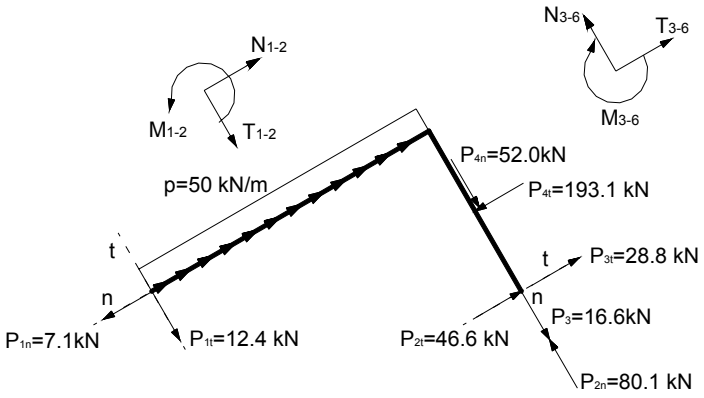
P3

$$P_{3n} = P_3 \times \cos 60° = 35.2 \times 0.5 \quad \textbf{= 16.6 kN}$$
$$P_{3t} = P_3 \times \cos 30° = 35.2 \times 0.87 \quad \textbf{= 28.8 kN}$$

P4

$$P_{4n} = P_4 \times \cos 75° = 200 \times 0.26 \quad \textbf{= 52.0 kN}$$
$$P_{4t} = P_4 \times \cos 15° = 200 \times 0.97 \quad \textbf{= 193.2 kN}$$

Proračun unutarnjih sila:



Presjek 1-1

$\sum x = 0$	N_1-7.1=0	**N_1=7.1 kN**
$\sum y = 0$	-T_1-12.4=0	**T_1=-12.4 kN**
$\sum M = 0$	M_1+0=0;	**M_1=0**

Presjek 2-2

$\sum x = 0$	N_2-7.1+50×2.5=0	**N_2=-117.9 kN**
$\sum y = 0$	-T_2-12.4=0	**T_2=-12.4 kN**
$\sum M = 0$	M_2+12.4×2.5=0;	**M_2=-31.0 kNm**

Presjek 6-6

$\sum x = 0$	-N_6-80.1+16.6=0	**N_6=-63.5 kN**
$\sum y = 0$	T_6+46.6+28.8=0 ;	**T_6=-75.4 kN**
$\sum M = 0$	-M_6+0=0;	**M_6=0**

Presjek 5-5

$\sum x = 0$	-N_5-80.1+16.6=0	**N_5=-63.5 kN**
$\sum y = 0$	T_5+46.6+28.8=0 ;	**T_5=-75.4 kN**
$\sum M = 0$	-M_5+46.6×0.72+28.8×0.72=0;	**M_5=54.3 kNm**

Presjek 4-4

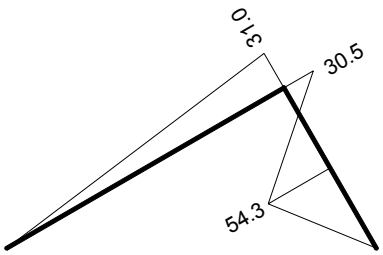
$\sum x = 0$	-N_4-80.1+16.6+52=0	**N_4=-11.5 kN**
$\sum y = 0$	T_4+46.6+28.8-193.2=0	**T_4=117.8 kN**
$\sum M = 0$	-M_4+46.6×0.72+28.8×0.72=0;	**M_4=54.3 kNm**

Presjek 3-3

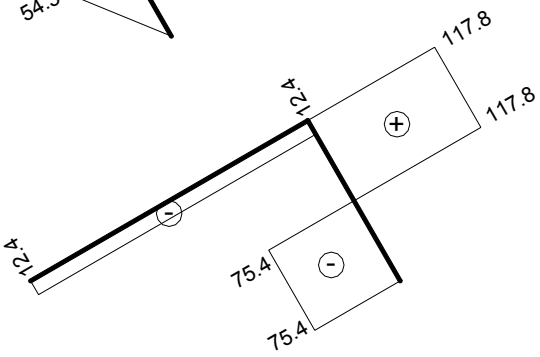
$\sum x = 0$	-N_3-80.1+16.6+52=0	**N_3=-11.5 kN**
$\sum y = 0$	T_3+46.6+28.8-193.2=0	**T_3=117.8 kN**
$\sum M = 0$	-M_3+46.6×1.44+28.8×1.44-193.2×0.72=0;	**M_3=-30.5 kNm**

Dijagrami :

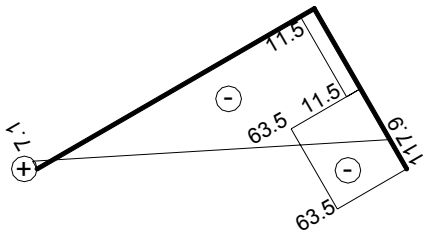
M



T



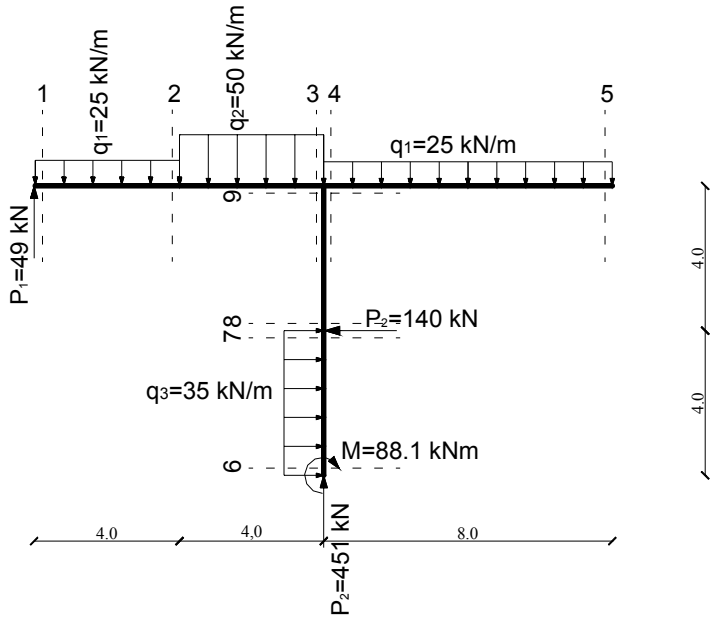
N



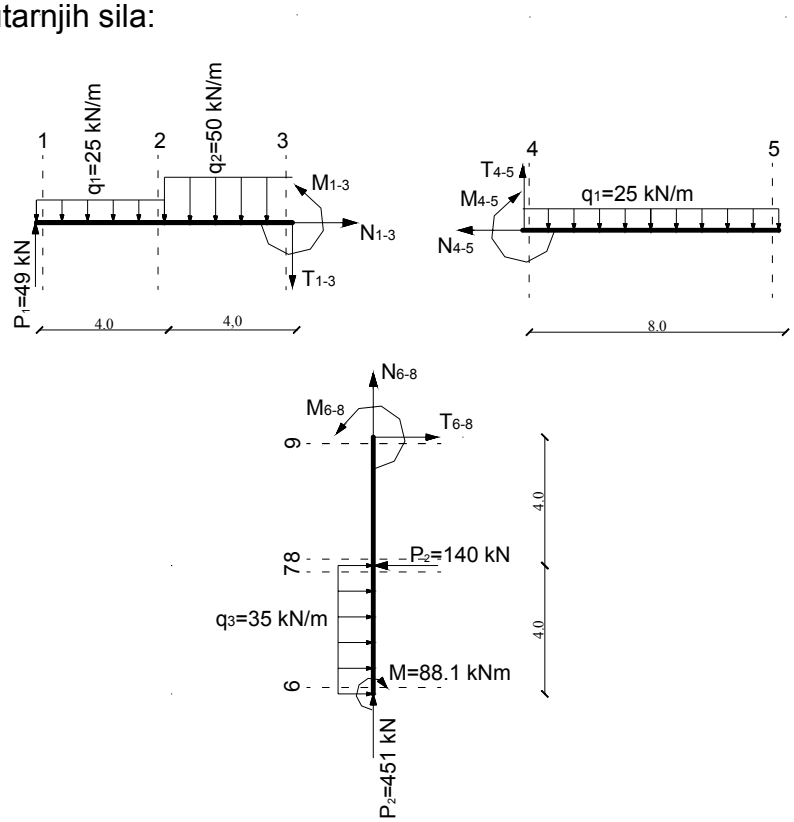
- Ravnoteža čvora zadovoljena, prisutne su male razlike zbog zaokruživanja decimala.

Zadatak 39:

Za prikazani uravnoteženi sustav nacrtati dijagrame unutarnjih sila M,T i N



Postupak:
Smjerovi unutarnjih sila:



Proračun unutarnjih sila:

Presjek 1-1

$\sum x = 0$	$N_1+0=0$	**$N_1=0$**
$\sum y = 0$	$-T_1+49=0$	**$T_1=49$ kN**
$\sum M = 0$	$M_1+0=0$;	**$M_1=0$**

Presjek 2-2

$\sum x = 0$	$N_2+0=0$	**$N_2=0$**
$\sum y = 0$	$-T_2+49-25\times4.0=0$	**$T_2=-51$ kN**
$\sum M = 0$	$M_2-49\times4.0+25\times4.0\times2.0=0$;	**$M_2=-4$ kNm**

Presjek 3-3

$\sum x = 0$	$N_3+0=0$	**$N_3=0$**
$\sum y = 0$	$-T_3+49-25\times4.0-50\times4.0=0$	**$T_3=-251.0$ kN**
$\sum M = 0$	$M_3-49\times8.0+25\times4.0\times6.0+50\times4.0\times2.0=0$;	**$M_3=-608.0$ kNm**

Presjek 5-5

$\sum x = 0$	$-N_5+0=0$	**$N_5=0$**
$\sum y = 0$	$T_5+0=0$;	**$T_5=0$**
$\sum M = 0$	$-M_5+0=0$;	**$M_5=0$**

Presjek 4-4

$\sum x = 0$	$-N_4+0=0$	**$N_4=0$**
$\sum y = 0$	$T_4-25\times8.0=0$	**$T_4=200$ kN**
$\sum M = 0$	$-M_4-25\times8.0\times4.0=0$;	**$M_4=-800$ kNm**

Presjek 6-6

$\sum x = 0$	T_6+0=0	**T_6=0**
$\sum y = 0$	N_6+451.0=0 ;	**N_6=-451.6 kN**
$\sum M = 0$	M_6-88.1=0;	**M_6=88.1 kNm**

Presjek 7-7

$\sum x = 0$	T_7+35×4.0=0	**T_7=-140.0**
$\sum y = 0$	N_7+451.0=0 ;	**N_7=-451.6 kN**
$\sum M = 0$	M_7-88.1+35×4.0×2.0=0;	**M_7=-191.9 kNm**

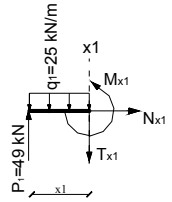
Presjek 8-8

$\sum x = 0$	T_8+35×4.0-140.0=0	**T_8=0**
$\sum y = 0$	N_8+451.0=0 ;	**N_8=-451.6 kN**
$\sum M = 0$	M_8-88.1+35×4.0×2.0=0;	**M_8=-191.9 kNm**

Presjek 9-9

$\sum x = 0$	T_9+35×4.0-140.0=0	**T_9=0**
$\sum y = 0$	N_9+451.0=0 ;	**N_9=-451.6 kN**
$\sum M = 0$	M_9-88.1+35×4.0×6.0-140×4.0=0;	**M_9=-191.9 kNm**

Presjek x1

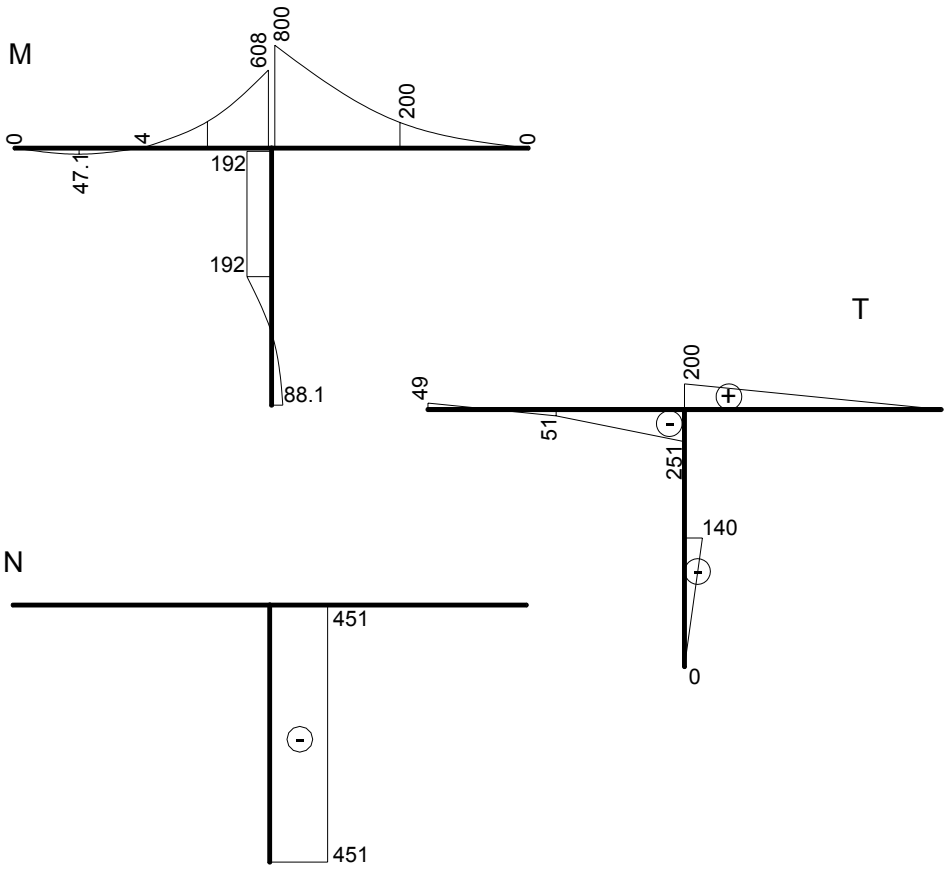


uvjet → T_x=0

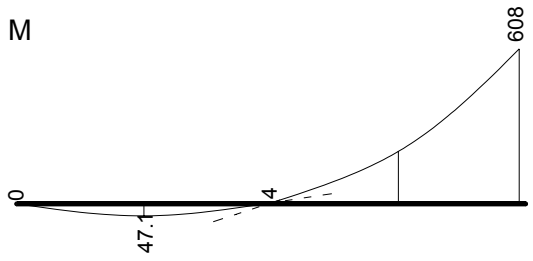
49-25×x-T_x=0 → x = 1.96m

$\sum x = 0$	N_x+0=0	**N_x=0**
$\sum y = 0$	49-25×1.96-T_x=0 ;	**T_x=0**
$\sum M = 0$	-49×1.96+25×1.96+M_x=0;	**M_x=47.1 kNm**

Dijagrami :

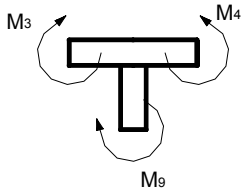


- različito kontinuirano opterećenje na dijagramu daje različiti nagib tangenti :



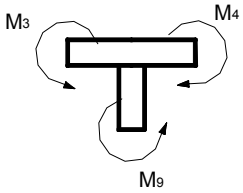
RAVNOTEŽA ČVORA :

Pozitivni predznaci na presjeku :



M_3= -608kNm
M_4= -800kNm
M_9= -192kNm

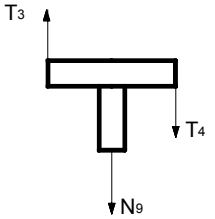
Pravi smjerovi na presjeku :



Ravnoteža momenata :

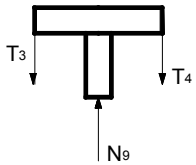
$$\sum M = 0 \qquad 608+192-800=0; \quad \text{- zadovoljena}$$

Pozitivni predznaci na presjeku :



T_3= -251kN
T_4= 200kN
N_9= -451kN

Pravi smjerovi na presjeku :

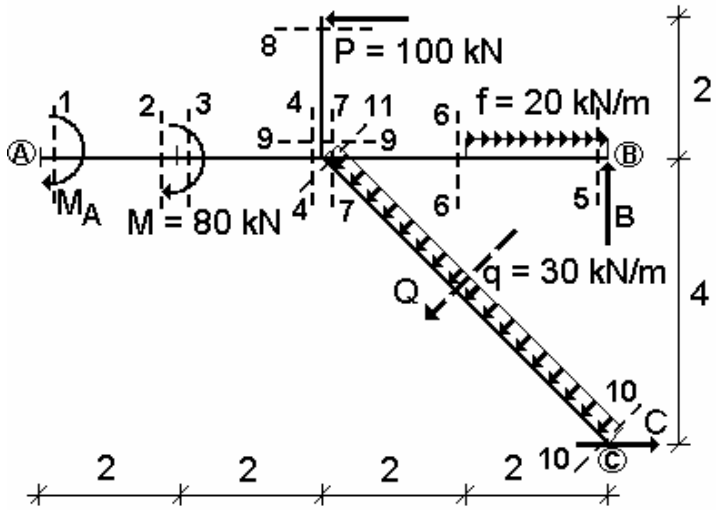


Ravnoteža sila :

$$\sum y = 0 \qquad -251 - 200 + 451=0 \text{ - zadovoljena}$$

Zadatak 40:

1.) Uravnotežiti zadani sustav sila momentom u točki A te silama na pravcima b i c
2.) Za tako uravnoteženi sustav sila nacrtati dijagrame unutarnjih sila M , T i N



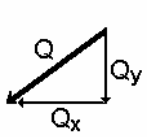
Postupak:

$F = l \cdot f = 2 \cdot 20 = 40$ kN

$Q = l \cdot q = \sqrt{4^2 + 4^2} \cdot 30 = 5{,}657 \cdot 30 = 169{,}71$ kN

$\mathbf{Q_x = q \cdot l_y} = 30 \cdot 4 = 120$ kN

$\mathbf{Q_y = q \cdot l_x} = 30 \cdot 4 = 120$ kN



Pretpostavimo smjerove sila B i C i momenta M_A

$\sum \mathbf{M_{(C)} = 0}$ $\quad -M_A - M + P \cdot 6 - f \cdot l \cdot 4 + Q_x \cdot 2 + Q_y \cdot 2 = 0$

$\mathbf{M_A} = -80 + 100 \cdot 6 - 40 \cdot 4 + 120 \cdot 2 + 120 \cdot 2 = \mathbf{840\ kNm}$

$\mathbf{\Sigma x = 0}$ $\quad C - P - Q_x = 0 \Rightarrow \mathbf{C} = P + Q_x - F = 100 + 120 - 40 = \mathbf{180\ kN}$

$\mathbf{\Sigma y = 0}$ $\quad B - Q_y = 0 \quad \Rightarrow \quad \mathbf{B = 120\ kN}$

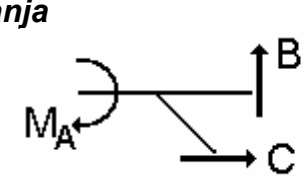
KONTROLA !!!!! $\Rightarrow$ obavezno provesti kontrolu prije nego krenemo na određivanje unutarnjih sila

$\sum \mathbf{M_{(B)} = 0}$ $\quad -M_A - M + P \cdot 2 - Q_x \cdot 2 + Q_y \cdot 2 + C \cdot 4 = 0$

$- 840 - 80 + 100 \cdot 2 + 180 \cdot 4 = 0$ zadovoljava

Bilo bi bolje odabrati $\sum M_{(T)} = 0$ tj. na točku kroz koju ne prolazi niti jedna nepoznanica, npr. neka proizvoljna točka T *(može biti bilo gdje, ne treba biti na konstrukciji)***.**

Skica stvarnog djelovanja



M_A	840 kNm
B	120 kN
C	180 kN

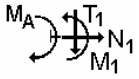
▪ **Imamo 11 karakterističnih presjeka** – označeni proizvoljnim redoslijedom

Presjek 1.

Σ x = 0 $\mathbf{N_1} = 0$

Σ y = 0 $\mathbf{T_1} = 0$

$\sum \mathbf{M_{(1)} = 0}$ $M_1 - M_A = 0$ $\mathbf{M_1} = 840$ kNm

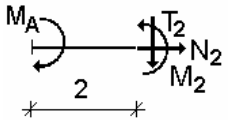


Presjek 2.

Σ x = 0 $N_2 = 0$

Σ y = 0 $T_2 = 0$

$\sum \mathbf{M_{(2)} = 0}$ $M_2 - M_A = 0$ $M_2 = 840$ kNm



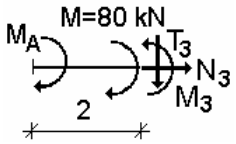
Presjek 3.

Σ x = 0 $\mathbf{N_3} = 0$ kN

Σ y = 0 $\mathbf{T_3} = 0$

$\sum \mathbf{M_{(3)} = 0}$ $M_3 - M_A - M = 0$

$\mathbf{M_3} = 840 + 80 = 920$ kNm



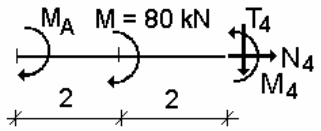
Presjek 4.

Σ x = 0 $\mathbf{N_4} = 0$

Σ y = 0 $\mathbf{T_4} = 0$

$\sum \mathbf{M_{(4)} = 0}$ $M_4 - M_A - M = 0$

$\mathbf{M_4} = 840 + 80 = 920$ kNm



Presjek 5.

Σ x = 0 $\mathbf{N_5} = 0$

Σ y = 0 $T_5 + B = 0$ $\mathbf{T_5} = -B = -120$ kN

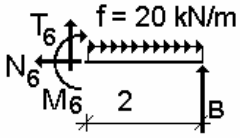
$\sum M_{(5)} = 0$ $\mathbf{M_5} = 0$

Presjek 6.

Σ x = 0 $-N_6 + f \cdot 2 = 0$ $\mathbf{N_6} = 20 \cdot 2 = 40$ kN

Σ y = 0 $T_6 + B = 0$ $\mathbf{T_6} = -B = -120$ kN

$\sum \mathbf{M_{(6)} = 0}$ $-M_6 + B \cdot 2 = 0$ $\mathbf{M_6} = 240$ kNm

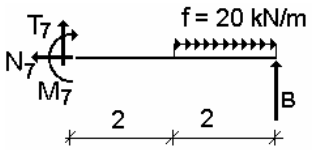


Presjek 7.

Σ x = 0 $-N_7 + f \cdot 2 = 0$ $\mathbf{N_7} = 40$ kN

Σ y = 0 $T_7 + B = 0$ $\mathbf{T_7} = -120$ kN

$\sum \mathbf{M_{(7)} = 0}$ $-M_7 + B \cdot 4 = 0$ $\mathbf{M_7} = 120 \cdot 4 = 480$ kNm

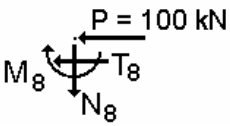


Presjek 8.

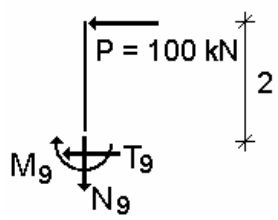
Σ x = 0 $-T_8 - P = 0$ kN $\mathbf{T_8} = -100$ kN

Σ y = 0 $\mathbf{N_8} = 0$

$\sum \mathbf{M_{(8)} = 0}$ $\mathbf{M_8} = 0$



Presjek 9.

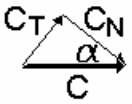


Σ x = 0 $-T_9 - P = 0$ **T_9** = -100 kN

Σ y = 0 **N_9** = 0

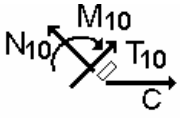
∑ $M_{(9)}$ = 0 $-M_9 + P \cdot 2 = 0$ **M_9** = 200 kNm (vlak desno)

Presjek 10.



$C_T = C_N = C \cdot \sin\alpha = C \cdot 0{,}7071$

$C_T = C_N = 180 \cdot 0{,}7071 = 127{,}28$ kN

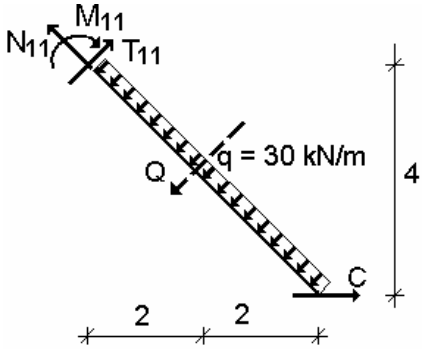


∑ N = 0 $N_{10} - C_N = 0$ **N_{10}** = 127,28 kN

∑ T = 0 $T_{10} + C_T = 0$ **T_{10}** = -127,28 kN

∑ $M_{(10)}$ = 0 **M_{10}**= 0

Presjek 11.



Σ N = 0 $N_{11} - C_N = 0$ **N_{11}**= 127,28 kN

Σ T = 0 $T_{11} - q \cdot l + C_T = 0$

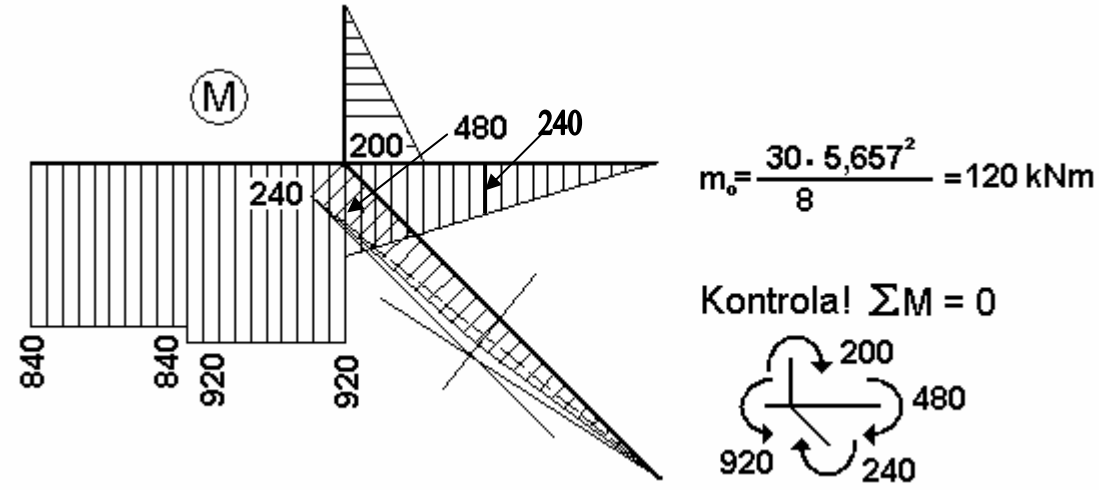
$T_{11} = -127{,}28 + 30 \cdot 5{,}66$ **T_{11}** = 42,42 kN

∑ $M_{(11)}$ = 0 $-M_{11} + C \cdot 4 - q \cdot 5{,}66 \cdot \frac{5{,}56}{2} = 0$

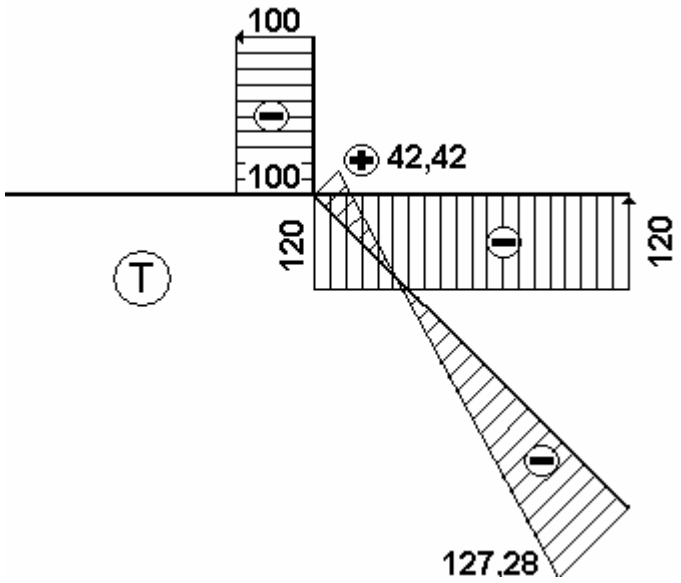
M_{11} = 239,98 kNm

Dijagrami unutarnjih sila M,T i N

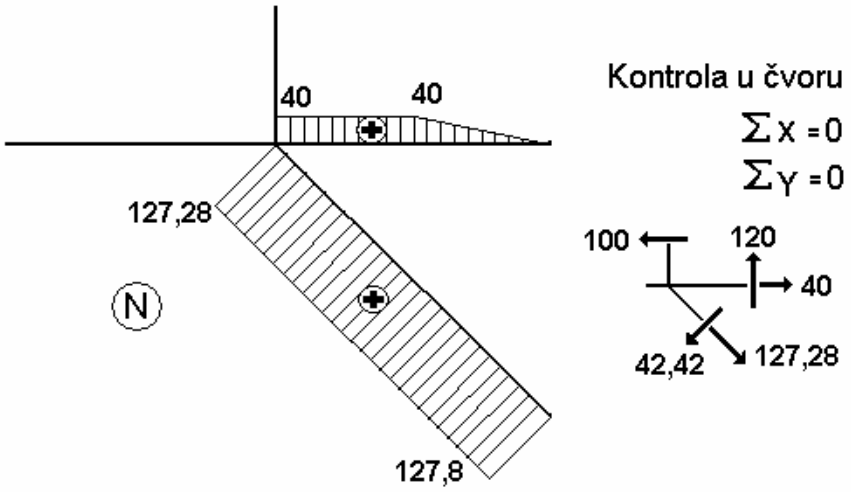
M – dijagram : Mj.sila 1cm = 100 kNm



T – dijagram : Mj.sila 1cm = 100 kNm

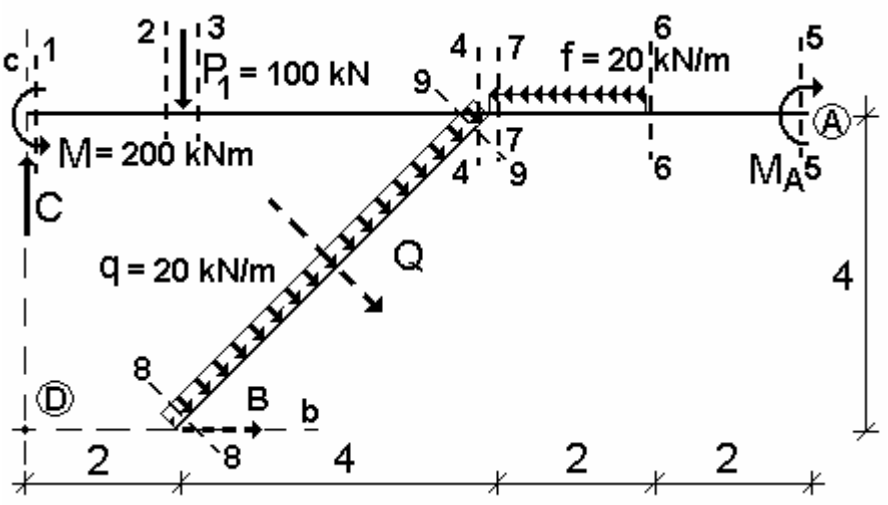


N – dijagram :



Zadatak 41:

1.) Uravnotežiti zadani sustav sila momentom u točki A te silama na pravcima b i c
2.) Za tako uravnoteženi sustav sila nacrtati dijagrame unutarnjih sila M , T i N



Postupak:

$F = l \cdot f = 2 \cdot 20 = 40$ kN

$Q = l \cdot q = \sqrt{4^2 + 4^2} \cdot 20 =$
$5{,}657 \cdot 20 = 113{,}14$ kN

$\mathbf{Q_x = q \cdot l_y} = 20 \cdot 4 = 80$ kN
$\mathbf{Q_y = q \cdot l_x} = 20 \cdot 4 = 80$ kN

Pretpostavimo smjerove nepoznatih sila i momenta

$\sum \mathbf{M_{(D)} = 0}$ $\quad -M_A + M - P \cdot 2 + f \cdot l \cdot 4 - Q_x \cdot 2 - Q_y \cdot 4 = 0 \Rightarrow$ $\mathbf{M_A = -320\ kNm}$

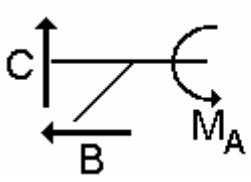
$\mathbf{\Sigma x = 0}$ $\quad B + Q_x - F = 0 \Rightarrow$ $\mathbf{B = -40\ kN}$

$\mathbf{\Sigma y = 0}$ $\quad C - P - Q_y = 0 \Rightarrow$ $\mathbf{C = 180\ kN}$

KONTROLA !

$\sum \mathbf{M_{(A)} = 0}$ $\quad -M_A + M + P \cdot 8 + Q_x \cdot 2 + Q_y \cdot 6 + B \cdot 4 - C \cdot 10 = 0$
$+320 + 200 + 800 + 80 \cdot 2 + 80 \cdot 6 - 40 \cdot 4 - 180 \cdot 10 = 0$ zadovoljava

Skica stvarnog djelovanja



M_A	320 kNm
B	40 kN
C	180 kN

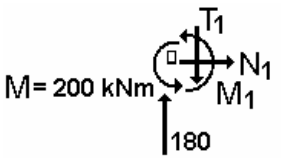
▪ **Imamo 9 karakterističnih presjeka**

Presjek 1.

Σ x = 0 $\mathbf{N_1} = 0$

Σ y = 0 $-T_1 + C = 0$ $\mathbf{T_1} = 180$ kN

$\sum \mathbf{M_{(1)} = 0}$ $M_1 + 200 = 0$ $\mathbf{M_1} = -200$ kNm (vlak gore)

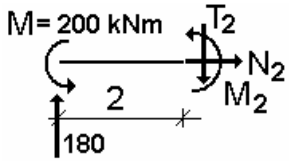


Presjek 2.

Σ x = 0 $\mathbf{N_2} = 0$

Σ y = 0 $180 - T_2 = 0$ $\mathbf{T_2} = 180$ kN

$\sum \mathbf{M_{(2)} = 0}$ $M_2 + 200 - 180 \cdot 2 = 0$ $\mathbf{M_2} = 160$ kNm (vlak dolje)



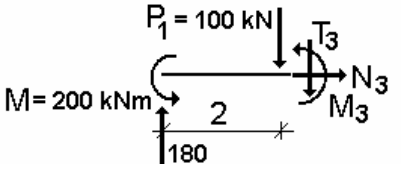
Presjek 3.

Σ x = 0 $\mathbf{N_3} = 0$ kN

Σ y = 0 $180 - 100 - T_3 = 0$ $\mathbf{T_3} = 80$ kN

$\sum \mathbf{M_{(3)} = 0}$ $M_3 + 200 - 180 \cdot 2 = 0$ $\mathbf{M_3} = 160$ kNm

$M_3 = M_2$



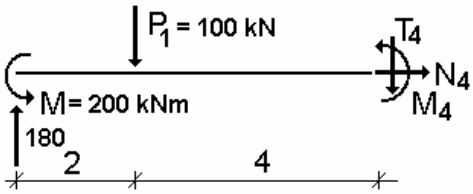
Presjek 4.

Σ x = 0 $\mathbf{N_4} = 0$

Σ y = 0 $180 - 100 - T_4 = 0$ $\mathbf{T_4} = 80$ kN

$\sum \mathbf{M_{(4)} = 0}$ $M_4 + 200 - 180 \cdot 6 + 100 \cdot 4 = 0$

$\mathbf{M_4} = 480$ kNm



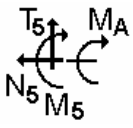
Presjek 5.

Σ x = 0 $\mathbf{N_5} = 0$

Σ y = 0 $\mathbf{T_5} = 0$

$\sum \mathbf{M_{(5)} = 0}$ $-M_5 - M_A = 0$ $-M_5 - (-320) = 0$ $\mathbf{M_5} = 320$ kNm

(ucrtali smo pretpostavljeni smjer M_A pa moramo uzeti stvarnu vrijednost)



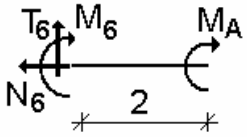
Presjek 6.

Σ x = 0 $\mathbf{N_6} = 0$

Σ y = 0 $\mathbf{T_6} = 0$

$\sum \mathbf{M_{(6)} = 0}$ $-M_6 - M_A = 0$ $-M_6 - (-320) = 0$

$\mathbf{M_6} = 320$ kNm

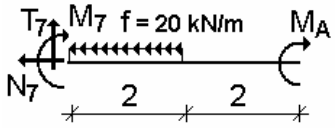


Presjek 7.

Σ x = 0 $-N_7 - f \cdot 2 = 0$ $\mathbf{N_7} = -40$ kN

$\sum \mathbf{M_{(7)} = 0}$ $-M_7 - M_A = 0$ $\mathbf{M_7} = 320$ kNm

Σ y = 0 $\mathbf{T_7} = 0$

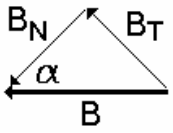


Presjek 8.

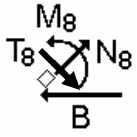
tgα = 1 α = 45°

sinα = cosα = 0,707

$B_T = B_N = 40 \cdot 0{,}707 = 28.28$ kN



∑ N = 0	$N_8 - B_N = 0$	$\mathbf{N_8} = B_N = 28{,}28$ kN
∑ T = 0	$T_8 - B_T = 0$	$\mathbf{T_8} = +28{,}28$ kN
∑ $M_{(8)}$ = 0	$\mathbf{M_8} = 0$	



Presjek 9.

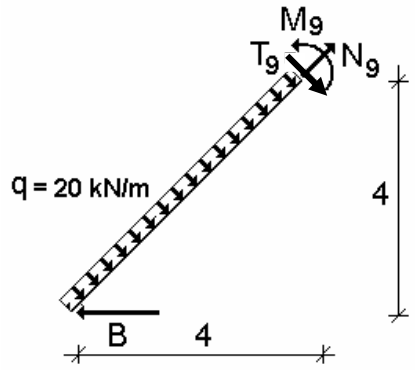
∑ N = 0 $N_9 - B_N = 0$ $\mathbf{N_9} = 28{,}28$ kN

∑ T = 0 $T_9 - B_T + q \cdot l = 0$ $\mathbf{T_9} = -84{,}86$ kN

∑ $M_{(9)}$ = 0 $M_9 - B \cdot 4 + q \cdot 5{,}657 \cdot \frac{5{,}567}{2} = 0$

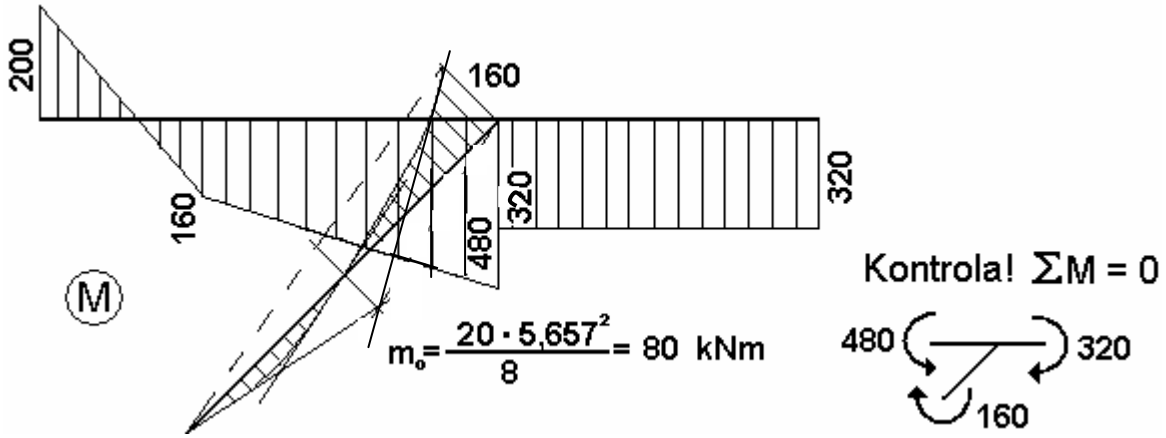
$\mathbf{M_9} = 40 \cdot 4 - 20 \cdot \frac{5{,}657^2}{2} = -160$ kNm

(vlak gore)

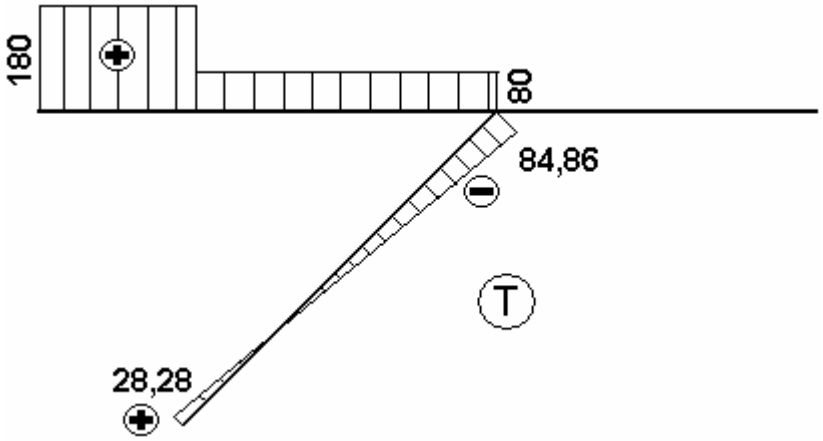


Dijagrami unutarnjih sila M,T i N

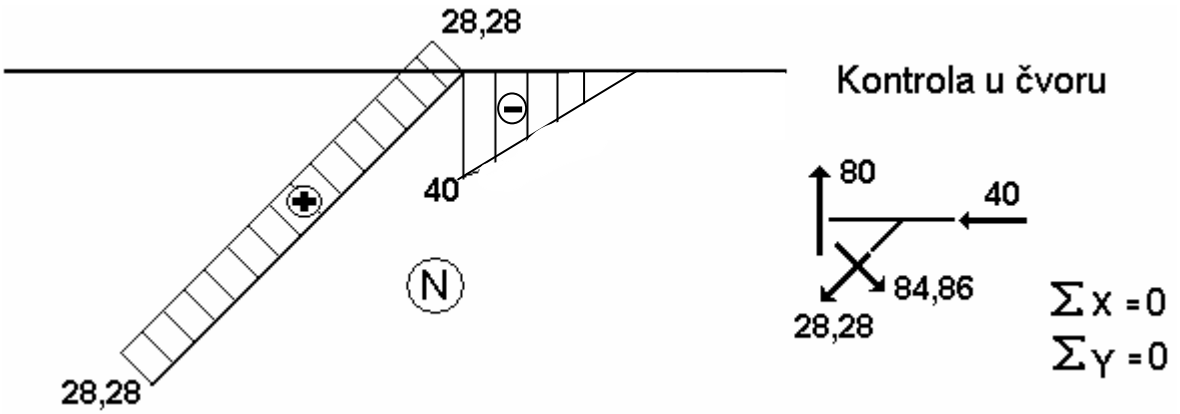
M – dijagram : Mj.sila 1cm = 200 kNm



T – dijagram : Mj.sila 1cm = 200 kNm



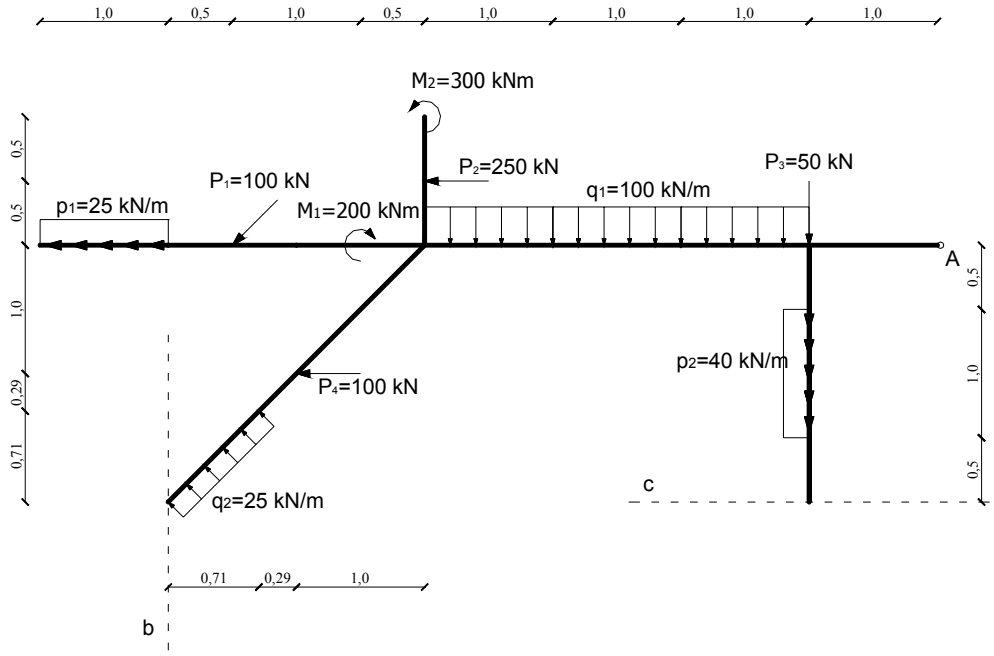
N – dijagram :



DODATNI ZADACI ZA VJEŽBU

Zadatak 42: (ovo je samo za one koji su stvarno uporni ☺)

I.) prikazani sustav uravnotežiti momentom u točki A te silama na pravcima b i c.
II.) za uravnoteženi sustav nacrtati dijagrame unutarnjih sila



Postupak:

RASTAVLJANJE SILA NA KOMPONENTE

sin 45° = cos 45° = 0.707

Q_2

$Q_2 = 25 \times 1{,}0 = \underline{25.0\ kN}$

$Q_{2x} = Q_1 \times \cos 45° = 25.0 \times 0.707 =$ **17.6 kN**
$Q_{2y} = Q_1 \times \cos 45° = 25.0 \times 0.707 =$ **17.6 kN**

P_1

$P_{1x} = P_1 \times \cos 45° = 100.0 \times 0.707 =$ **70.7 kN**
$P_{1y} = P_1 \times \sin 45° = 100.0 \times 0.707 =$ **70.7 kN**

ODREĐIVANJE URAVNOTEŽAVAJUĆIH SILA

Pretpostavljamo smjerove sila i koncentriranog momenta

$\sum M_{(1)} = 0$

$p_1 \times 1.0 \times 2.0 - P_{1y} \times 0.5 + P_{1x} \times 2.0 - M_1 + M_2 + P_2 \times 2.5 - q_1 \times 3.0 \times 3.5 - P_3 \times 5.0 - p_2 \times 1.0 \times 5.0 + P_4 \times 1.0 + Q_2 \times 0.5 + M_A = 0$

$25 \times 1.0 \times 2.0 - 70.7 \times 0.5 + 70.7 \times 2.0 - 200 + 300 + 250 \times 2.5 - 100 \times 3.0 \times 3.5 - 50 \times 5.0 - 40 \times 1.0 \times 5.0 + 100 \times 1.0 + 25 \times 0.5 + M_A = 0$

$$\underline{M_A = 506 \text{ kNm}}$$

$\sum y = 0$

$-P_{1y} - q_1 \times 3.0 - P_3 - p_2 \times 1.0\ Q_2 + B = 0$

$$-70.7 - 100 \times 3.0 - 50 - 40 \times 1.0 + 17.6 + B = 0$$

$$\underline{B = 443.1 \text{ kN}}$$

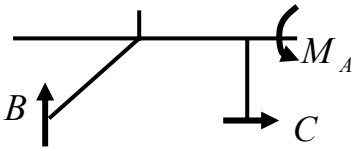
$\sum x = 0$

$-p_1 \times 1.0 - P_{1x} - P_2 - P_4 - Q_{2x} + C = 0$

$-25 \times 1.0 - 70.7 - 250 - 100 - 17.6 + C = 0$

$$\underline{C = 463.3 \text{ kN}}$$

Skica stvarnog djelovanja



C

$B_n = B_n \times \cos 45° = 443.1 \times 0.707$ **= 313.2 kN**

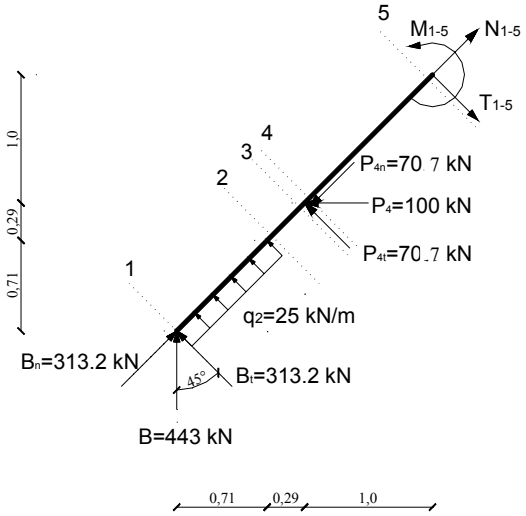
$B_t = B_t \times \cos 45° = 443.1 \times 0.707$ **= 313.2 kN**

P_4

$P_{4n} = P_4 \times \cos 45° = 100 \times 0.707$ **= 70.7 kN**

$P_{4t} = P_4 \times \cos 45° = 100 \times 0.707$ **= 70.7 kN**

Proračun unutarnjih sila:



Presjek 1-1

$\sum x = 0$	N_1+313.2 = 0	**N_1=-313.2 kN**
$\sum y = 0$	-T_1+313.2=0 ;	**T_1=313.2 kN**
$\sum M = 0$	M_1+0=0;	**M_1=0**

Presjek 2-2

$\sum x = 0$	N_2+313.2 = 0	**N_2=-313.2 kN**
$\sum y = 0$	-T_2+313.2+25×1.0=0 ;	**T_2=338.2 kN**
$\sum M = 0$	M_2-313.2×1.0+25×1.0×0.5=0;	**M_2=325.7 kNm**

Presjek 3-3

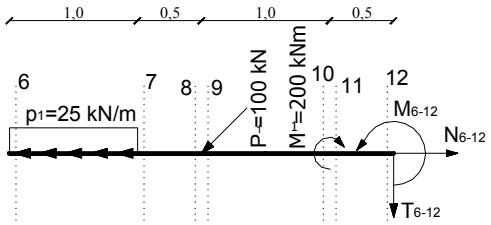
$\sum x = 0$	N_3+313.2 = 0	**N_3=-313.2 kN**
$\sum y = 0$	-T_3+313.2+25×1.0=0	**T_2=338.2 kN**
$\sum M = 0$	M_2-313.2×1.41-25×1.0×0.91=0;	**M_2=463.4 kNm**

Presjek 4-4

$\sum x = 0$	N_4+313.2-70.7 = 0	**N_4=-243.1 kN**
$\sum y = 0$	-T_4+313.2+25×1.0+70.7=0	**T_4=408.3 kN**
$\sum M = 0$	M_4-313.2×1.41-25×1.0×0.91=0;	**M_4=463.4 kNm**

Presjek 5-5

$\sum x = 0$	N_5+313.2-70.7 = 0	**N_5=-243.1 kN**
$\sum y = 0$	-T_5+313.2+25×1.0+70.7=0	**T_5=408.3 kN**
$\sum M = 0$	M_5-313.2×2.82-25×1.0×2.31-70.7×1.41=0	**M_5=1039.8 kNm**



Presjek 6-6

$\sum x = 0$ $N_6+0=0$ **$N_6=0$**

$\sum y = 0$ $-T_6+0=0$; **$T_6=0$**

$\sum M = 0$ $M_6+0=0$; **$M_6=0$**

Presjek 7-7

$\sum x = 0$ $N_7-25\times1.0=0$ **$N_7=25kN$**

$\sum y = 0$ $-T_7+0=0$ **$T_7=0$**

$\sum M = 0$ $M_7+0=0$; **$M_7=0$**

Presjek 8-8

$\sum x = 0$ $N_8-25\times1.0=0$ **$N_8=25kN$**

$\sum y = 0$ $-T_8+0=0$; **$T_8=0$**

$\sum M = 0$ M_8+0 **$M_8=0$**

Presjek 9-9

$\sum x = 0$ $N_9-25\times1.0-70.7=0$ **$N_8=95.7kN$**

$\sum y = 0$ $-T_9-70.1=0$; **$T_9=-70.1$**

$\sum M = 0$ $M_9+0=0$; **$M_9=0$ kNm**

Presjek 10-10

$\sum x = 0$ $N_{10}-25\times1.0-70.7=0$ **$N_{10}=95.7kN$**

$\sum y = 0$ $-T_{10}-70.7=0$; **$T_{10}=-70.7$**

$\sum M = 0$ $M_{10}+70.7\times1.0=0$; **$M_{10}=-70.7$ kNm**

Presjek 11-11

$\sum x = 0$ $N_{11}-25\times1.0-70.7=0$ **$N_{11}=95.7kN$**

$\sum y = 0$ $-T_{11}-70.7=0$; **$T_{11}=-70.7$**

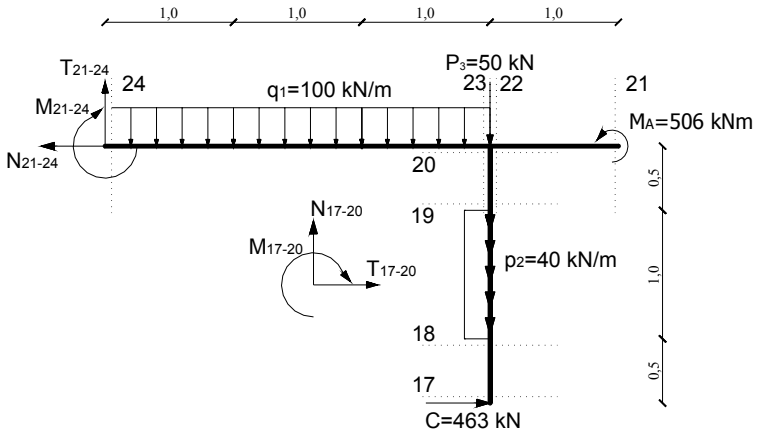
$\sum M = 0$ $M_{11}+70.7\times1.0-200=0$; **$M_{10}=129.9$ kNm**

Presjek 12-12

$\sum x = 0$ $N_{12}-25\times1.0-70.7=0$ **$N_{12}=95.7kN$**

$\sum y = 0$ $-T_{12}-70.7=0$; **$T_{12}=-70.7$**

$\sum M = 0$ $M_{12}+70.7\times1.5-200=0$; **$M_{10}=94.9$ kNm**



Presjek 17-17

$\sum x = 0$ $-N_{17}+0=0$ **$N_{17}=0$**

$\sum y = 0$ $T_{17}+463.0=0$; **$T_{17}=-463.0$ kN**

$\sum M = 0$ $-M_{17}+0=0$; **$M_{17}=0$**

Presjek 18-18

$\sum x = 0$ $-N_{18}+0=0$ **$N_{18}=0$**

$\sum y = 0$ $T_{18}+463.0=0$; **$T_{18}=-463.0$ kN**

$\sum M = 0$ $-M_{18}+463\times0.5=0$; **$M_{18}=231.5$ kNm**

Presjek 19-19

$\sum x = 0$ $-N_{19}+40\times1.0=0$ **$N_{19}=40$ kN**

$\sum y = 0$ $T_{19}+463.0=0$; **$T_{19}=-463.0$ kN**

$\sum M = 0$ $-M_{19}+463\times1.5=0$; **$M_{18}=694.5$ kNm**

Presjek 20-20

$\sum x = 0$ $-N_{20}+40\times1.0=0$ **$N_{19}=40$ kN**

$\sum y = 0$ $T_{20}+463.0=0$; **$T_{20}=-463.0$ kN**

$\sum M = 0$ $-M_{20}+463\times2.0=0$; **$M_{18}=926.0$ kNm**

Presjek 21-21

$\sum x = 0$ $-N_{21}+0=0$ **$N_{21}=0$**

$\sum y = 0$ $T_{21}+0=0$; **$T_{21}=-0$**

$\sum M = 0$ $-M_{21}+506.0=0$; **$M_{18}=506.0$ kNm**

Presjek 22-22

$\sum x = 0$ $-N_{22}+0=0$ **$N_{22}=0$**

$\sum y = 0$ $T_{22}+0=0$; **$T_{22}=-0$**

$\sum M = 0$ $-M_{22}+506.0=0$; **$M_{22}=506.0$ kNm**

Presjek 23-23

$\sum x = 0$ $-N_{23}+463=0$ **$N_{23}=463.0$ kN**

$\sum y = 0$ $T_{23}-40\times1.0-50=0$; **$T_{23}=90$ kN**

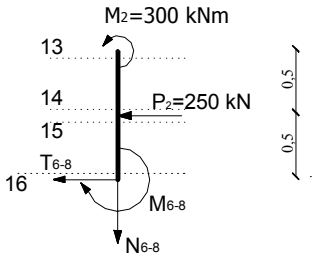
$\sum M = 0$ $-M_{23}+463\times2.0+506=0$; **$M_{23}=1432.0$ kNm**

Presjek 24-24

$\sum x = 0$ $-N_{24}+463=0$ **$N_{24}=463.0$ kN**

$\sum y = 0$ $T_{24}-40\times1.0-50-100\times3.0=0$; **$T_{24}=390$ kN**

$\sum M = 0$ $-M_{24}+463\times2.0+506-100\times3\times1.5-50\times3.0-40\times3=0$; **$M_{24}=712$ kNm**



Presjek 13-13

$\sum x = 0$ $-N_{13}+0=0$ **$N_{13}=0$**

$\sum y = 0$ $T_{13}+0=0$; **$T_{13}=0$**

$\sum M = 0$ $-M_{13}+300=0$; **$M_{13}=300$ kNm**

Presjek 14-14

$\sum x = 0$ $-N_{14}+0=0$ **$N_{14}=0$**

$\sum y = 0$ $T_{14}+0=0$; **$T_{14}=0$**

$\sum M = 0$ $-M_{14}+300=0$; **$M_{14}=300$ kNm**

Presjek 15-15

$\sum x = 0$ $-N_{15}+0=0$ **$N_{15}=0$**

$\sum y = 0$ $T_{15}+250=0$; **$T_{15}=-250$ kN**

$\sum M = 0$ $-M_{15}+300=0$; **$M_{15}=300$ kNm**

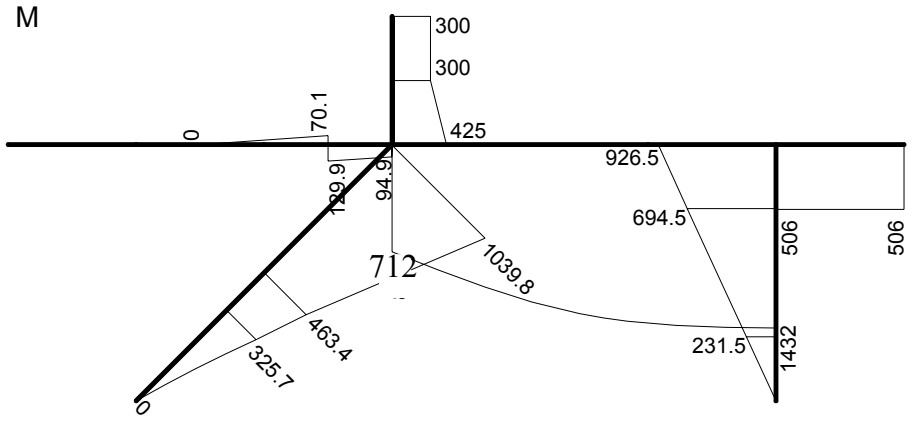
Presjek 16-16

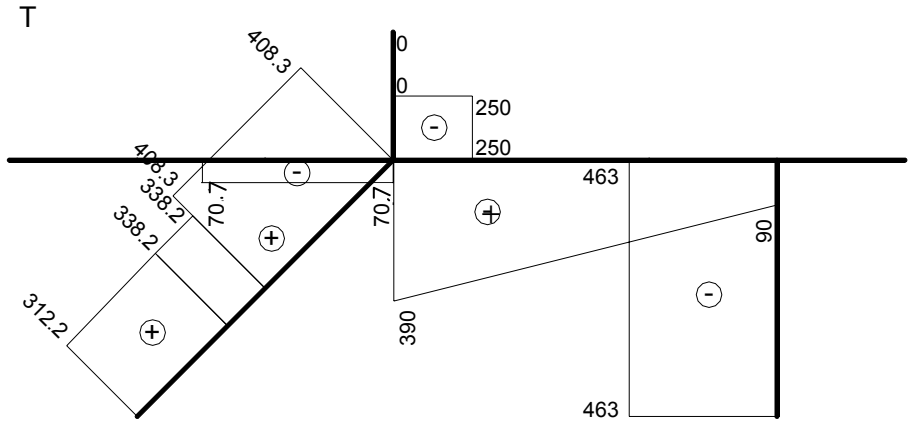
$\sum x = 0$ $-N_{16}+0=0$ **$N_{16}=0$**

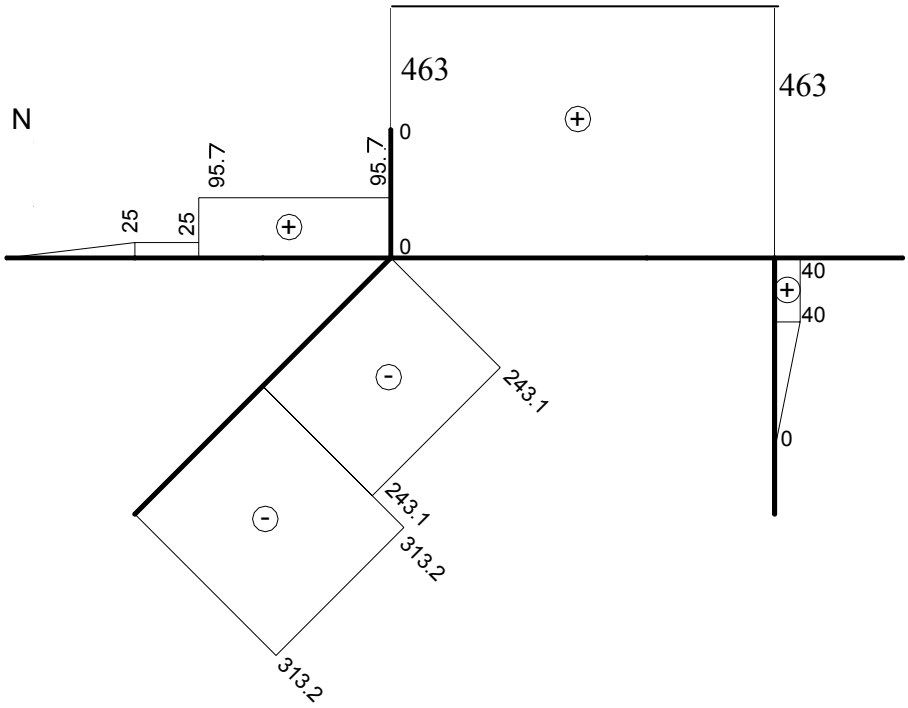
$\sum y = 0$ $T_{16}+250=0$; **$T_{16}=-250$ kN**

$\sum M = 0$ $-M_{16}+300+250\times0.5=0$; **$M_{16}=425$ kNm**

Dijagrami unutarnjih sila:

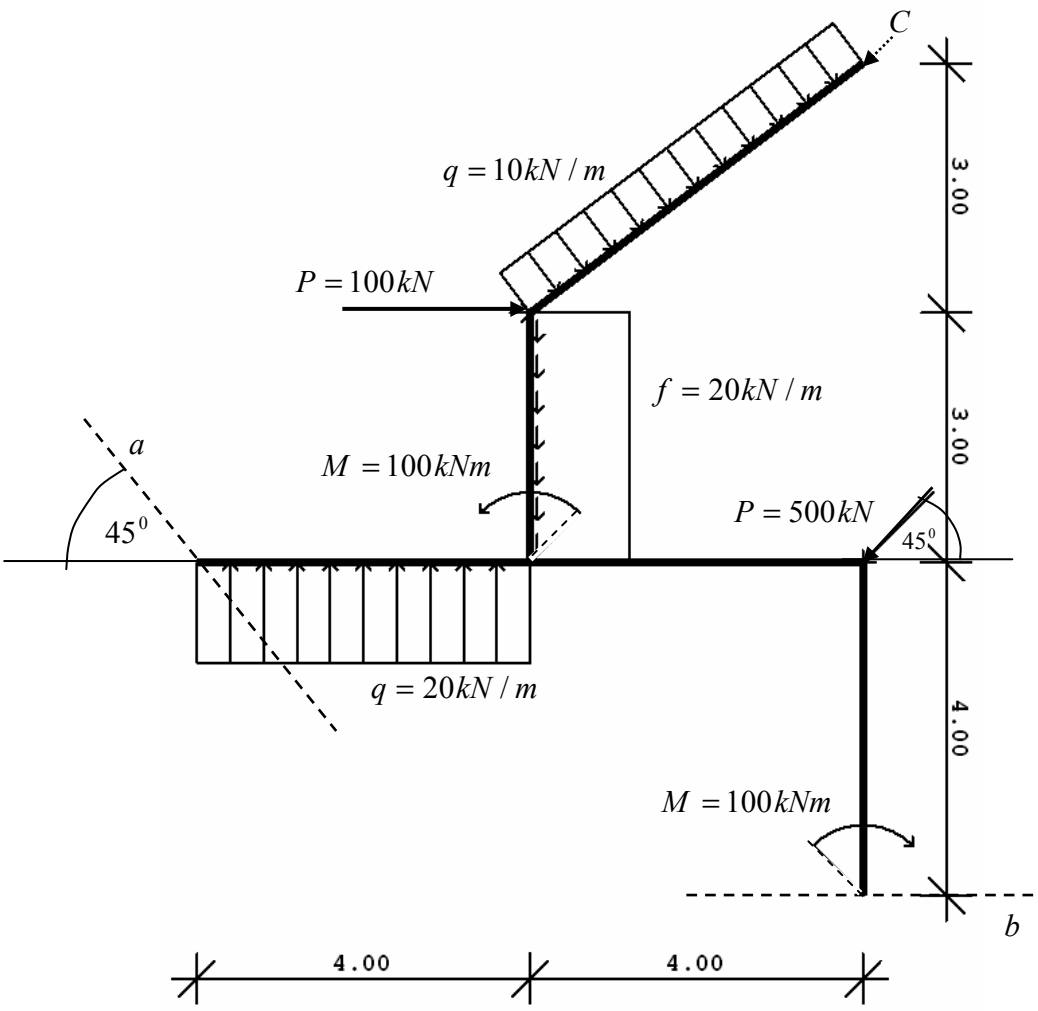






Zadatak 43: koncentrirani moment u čvoru (ispitni zadatak 20. 6. 2006.)

1.) **Zadani sustav u ravnini uravnotežiti silama A, B na pravcima *"a"* i *"b"* te momentom M_c u točki C**

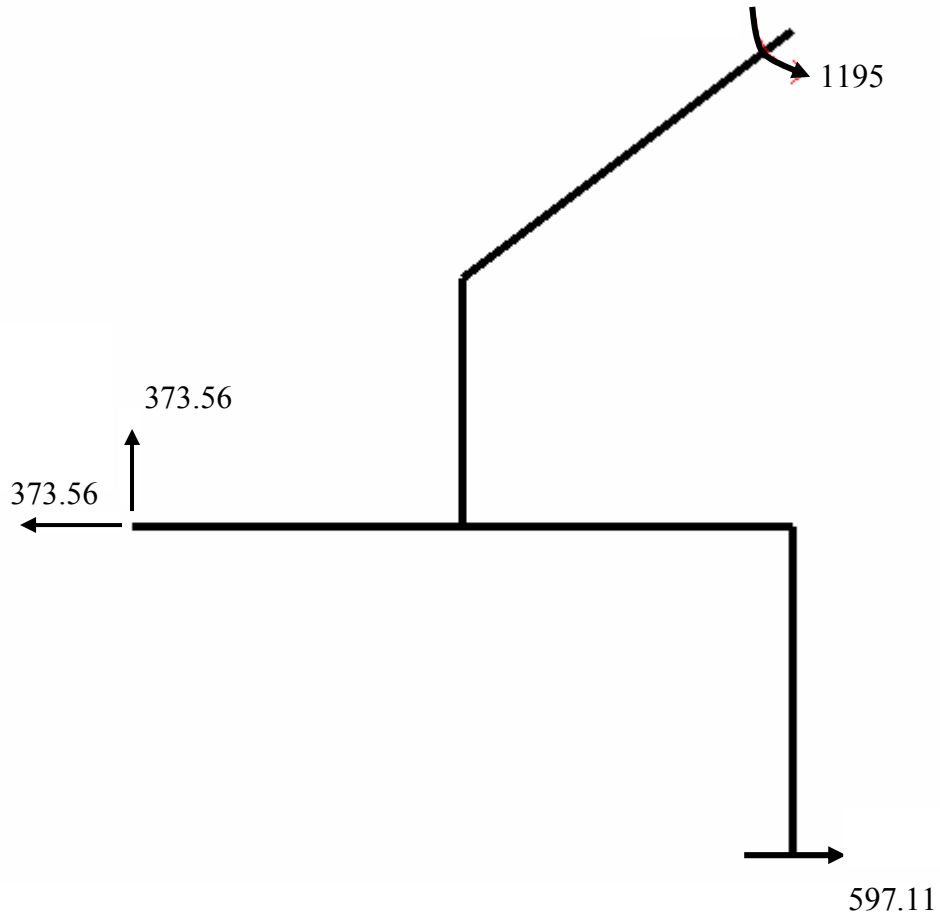


(dimenzije u metrima)

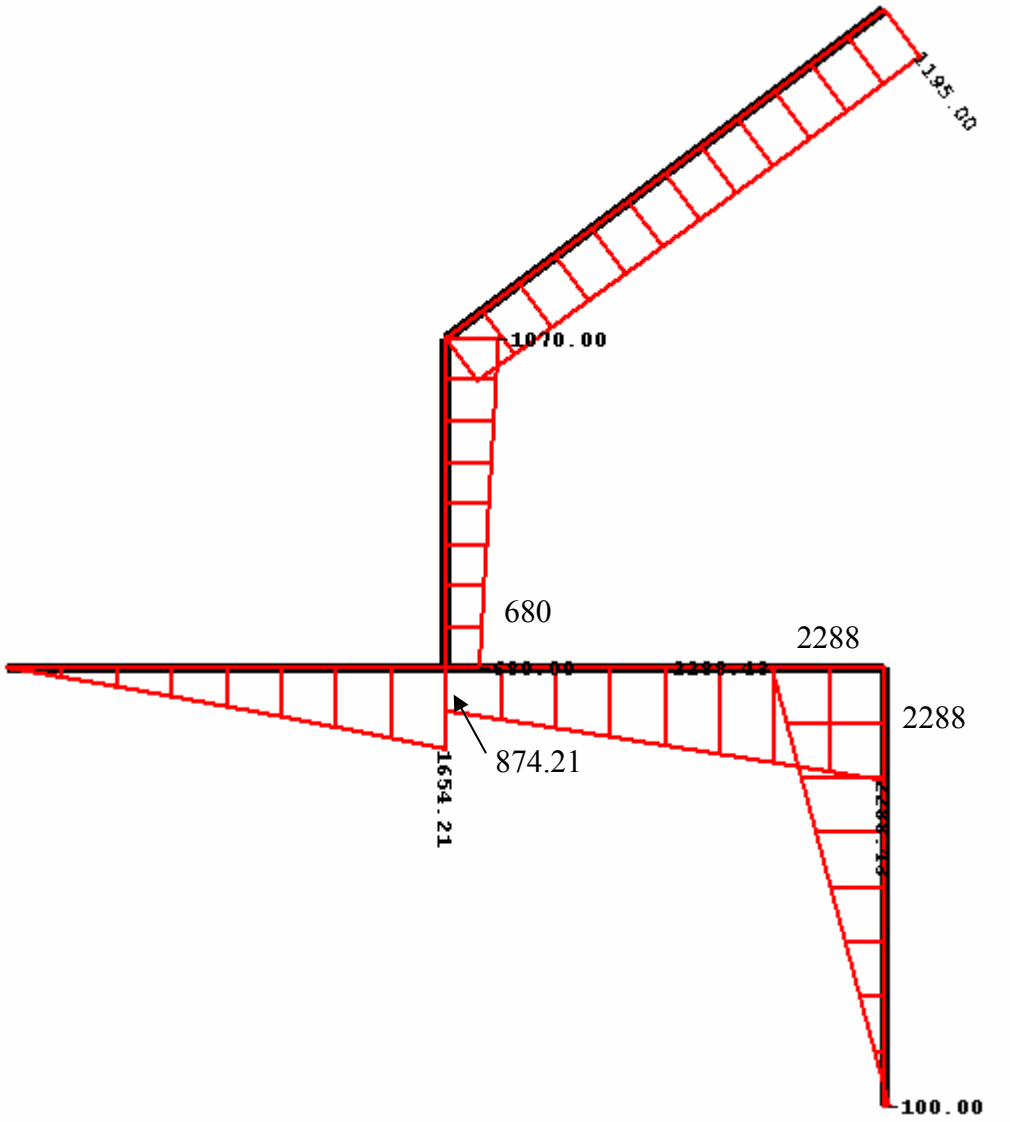
2.) Za uravnoteženi sustav iz prvog zadatka izračunati i nacrtati dijagrame unutarnjih sila.

Rješenja:

1. Stvarni smjerovi sila A, B i momenta Mc *(kompjutorsko rješenje, dozvoljena greška kod ručnog proračuna 2-3 %)*

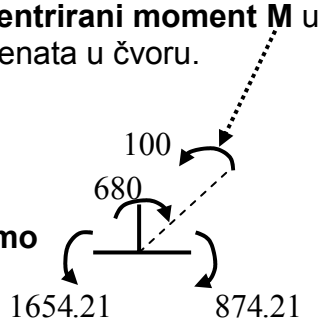


2. Dijagrami unutarnjih sila
M dijagram



Napomena:

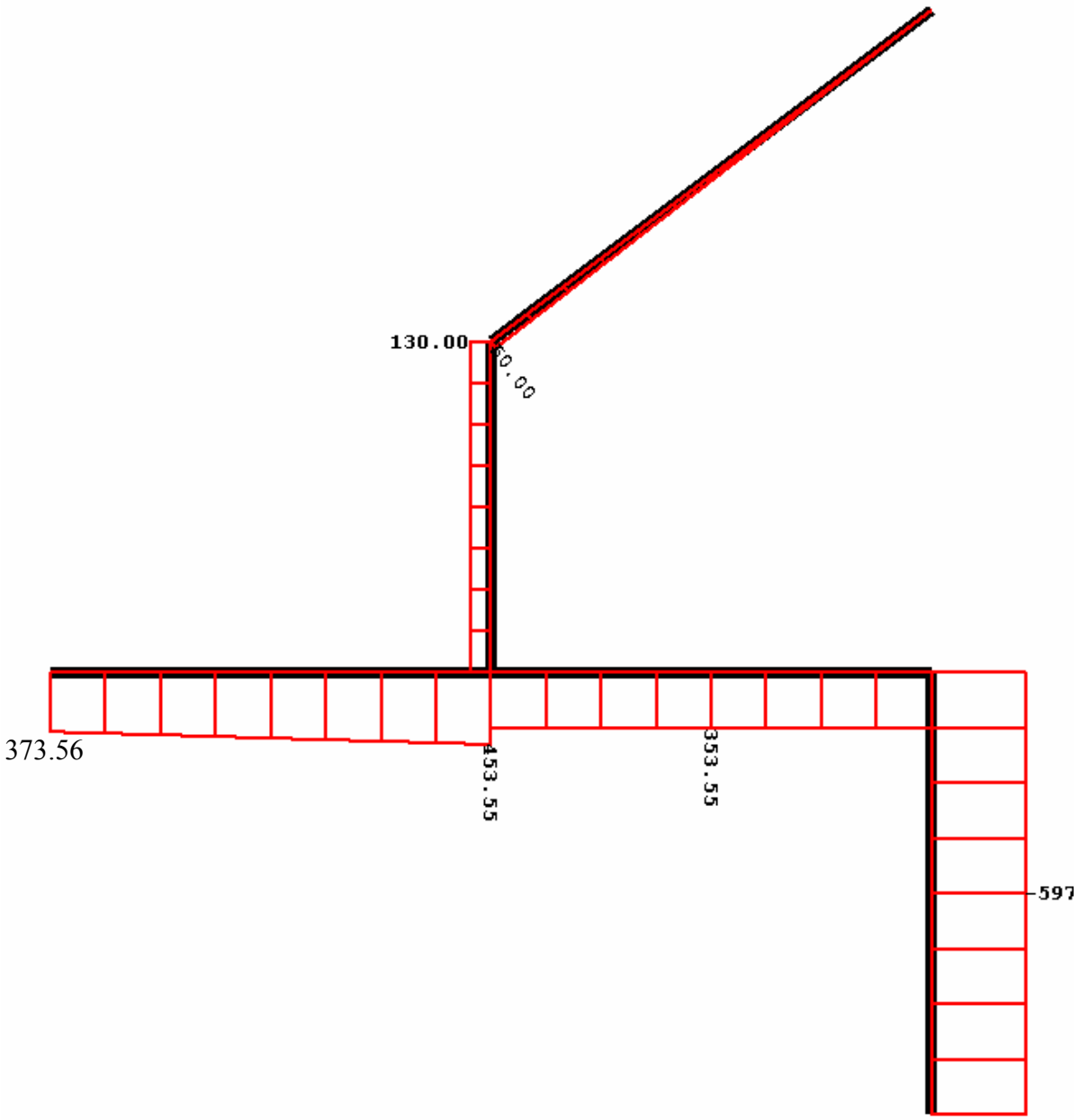
Na spoju 3 elemeta ***(u čvoru)*** zadan je **koncentrirani moment M** u iznosu od **100 kNm**. To je bitno zapamtiti kod kontrole momenata u čvoru.



Naučili smo da nam u čvoru mora biti zadovoljena ravnoteža momenata pa pišemo
ΣM=0
-680-874.21+1654.21=**100**

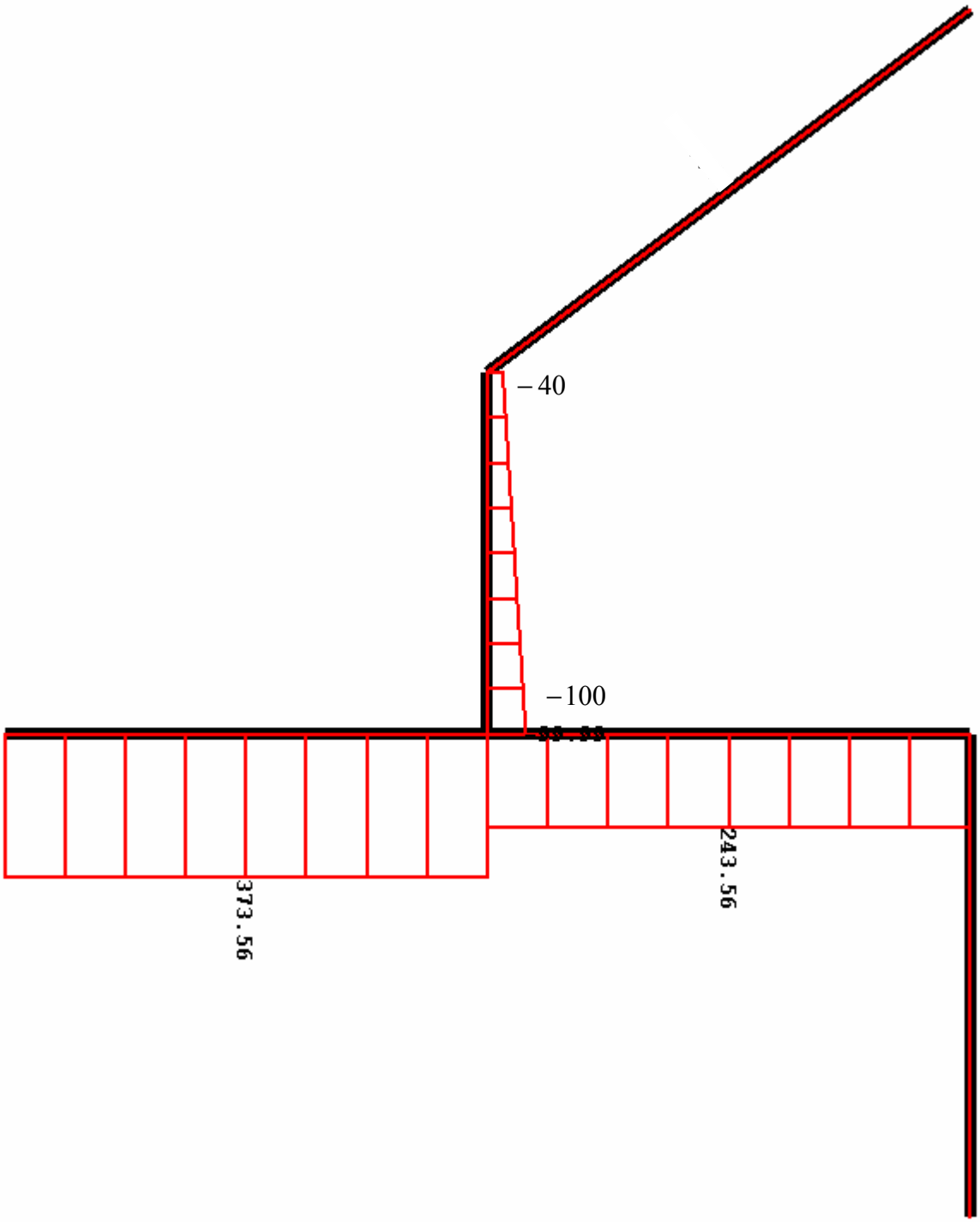
*Nismo dobili **0** kako smo naučili da treba biti, nego nam je iznos momenta, koji vrte suprotno od smjera kazaljke na satu, **veći za iznos 100**, a upravo toliko iznosi zadani koncentrirani moment. Znači, **pojavio se "skok" za 100 kNm** u smjeru zadanog momenta, jer smo naučili da **na mjestu koncentriranog momenta uvijek** imamo **skok** u dijagramu momenata u smjeru zadanog momenta.*

T dijagram



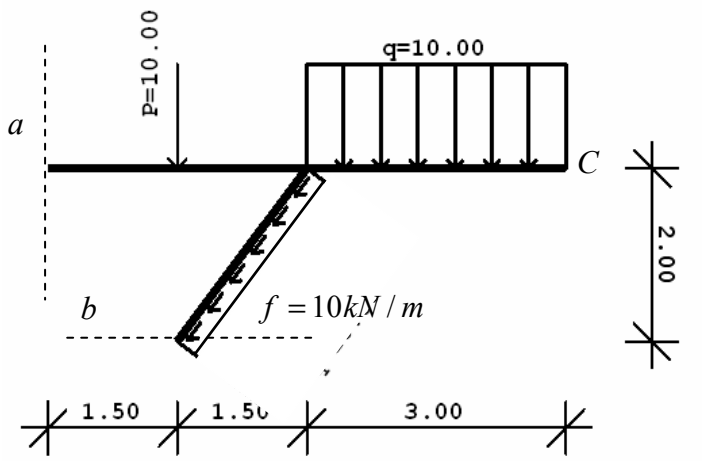
Ovo je kompjutorski nacrtan dijagram poprečnih sila. Predznaci odgovaraju, ali su dijagrami nacrtani na suprotnim stranama od onih na koje smo mi navikli u prethodnim zadacima.
Međutim, to nije greška te ako vi nacrtate dijagram na krivoj strani, ali upišete točan predznak, rješenje će biti dobro.
Ako ove dijagrame zrcalite oko osi štapa izgledat će onako kako smo mi naučili na vježbama iz Tehničke mehanike.

N dijagram

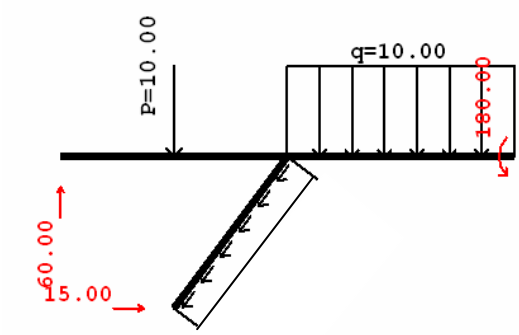


Zadatak 44: kosi štap i horizontalna sila.

Zadani sustav uravnotežite silama na pravcima a i b te momentom u točki C. Za tako uravnoteženi sustav nacrtajte dijagrame unutarnjih sila.

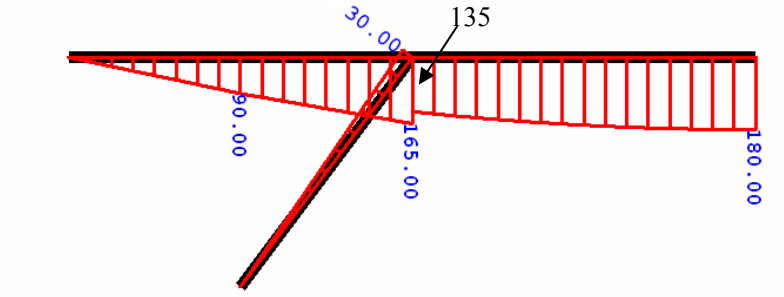


Ako krivo projicirate horizontalnu silu na kosi štap dobit ćete krive vrijednosti za T i N, a to znači i krive dijagrame poprečnih i uzdužnih sila. !!!

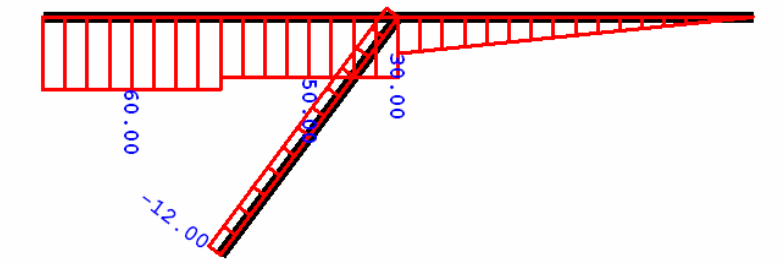


Rješenje:
Uravnotežavajuće sile
A= 60 kN, B= 15 kN i
uravnotežavajući moment
M_c= 180 kNm

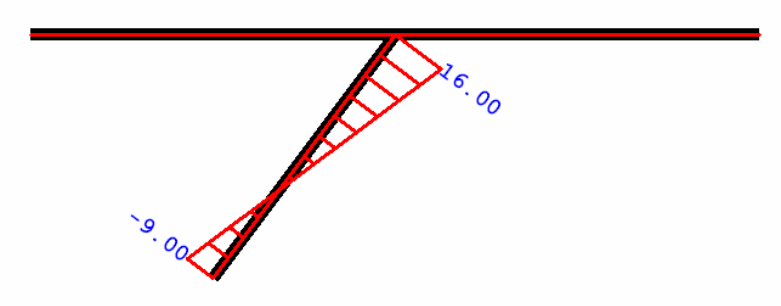
M dijagram



T dijagram



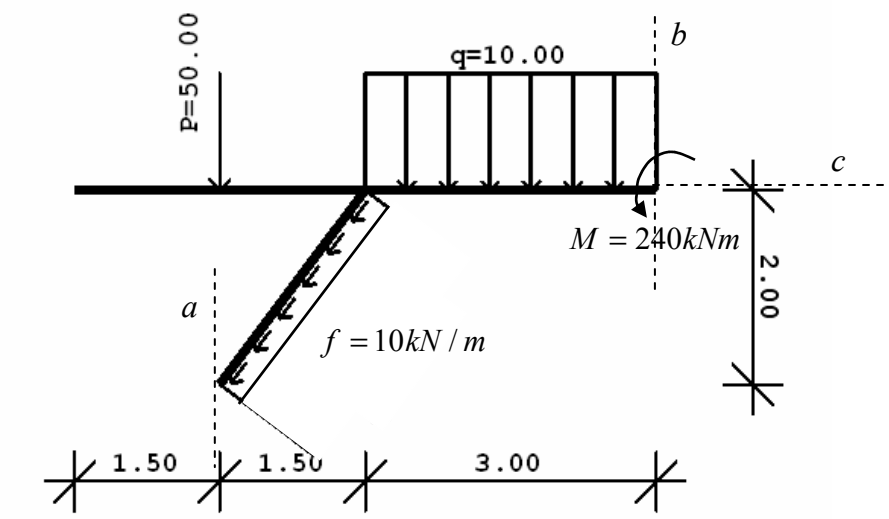
(dijagrame treba zrcaliti oko osi pojedinih štapova pa će izgledati kako smo naučili na vježbama)

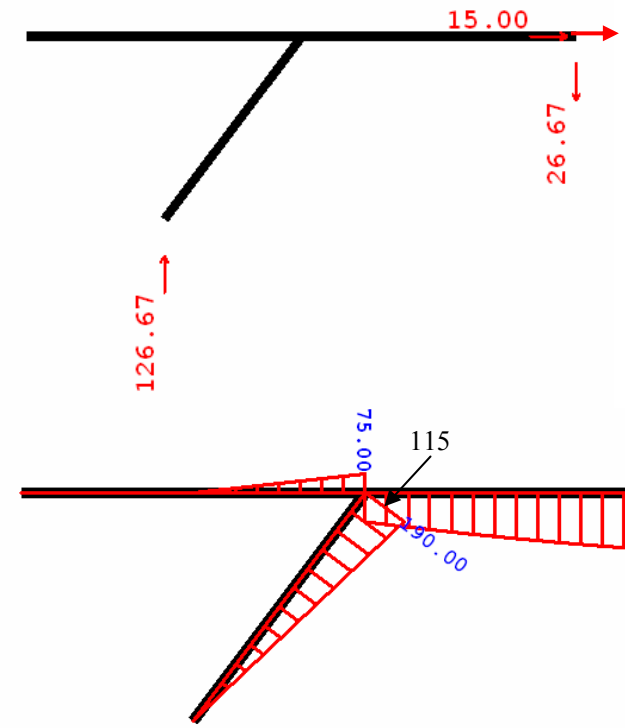


N dijagram

Zadatak 45: kosi štap i vertikalna sila.

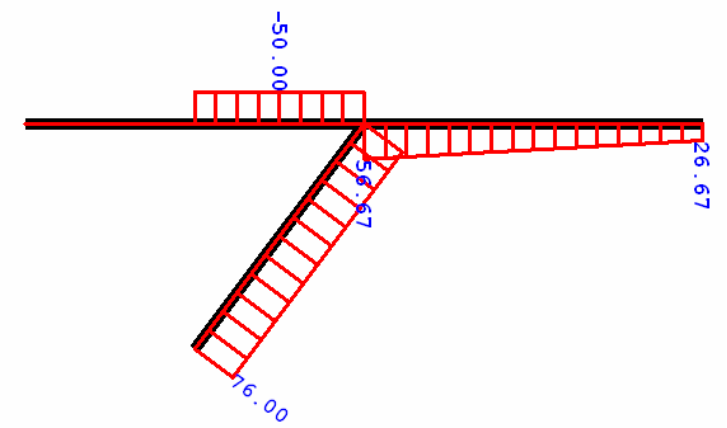
Zadani sustav uravnotežite silama na pravcima a, b i c. Za tako uravnoteženi sustav nacrtajte dijagrame unutarnjih sila.





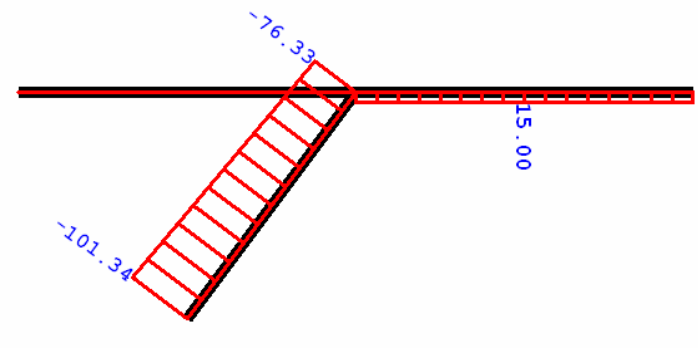
Rješenje:
Uravnotežavajuće sile:
A=126.67 kN, B= 26,67 kN
C=15 kN

M dijagram



T dijagram

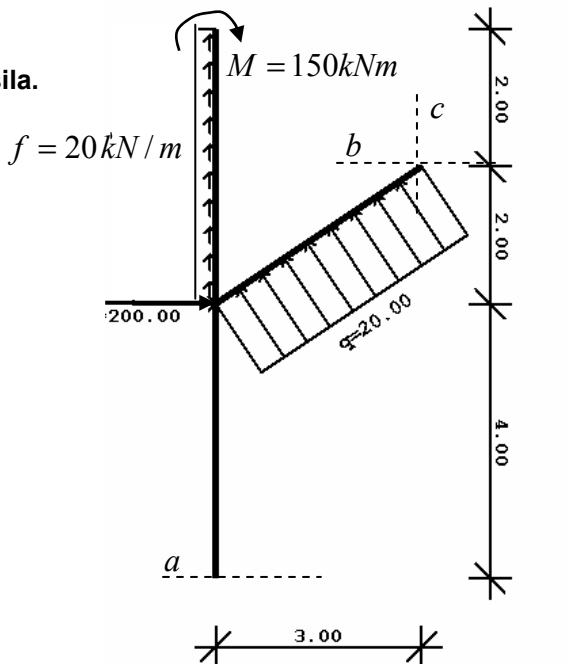
(dijagrame treba zrcaliti oko osi pojedinih štapova pa će izgledati kako smo naučili na vježbama)

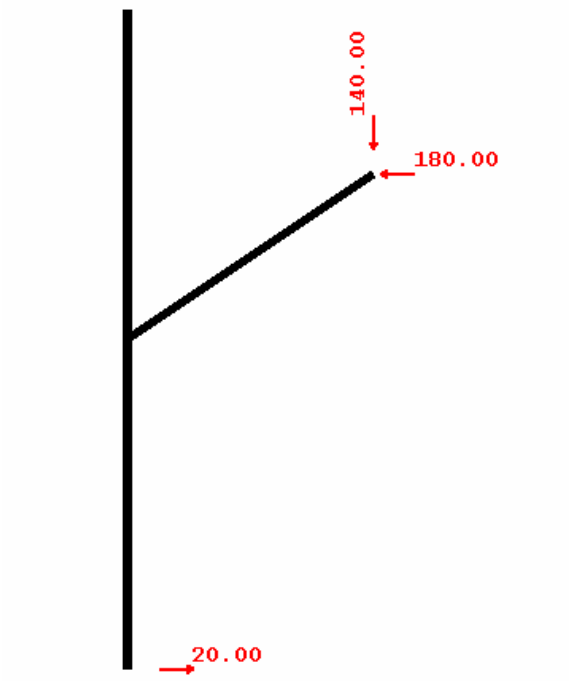


N dijagram

Zadatak 46: kosi štap s vertikalnom i horizontalnom silom

Zadani sustav uravnotežite silama na pravcima a, b i c. Za tako uravnoteženi sustav nacrtajte dijagrame unutarnjih sila.

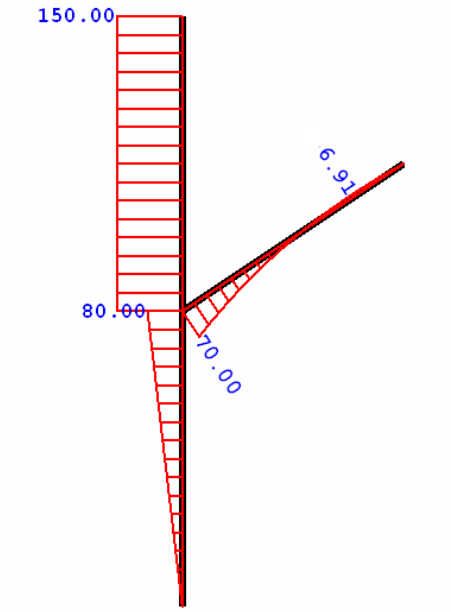




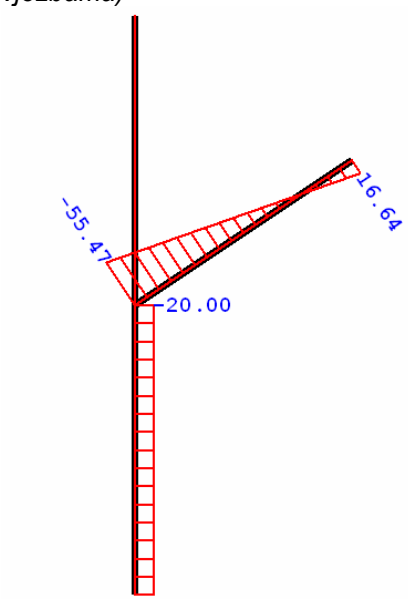
Rješenje:

Uravnotežavajuće sile:
A= 20 kN B= 180 kN C= 140 kN

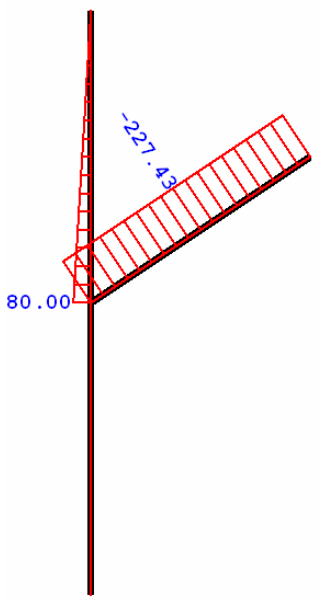
M dijagram



T dijagram *(dijagrame treba zrcaliti oko osi pojedinih štapova pa će izgledati kako smo naučili na vježbama)*

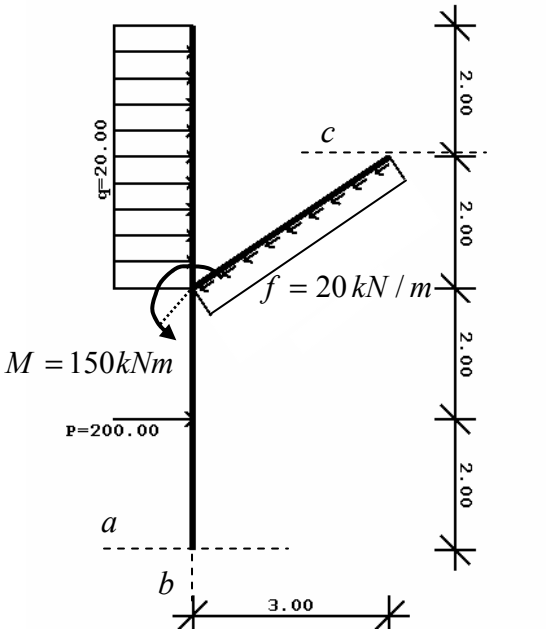


N dijagram



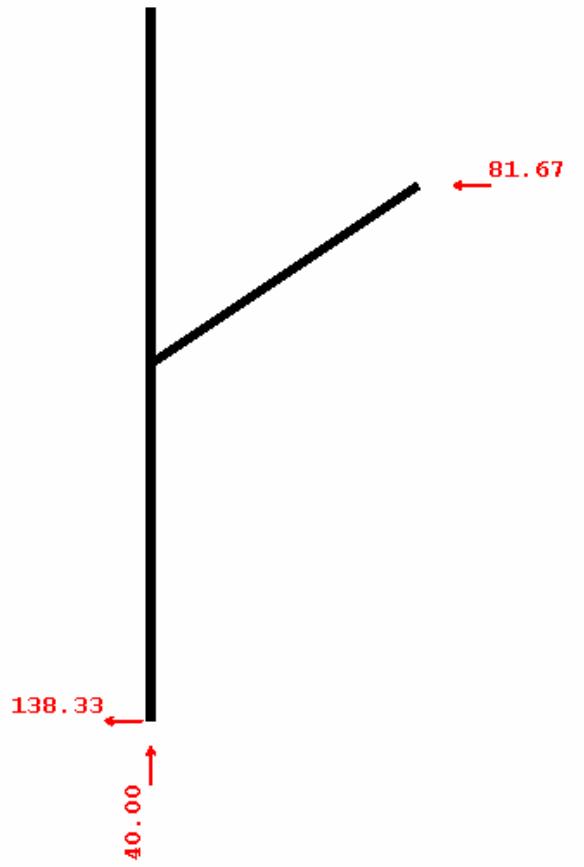
Zadatak 47: koncentrirani moment u čvoru

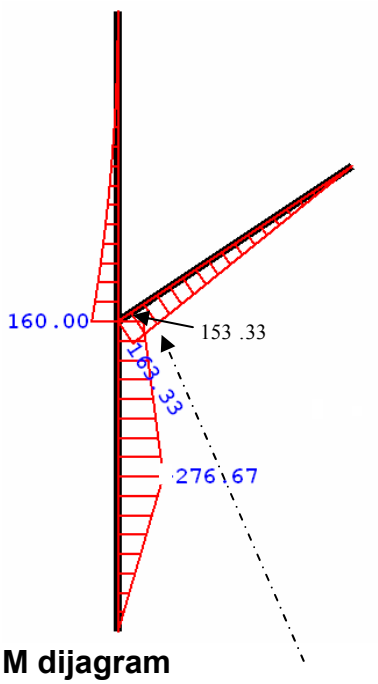
Zadani sustav uravnotežite silama na pravcima a, b i c.
Za tako uravnotežen sustav nacrtajte dijagrame unutarnjih sila.



Rješenje:

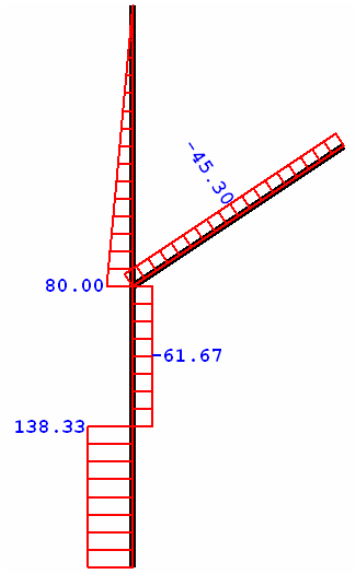
Uravnotežavajuće sile :
A= 138.33 kN B=40 kN C= 81.67 kN





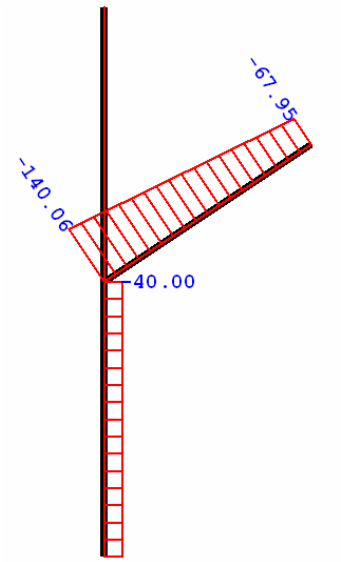
M dijagram

(u spoju tri štapa suma momenata nije jednaka 0, nego u smjeru zadanog momenta imamo skok za 150 kNm; tj. u smjeru kazaljke na satu imamo moment od 163.33 kNm, a suprotno od kazaljke na satu 313.33 kNm)



T dijagram

(dijagrame treba zrcaliti oko osi pojedinih štapova pa će izgledati kako smo naučili na vježbama)

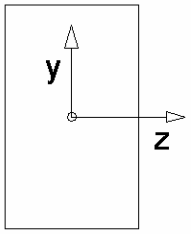


N dijagram

3. DIO – primjeri i zadaci za 3. kolokvij

KARAKTERISTIKE POPREČNOG PRESJEKA ŠTAPA

U štapnoj mehanici pojavljuju se sljedeće karakteristike:

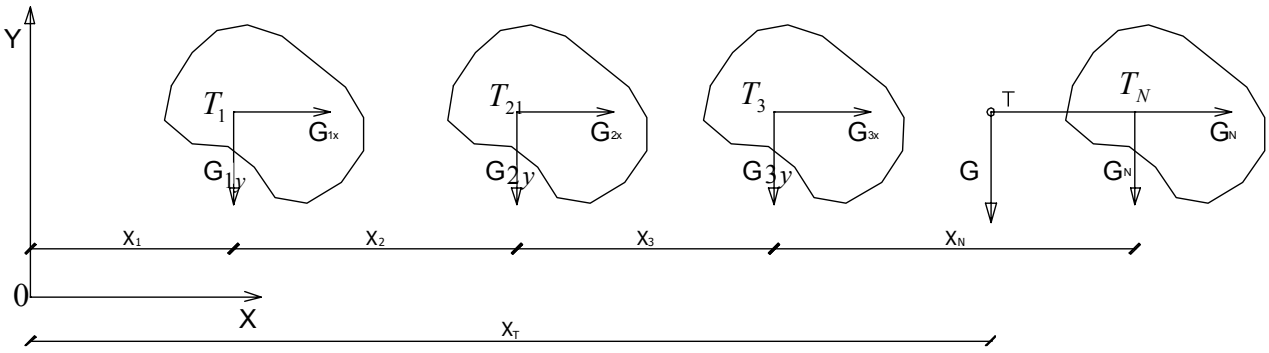


- površina $F = \int_F dF$
- težište T

- statički moment površine na os z ili os y $S_z = \iint_F y dF$; $S_y = \iint_F z dF$
- momenti inercije $I_z = \iint_F y^2 dF$; $I_y = \iint_F z^2 dF$, $I_{zy} = \iint_F zy dF$

TEŽIŠTE

Možemo ga tražiti kao da tražimo "težište" sustava paralelnih sila → tj. težište je točka kroz koju treba proći rezultanta sustava paralelnih sila



$\sum y = G \qquad G = G_1 + G_2 + G_3 + ... + G_N$

$$\sum^{traženo} M_{(O)} = \sum^{zadano} M_{(O)}$$

$$G \times x_T = G_1 \times x_1 + G_2 \times x_2 + G_3 \times x_3 + ... + G_N \times x_N$$

$$x_T = \frac{G_1 \times x_1 + G_2 \times x_2 + G_3 \times x_3 + ... + G_N \times x_N}{G}$$

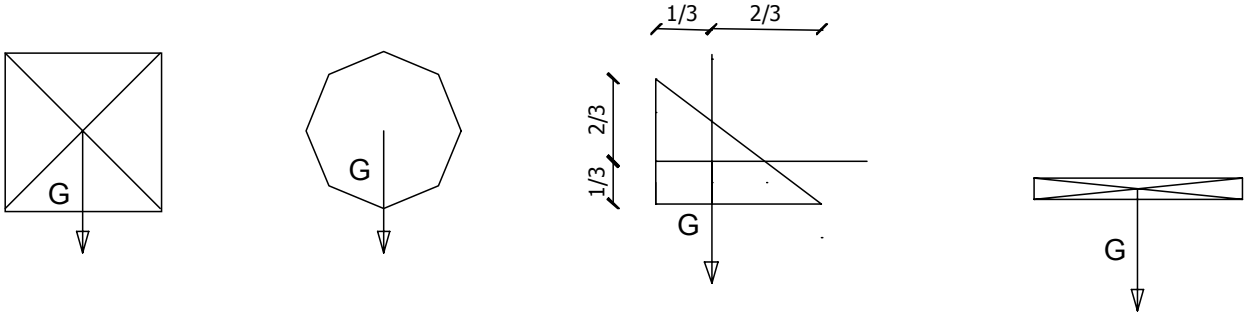
isto vrijedi i za "y" smjer

$$y_T = \frac{G_1 \times y_1 + G_2 \times y_2 + G_3 \times y_3 + ... + G_N \times y_N}{G}$$

za "z" smjer

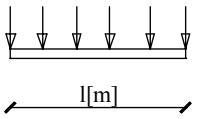
$$z_T = \frac{G_1 \times z_1 + G_2 \times z_2 + G_3 \times z_3 + ... + G_N \times z_N}{G}$$

Zamjenjujuće težine, tj. **površine** uvijek zamišljamo da djeluju u težištu pojedinog poznatog lika :

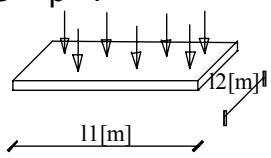


Izračun težina za :

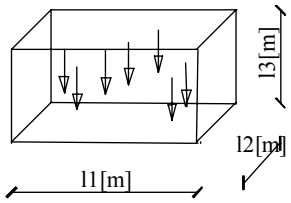
Linijsko tijelo:
q-linijska težina [N/m]
$G = q \cdot l$



Plošno tijelo:
p – plošna težina [N/m²]
$G = p \cdot F$

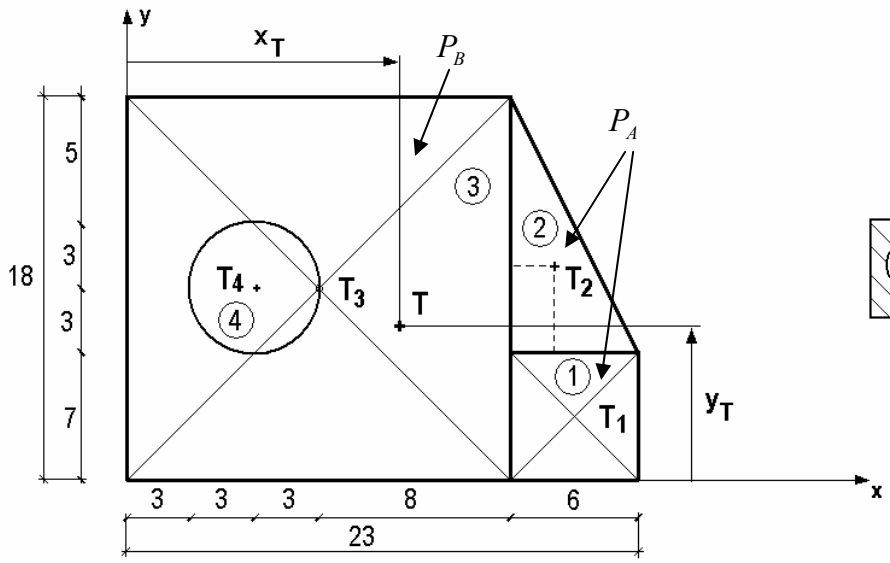


Tijelo u prostoru:
γ - volumenska težina [N/m³]
$G = \gamma \cdot V$



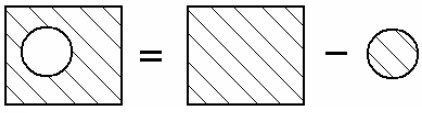
ZADATAK 48:

Tijelo prikazano na slici izrađeno je od dva različita materijala. Treba odrediti položaj težišta tijela.



P_A= 20 N/m^2

P_B = 12 N/m^2



F = F_3 – **F_4**

Površine :

$F_1 = 6 \times 7 = 42 m^2$

$F_2 = (6 \times 11)/2 = 33\ m^2$

$F_3 = 17 \times 18 = 306\ m^2$

F_4 = $-3^2 \times \pi = -28{,}26\ m^2$ (rupa)

Težine:

$G_1 = P_A \times F_1 = 20 \times 42 = 840$ N

$G_2 = P_A \times F_2 = 20 \times 33 = 660$ N

$G_3 = P_B \times F_3 = 12 \times 306 = 3672$ N

$G_4 = P_B \times F_4 = -12 \times 28{,}26 = -339{,}12$ N

$\sum G = 4.832{,}88$ N

Udaljenosti do osi :

x_1=23 - 6/2 = 20 m y_1= 7/2 = 3,5 m

x_2=17 + 6/3 = 19 m y_2= 7 + 11/3 = 10,66 m

x_3=17/2 = 8,5 m y_2= 18/2 = 9 m

x_4= 6 m y_4= 10 m

$$x_T = \frac{G_1 \times x_1 + G_2 \times x_2 + G_3 \times x_3 + G_4 \times x_4}{G} = \frac{840 \times 20 + 660 \times 19 + 3672 \times 8{,}5 - 339{,}12 \times 6}{4832{,}88} =$$

x_T = 12,10 m

$$y_T = \frac{G_1 \times y_1 + G_2 \times y_2 + G_3 \times y_3 + G_4 \times y_4}{G} = \frac{840 \times 3{,}5 + 660 \times 10{,}66 + 3672{,}9 \times 9 - 339{,}12 \times 10}{4832{,}88} =$$

y_T = 8,2 m

Kontrola : $\sum M_{(T)}^{traženo} = \sum M_{(T)}^{zadano} = 0$ *(tražimo sumu momenata oko težišta zadanih "sila")*

$$G_1 \times (x_T - x_1) + G_2 \times (x_T - x_2) + G_3 \times (x_T - x_3) + G_4 \times (x_T - x_4) = 0$$

840 × (12,1-20) + 660 × (12,1-19) + 3672 × (12,1-8,5) - 339,12 × (12,1-6) =
840 × (-7,9) + 660 × (-6,9) + 3672 × (3,6) - 339,12 × (6,1) =
-6636 – 4554 + 13219,2 – 2068,63 = -39,43

Greška** : (39,43/13219)×100 =* ***0,3 % *- (postotak se računa s obzirom na najveći moment u zbroju)*

**** provjerite sami kontrolu za y smjer***

ZADATAK 49:

Zadano je tijelo istog oblika kao u prethodnom zadatku, ali je izrađeno od jednog materijala. Odredite položaj težišta:

Kad je cijelo tijelo od istog materijala možemo uzeti
$p_1 = p_2 = p_3 = p_4 = 1\ N/m^2$ pa je tada

$$\left.\begin{aligned} G_1 &= F_1 = 42N \\ G_2 &= F_2 = 33N \\ G_3 &= F_3 = 306N \\ G_4 &= F_4 = -28{,}26N \end{aligned}\right\} F = F_1 + F_2 + F_3 + F_4 = 352{,}74m^2$$

$$x_T = \frac{F_1 \times x_1 + F_2 \times x_2 + F_3 \times x_3 + F_4 \times x_4}{F} = 11{,}05\ \text{m}$$

$$y_T = \frac{F_1 \times y_1 + F_2 \times y_2 + F_3 \times y_3 + F_4 \times y_4}{F} = 8{,}42\ \text{m}$$

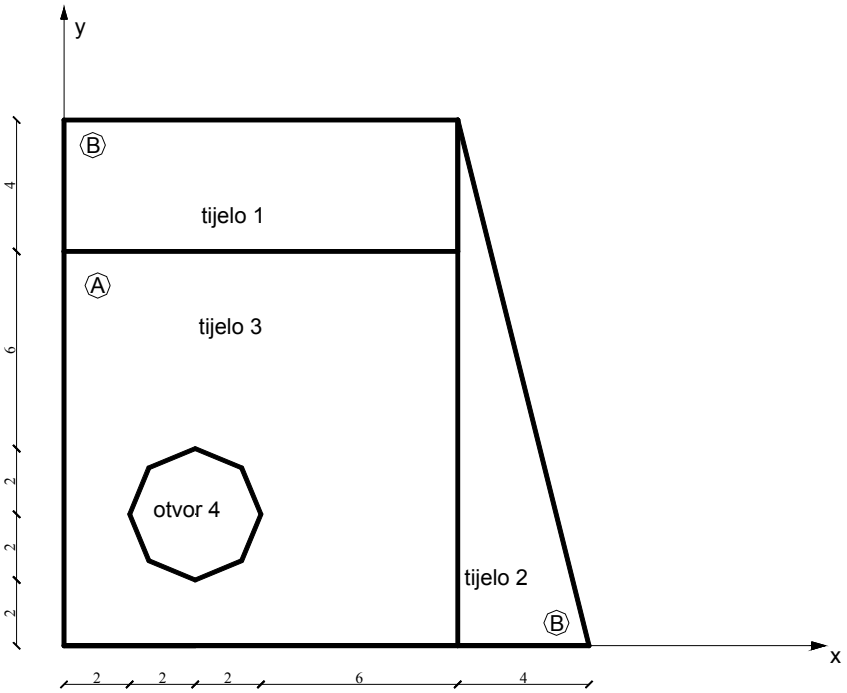
Kontrola! $\sum M_{(T)} = 0$ - provjerite sami

ZADATAK 50:

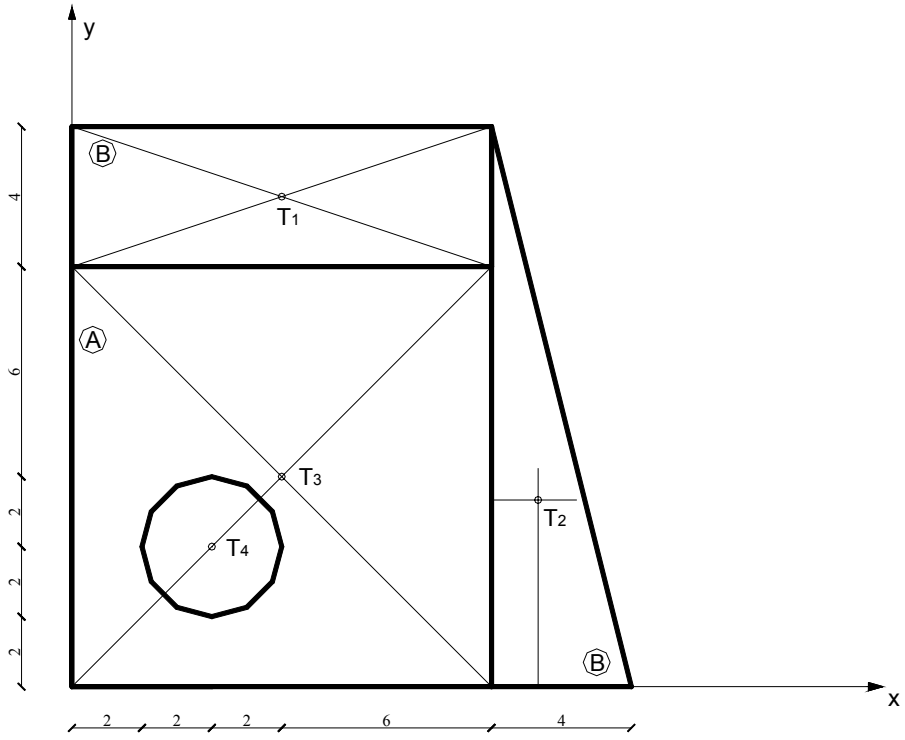
Za tijelo na slici odrediti položaj težišta :

a) za slučaj kada je tijelo izrađeno od jednog materijala p=20 N/m^2

b) za slučaj kada je tijelo izrađeno od različitih materijala
p_A=15 N/m^2
p_B=10 N/m^2



Određivanje težišta pojedinih tijela :



a).

Površine :
$F_1 = 12\times4 = 48m^2$
$F_2 = (4\times16)/2 = 32m^2$
$F_3 = 12\times12 = 144m^2$
$F_4 = -2^2\times\pi = -12.56m^2$

Težine:
$G_1 = p\times F_1 = 20\times48 = 960$ N
$G_2 = p\times F_2 = 20\times32 = 640$ N
$G_3 = p\times F_3 = 20\times144 = 2880$ N
$G_4 = -p\times F_4 = 20\times12.56 = 251.2$ N

$\sum G =$ 4228,8 N

Udaljenosti do osi :
x_1=12/2 = 6m
x_2=12 + 4×1/3 = 13.3m
x_3=12/2 = 6m
x_4= 4m

y_1=12+4/2 = 14m
y_2=16×1/3 = 5.3m
y_3=12/2 = 6m
y_4= 4m

$$x_T = \frac{G_1\times x_1 + G_2\times x_2 + G_3\times x_3 + G_4\times x_4}{G} = \frac{960\times6 + 640\times13.3 + 2880\times6 - 251\times4}{4228.8} = 7.2 \text{ m}$$

$$y_T = \frac{G_1\times y_1 + G_2\times y_2 + G_3\times y_3 + G_4\times y_4}{G} = \frac{960\times14 + 640\times5.3 + 2880\times6 - 251\times4}{4228.8} = 7.8 \text{ m}$$

Kontrola :

$$\sum^{\text{traženo}} M_{(T)} = \sum^{\text{zadano}} M_{(T)}$$

$$G_1\times(x_T - x_1) + G_2\times(x_T - x_2) + G_3\times(x_T - x_3) + G_4\times(x_T - x_4) = 0$$

960×(7.2-6)+640×(7.2-13.3)+2880×(7.2-6)-251.2×(7.2-4)= - 100

Greška : 100/3904 = ***2%*** - rezultat podijeljen sa najvećim momentom u zbroju :
(3904=640×(7.2-13.3))

b).

Površine :
$F_1 = 12\times4 = 48m^2$
$F_2 = (4\times16)/2 = 32m^2$
$F_3 = 12\times12 = 144m^2$
$F_4 = -2^2\times\pi = -12.56m^2$

Težine:
$G_1 = p_B\times F_1 = 10\times48 = 480$ N
$G_2 = p_B\times F_2 = 10\times32 = 320$ N
$G_3 = p_A\times F_3 = 15\times144 = 2160$ N
$G_4 = -p_A\times F_4 = 15\times12.56 = 188.2$ N

$\sum G = 2771.6$ N

Udaljenosti do osi :
$x_1=12/2 = 6$m
$x_2=12 + 4\times1/3 = 13.3$m
$x_3=12/2 = 6$m
$x_4= 4$m

$y_1=12+4/2 = 14$m
$y_2=16\times1/3 = 5.3$m
$y_3=12/2 = 6$m
$y_4= 4$m

$$x_T = \frac{G_1\times x_1 + G_2\times x_2 + G_3\times x_3 + G_4\times x_4}{G} = \frac{480\times6 + 320\times13.3 + 2160\times6 - 188.2\times4}{2771.6}$$

=7.0 m

$$y_T = \frac{G_1\times y_1 + G_2\times y_2 + G_3\times y_3 + G_4\times y_4}{G} = \frac{480\times14 + 320\times5.3 + 2160\times6 - 188.2\times4}{2771.6}$$

=7.4m

Kontrola :

$$\sum M_{(T)}^{\text{traženo}} = \sum M_{(T)}^{\text{zadano}}$$

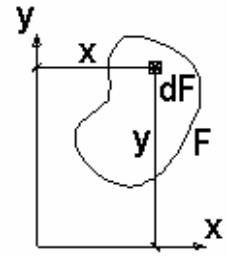
$$G_1\times(x_T - x_1) + G_2\times(x_T - x_2) + G_3\times(x_T - x_3) + G_4\times(x_T - x_4) = 0$$

480×(7.0-6)+320×(7.0-13.3)+2160×(7.0-6)-188.2×(7.0-4)= 59.4

Greška : 59.4/2160 = ***2.7%*** - rezultat podijeljen sa najvećim momentom u zbroju :
(2160=2160×(7.0-6))

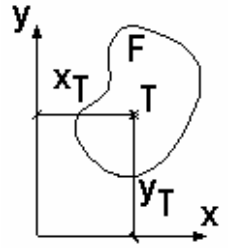
STATIČKI MOMENTI I MOMENTI INERCIJE RAVNIH PLOHA

STATIČKI MOMENTI POVRŠINE



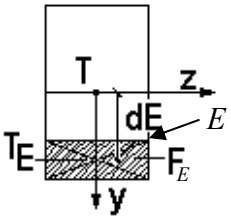
$$S_X = \iint_F y dF$$

$$S_y = \iint_F x dF$$



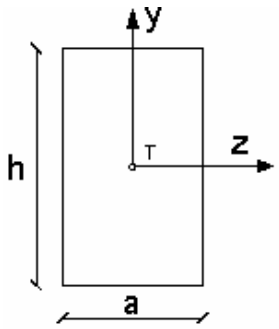
$S_X = F \times y_T$ *statički moment površine F s obzirom na osi x i y*

$S_y = F \times x_T$



$S_Z^E = F_E \times d_E$ *statički moment površine F_E (ispod točke E) s obzirom na težište poprečnog presjeka.*

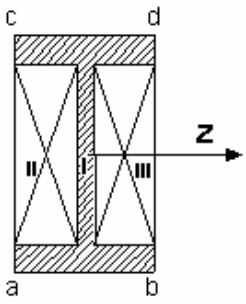
MOMENTI INERCIJE RAVNIH PLOHA



$$I_z = \iint_F y^2 dF \qquad I_y = \iint_F z^2 dF$$

$$I_z = \frac{a \times h^3}{12} \qquad I_y = \frac{h \times a^3}{12}$$

Kod složenih profila (ali s istim položajem težišta) rastavljamo presjek na jednostavnije dijelove.

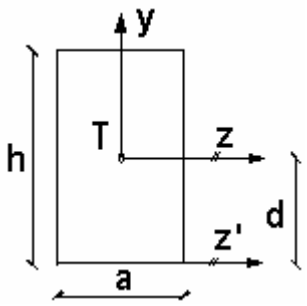


1.) $I_{ZI.}$ (pravokutnik a,b,c,d)
2.) $I_{ZII.}$ i $I_{Z\,III.}$ (mali pravokutnici). → $I_Z = I_{Z_I} - I_{Z_{II}} - I_{Z_{III}}$

STEINEROVO PRAVILO

Momenti inercije presjeka s obzirom na os z' koja je *paralelna* s osi kroz težište presjeka

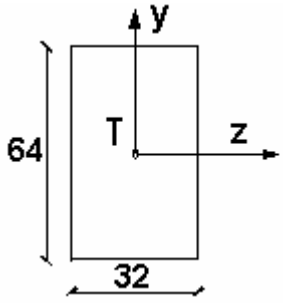
$$I_Z = \frac{a \times h^3}{12}$$



$$I_Z^{'} = I_Z + d^2 \times F$$

PRORAČUN POVRŠINE, TEŽIŠTA, MOMENTA INERCIJE TE STATIČKOG MOMENTA POVRŠINE ZA NEKE SIMETRIČNE PRESJEKE

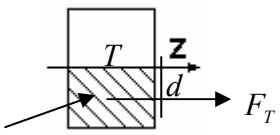
Primjer 1. (pravokutni presjek)



F = 32×64=2048 cm^2

$I_z = \frac{32 \times 64^3}{12}$ = 699 050,66 cm^4

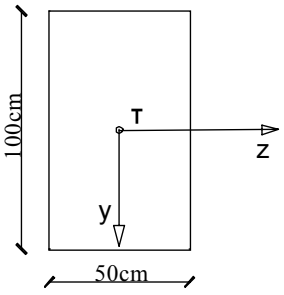
$I_y = \frac{64 \times 32^3}{12}$ = 174 762,70 cm^4



$S_z = F_T \times d = 32 \times 32 \times 16 = 16\,384$ cm^3

S_z = **S_{zmax}** *(statički moment površine oko težišta je maksimalan)*

Primjer 2. (pravokutni presjek)



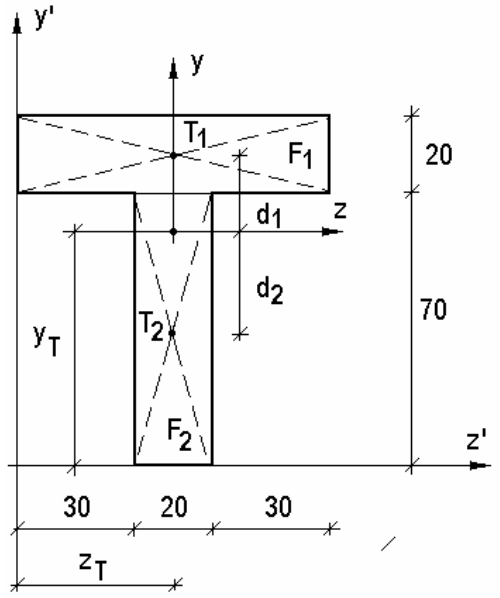
F = 50×100=5000 cm^2

$I_z = \frac{50 \times 100^3}{12}$ = 4 166 666,67 cm^4

$I_y = \frac{100 \times 50^3}{12}$ = 1 041 666,67 cm^4

$S_z = F_T \times d = 50 \times 50 \times 25 = 62\,500$ cm^3

Primjer 3. (T presjek)



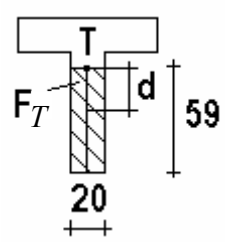
$F = F_1 + F_2 = 80\times20+70\times20=3000$ cm^2

$T_1 \Rightarrow z_1 = 40$ cm ; $y_1 = 80$ cm
$T_2 \Rightarrow z_2 = 40$ cm ; $y_1 = 35$ cm

$$z_T = \frac{F_1 \times z_1 + F_2 \times z_2}{F} = 40 \text{ cm}$$

$$y_T = \frac{F_1 \times y_1 + F_2 \times y_2}{F} = \frac{1600\times80+1400\times35}{3000} = 59 \text{ cm}$$

$S_Z = F_T \times d = 59 \times 20 \times \frac{59}{2} = 34\,810$ cm^3



$d_1 = y_1 - y_T = 80\text{-}59 = 21$ cm
$d_2 = y_2 - y_T = 35\text{-}59 = -24$ cm

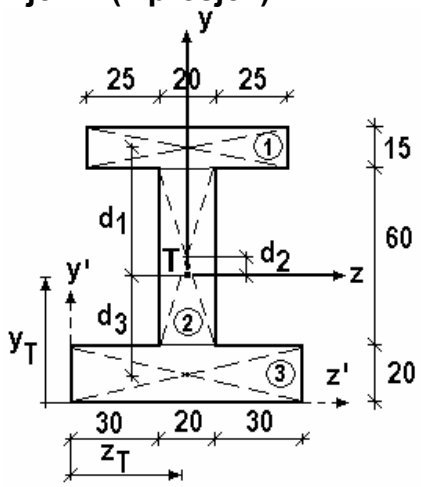
Pomoću Steinerrovog pravila:

$$I_Z = I_{Z1} + F_1 \times d_1^2 + I_{Z2} + F_2 \times d_2^2 = = \frac{80\times20^3}{12} + 80\times20\times21^2 + \frac{20\times70^3}{12} + 20\times70\times24^2 =$$

$= 53\,333{,}33 + 705600 + 571\,666{,}67 + 806\,400 = 2\,137\,000$ cm^4

$$I_y = I_{y1} + I_{y2} = \frac{20\times80^3}{12} + \frac{70\times20^3}{12} = 853\,333{,}33 + 46666{,}67 = 900\,000 \text{ cm}^4$$

Primjer 4. (I presjek)



$F_1 = 70\times15 = 1050$ cm^2
$F_2 = 60\times20 = 1200$ cm^2
$F_3 = 80\times20 = 1600$ cm^2
$\Sigma F = F_1+F_2+F_3 = 3850$ cm^2

$z_1 = 40$cm	$y_1 = 87{,}5$ cm
$z_2 = 40$cm	$y_2 = 50$ cm
$z_3 = 40$cm	$y_3 = 10$ cm

$$z_T = \frac{F_1 \times z_1 + F_2 \times z_2 + F_3 \times z_3}{F} = \frac{(1050+1200+1600)\times40}{3850} = 40 \text{ cm}$$

$$y_T = \frac{F_1 \times y_1 + F_2 \times y_2 + F_2 \times y_2}{F} = \frac{1050 \times 87{,}5 + 1200 \times 50 + 1600 \times 10}{3850} = 43{,}6\text{cm}$$

$d_1 = y_1 - y_T = 87{,}5 - 43{,}6 = 43{,}90$ cm
$d_2 = y_2 - y_T = 50 - 43{,}60 = 6{,}40$ cm
$d_3 = y_3 - y_T = 10 - 43{,}60 = -33{,}60$ cm

Pomoću Steinerrovog pravila:

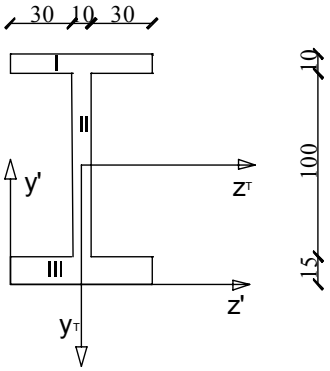
$$I_Z = I_{Z1} + F_1 \times d_1^2 + I_{Z2} + F_2 \times d_2^2 + I_{Z3} + F_3 \times d_3^2 = \frac{70 \times 15^3}{12} + 70 \times 15 \times 43{,}9^2 +$$

$$+ \frac{20 \times 60^3}{12} + 20 \times 60 \times 6{,}4^2 + \frac{80 \times 20^3}{12} + 80 \times 20 \times 33{,}60^2 = 4\,312\,079{,}33\ \text{cm}^2$$

$$I_Y = I_{Y1} + I_{Y2} + I_{Y3} = \frac{15 \times 70^3}{12} + \frac{60 \times 20^3}{12} + \frac{20 \times 80^3}{12} = 1\,322\,083\ \text{cm}^4$$

$$S_Z = (43{,}6-20) \times 20 \times \frac{43{,}60 - 20}{2} + 20 \times 80 \times (43{,}60-10) = 5569{,}6 + 53\,760 = 59\,329{,}6\ \text{cm}^3$$

Primjer 5. (T presjek)



$F_1 = 70 \times 10 = 700\ \text{cm}^2$
$F_2 = 10 \times 100 = 1000\ \text{cm}^2$
$F_3 = 70 \times 15 = 1050\ \text{cm}^2$
$F = 2750$

$z_1 = 30 + 10/2 = 35$cm
$z_2 = 35$cm
$z_3 = 35$cm

$y_1 = 15 + 100 + 10/2 = 120$ cm
$y_2 = 15 + 100/2 = 65$cm
$y_3 = 15/2 = 7.5$cm

$$z_T = \frac{F_1 \times z_1 + F_2 \times z_2 + F_3 \times z_3}{F} = \frac{700 \times 35 + 1000 \times 35 + 1050 \times 35}{2750} = 35\text{cm}$$

$$y_T = \frac{F_1 \times y_1 + F_2 \times y_2 + F_2 \times y_2}{F} = \frac{700 \times 120 + 1000 \times 65 + 1050 \times 7.5}{2750} = 57\text{cm}$$

$d_1 = y_1 - y_T = 120\text{-}57 = 63.0$cm
$d_2 = y_2 - y_T = 65\text{-}57 = 8$ cm
$d_3 = y_3 - y_T = 7.5\text{-}57 = -49.5$cm

S_Z = 15×70×(57-7.5) + (57-15)×10×(57-15)/2 = 60 795.0 cm^3

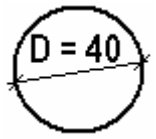
$$I_Z = I_{z1} + I_{Z2} + I_{Z3} = \frac{70\times10^3}{12} + 700\times63^2 + \frac{10\times100^3}{12} + 1000\times8^2 + \frac{70\times15^3}{12} + 1050\times(-49.5)^2 =$$

= 6 273922 cm^4

$$I_y = I_{y1} + I_{y2} + I_{y3} = \frac{10\times70^3}{12} + \frac{100\times10^3}{12} + \frac{15\times70^3}{12} =$$

= 722 916,67 cm^4

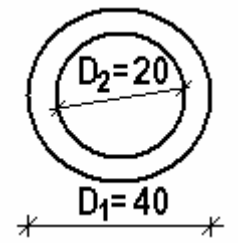
Primjer 6. (okrugli presjek)



$$F = r^2\times\pi = \frac{\pi\times D^2}{4} = 1256{,}64\ cm^2$$

$$I_Z = \frac{\pi\times D^4}{64} \approx \frac{D^4}{20} = 125\ 663{,}71\ cm^4 = I_Y$$

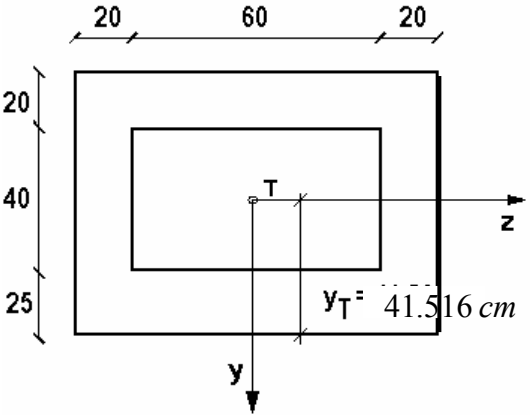
Primjer 7. (cjevasti presjek)



$$F = (D_1^2 - D_2^2)\times\frac{\pi}{4} = (40^2 - 20^2)\times\frac{\pi}{4} = 942{,}80\ cm^2$$

$$I_Z = I_{Z1} - I_{Z2} = \frac{\pi\times D_1^4}{64} - \frac{\pi\times D_2^4}{64} = (D_1^4 - D_2^4)\times\frac{\pi}{64} = 117\ 809{,}71\ cm^4$$

Primjer 8. (cjevasti pravokutni presjek)



F_1 =100×85 = 8500 cm^2
F_2 =60×40 = 2400 cm^2
F = F_1 - F_2 = 6100 cm^2

$T_1 \Rightarrow z_1$ = 50 cm ; y_1= 42,5 cm
$T_2 \Rightarrow z_2$ = 50 cm ; y_2= 25+20 cm

$$z_T = \frac{F_1 \times z_1 - F_2 \times z_2}{F} = \frac{8500 \times 50 - 2400 \times 50}{6100} = 50 \text{ cm}$$

$$y_T = \frac{F_1 \times y_1 - F_2 \times y_2}{F} = \frac{8500 \times 42{,}5 - 2400 \times 45}{6100} = 41{,}516 \text{ cm}$$

$d_1 = y_1 - y_T$ = 42,5 – 41,516 = 0,984 cm
$d_2 = y_2 - y_T$ = 45 – 41,516 = 3,484 cm

Pomoću Steinerrovog pravila:

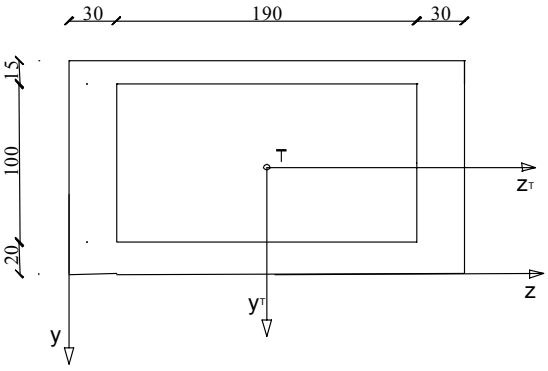
$$I_Z = I_{Z1} - I_{Z2} = \frac{100 \times 85^3}{12} + 100 \times 85 \times 0{,}984^2 - \left(\frac{60 \times 40^3}{12} + 60 \times 40 \times 3{,}484^2\right) =$$

= 5 117 708,33 + 8230,176 – (320 000 + 29 131,81) = 4 776 807 cm^4

$$I_y = I_{y1} - I_{y2} = \frac{85 \times 100^3}{12} - \frac{40 \times 60^3}{12} = 7\,083\,333{,}33 - 720\,000 = 6\,363\,333{,}33 \text{ cm}^4$$

$$S_Z = 41{,}516 \times 100 \times \frac{41{,}516}{2} - 60 \times (41{,}516 - 25) \times \frac{(41{,}516 - 25)}{2} = 78\,008{,}20 \text{ cm}^3$$

Primjer 9. (cjevasti pravokutni presjek)



F_1 = 250×135=33 750 cm^2 - cijeli pravokutnik
F_2 = 190×100=19 000 cm^2 - otvor

F = F_1 - F_2 =33750 – 19000 = 14 750 cm^2

z_1 = 250/2 = 125cm
z_2 = 250/2 = 125cm

y_1 = 135/2 = 67.5cm
y_2 = 100/2 +20= 70cm

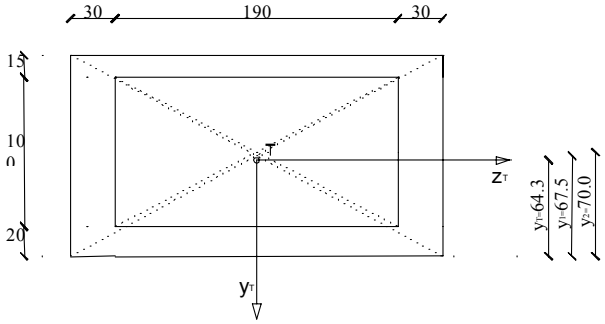
$$z_T = \frac{F_1 \times z_1 - F_2 \times z_2}{F} = \frac{33750 \times 125 - 19000 \times 125}{14750} = 125\text{cm}$$

$$y_T = \frac{F_1 \times y_1 - F_2 \times y_2}{F} = \frac{33750 \times 67.5 - 19000 \times 70}{14750} = 64.3\text{cm}$$

$S_Z = 64.3 \times 250 \times (64.3/2) - (64.3-20) \times 190 \times (64.3-20)/2 = 330.374{,}7\text{cm}^3$

$d_1 = y_1 - y_T = 67.5 - 64.3 = 3.2\text{cm}$

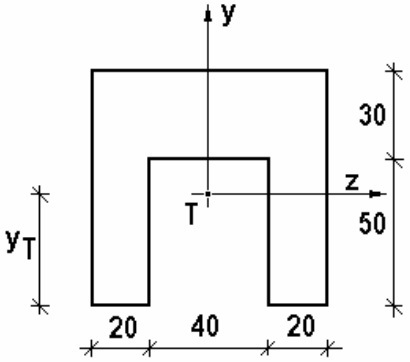
$d_2 = y_2 - y_T = 70 - 64.3 = 5.7\text{cm}$



$$I_Z = I_{z1} - I_{Z2} = \frac{250 \times 135^3}{12} + 33750 \times 3.2^2 - \frac{190 \times 100^3}{12} - 19000 \times 5.7^2 = 35\,152\,769{,}2\ \text{cm}^4$$

$$I_y = I_{y1} - I_{y2} = \frac{135 \times 250^3}{12} - \frac{100 \times 190^3}{12} = 118\,622\,916.67\ \text{cm}^4$$

Primjer 10. (otvoreni presjek)



$F_1 = 80 \times 80 = 6400\ \text{cm}^2$
$F_2 = 40 \times 50 = 2000\ \text{cm}^2$
$F = F_1 - F_2 = 4400\ \text{cm}^2$

$z_1 = z_2 = z_T = 40$ cm
$y_1 = 40$ cm $y_2 = 25$ cm

$$y_T = \frac{F_1 \times y_1 - F_2 \times y_2}{F} = \frac{6400 \times 40 - 2000 \times 25}{4400} = 46{,}82\ \text{cm}$$

$d_1 = 40 - 46{,}82 = -6{,}82$ cm
$d_2 = 25 - 46{,}82 = -21{,}82$ cm

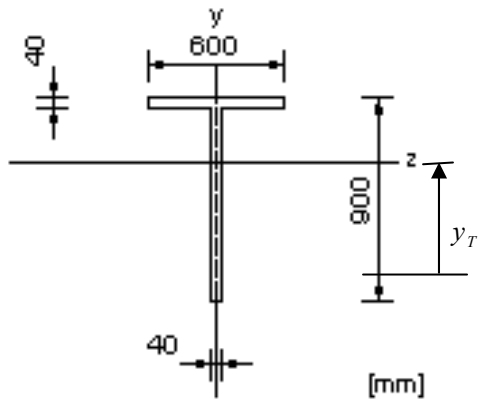
$$I_Z = I_{Z1} - I_{Z2} = \frac{80 \times 80^3}{12} + 6400 \times 6{,}82^2 - \left(\frac{40 \times 50^3}{12} + 40 \times 50 \times 21{,}82^2\right) = 2\,342\,121{,}3\ \text{cm}^4$$

$$I_y = I_{y1} - I_{y2} = \frac{80 \times 80^3}{12} - \frac{50 \times 40^3}{12} = 3\,146\,666{,}70\ \text{cm}^4$$

$$S_Z = 2 \times 46{,}82 \times 20 \times \frac{46{,}82}{2} = 43\,842{,}2\ \text{cm}^3$$

Primjeri za vježbu:

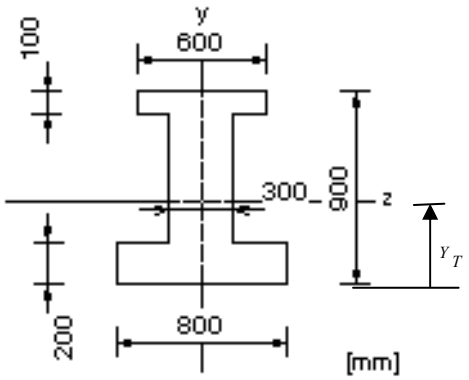
1.



Rješenje:

F= 0.0584 m^2 ;y_T= 0.615 m; I_z=0.0049861 m^4; I_y=0.000724586 m^4; S_z= 0.0075645 m^3

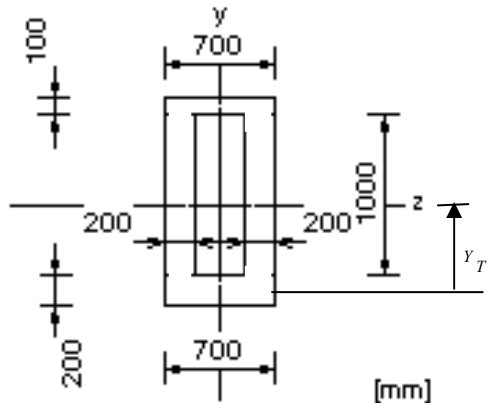
2



Rješenje:

F= 0.4 m^2 ;y_T= 0.3925 m; I_z=0.03431 m^4; I_y=0.011688 m^4; S_z= 0.052358 m^3

3.



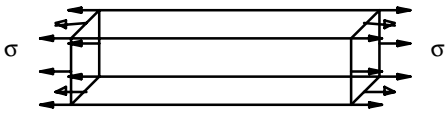
Rješenje:

F= 0.61 m^2 ;y_T= 0.6254 m; I_z=0.0102039 m^4; I_y=0.034908 m^4; S_z= 0.10976 m^3

NAPREZANJA

Općenito možemo reći da dokaz sigurnosti konstrukcije sadrži dokaz sigurnosti od sloma koji u materijalu nastupa kada sile između čestica u materijalu dosegnu neku graničnu vrijednost.

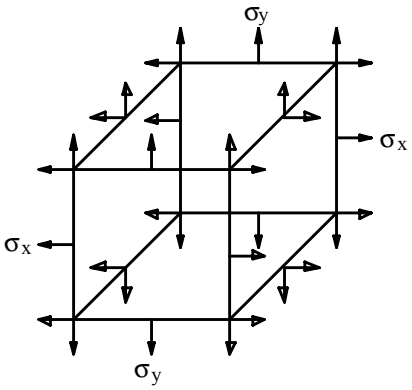
JEDNOOSNO POLJE NAPREZANJA

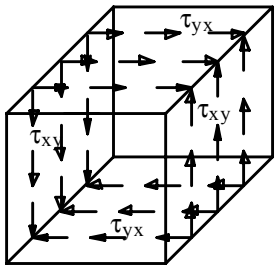


- na dvije stranice koje su okomite na karakterističnu os djeluje ukupno naprezanje čija se veličina označava sa σ.

DVOOSNO HOMOGENO POLJE NAPREZANJA

ukupno naprezanje = normalno naprezanje + posmično naprezanje
-prikaz na kvadru kojem su dvije stranice paralelne s karakterističnom ravninom.





Normalna naprezanja **σ**
dobivaju indeks po osi sa
kojom su paralelni σ_x i σ_y.

Posmična naprezanja **τ**
- označava se sa dva indeksa τ_{xy}

normala ravnine u kojoj djeluje

os s kojom su paralelni

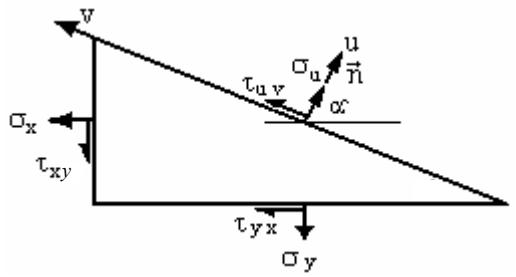
- Posmična naprezanja na okomitim presjecima su jednaka $\tau_{xy}=\tau_{yx}$

* specijalni slučaj – djeluju samo posmična naprezanja **- čisti posmik**

NAPREZANJA KAO FUNKCIJE σ_x,σ_y i τ_{xy}.

NAPREZANJA U PROIZVOLJNOJ RAVNINI ČIJA NORMALA ZATVARA S OSI x KUT α

(normala $\vec{n}$ i os koordinatnog sustava "u" su istog smjera)



$$\sigma_u = \sigma_x \times \cos^2 \alpha + \sigma_y \times \sin^2 \alpha + \tau_{xy} \times \sin 2\alpha$$

$$\tau_{uv} = \left(-\sigma_x + \sigma_y\right) \times \frac{\sin 2\alpha}{2} + \tau_{xy} \times \cos 2\alpha$$

GLAVNI NAPREZANJA – RAVNINE U KOJIMA IŠČEZAVAJU POSMIČNA NAPREZANJA AKO IMAMO ZADANO σ_x , σ_y i τ_{xy}

Kut ravnine $tg2\alpha_G = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$$\sigma_1 = \sigma_x \times \cos^2 \alpha_G + \sigma_y \times \sin^2 \alpha_G + \tau_{xy} \times \sin 2\alpha_G$$

$$\sigma_2 = \sigma_x \times \sin^2 \alpha_G + \sigma_y \times \cos^2 \alpha_G - \tau_{xy} \times \sin 2\alpha_G$$

Kontrola !:

$$\sigma_x + \sigma_y = \sigma_u + \sigma_v = \sigma_1 + \sigma_2$$

-za stanje čistog posmika

$\sigma_1 = \tau_{xy}$ $\quad$ $\sigma_2 = -\tau_{xy}$

MAKSIMALNA POSMIČNA NAPREZANJA I PRIPADAJUĆA NORMALNA NAPREZANJA

$$\sigma_3 = \sigma_4 = \sigma_S$$

$$\tau_{max} = \tau_{34} = \frac{\sigma_2 - \sigma_1}{2} \qquad \sigma_S = \frac{\sigma_2 + \sigma_1}{2}$$

MOHROVA KRUŽNICA ZA RAVNINSKO STANJE NAPREZANJA

ako imamo σ_x,σ_y i τ_{xy} vrijedi $\tau_{xy} = -\tau_{yx}$

ZADATAK 51:

Zadano je : σ_x=65 kN/cm^2, σ_y=18 kN/cm^2 , τ_{xy}= 18 kN/cm^2 , potrebno je odrediti :

a.) Naprezanja u proizvoljnoj ravnini čija normala "n" zatvara s osi x kut α=17°$\rightarrow \sigma_u, \tau_{uv}$

b.) Glavna naprezanja σ_1 i σ_2

c.) Maksimalna posmična naprezanja τ_{max} i njemu odgovarajuće normalno naprezanje

Sve dobivene veličine prikazati pomoću Mohrove kružnice za ravninsko stanje naprezanja

Postupak:

a.) $\sigma_u = \sigma_x \times \cos^2\alpha + \sigma_y \times \sin^2\alpha + \tau_{xy} \times \sin 2\alpha =$

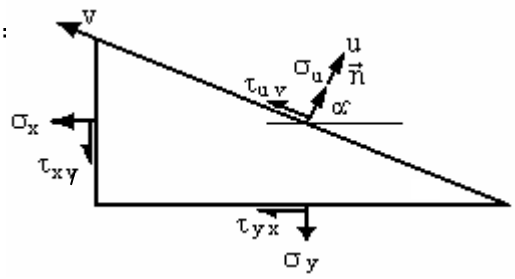
$= 65 \times \cos^2 17 + 18 \times \sin^2 17 + 18 \times \sin 34 =$

$= 59{,}44 + 1{,}54 + 10{,}06 = 71{,}06$ kN/cm^2

$$\tau_{uv} = \left(-\sigma_x + \sigma_y\right) \times \frac{\sin 2\alpha}{2} + \tau_{xy} \times \cos 2\alpha =$$

$$= \left(-65 + 18\right) \times \frac{\sin 34}{2} + 18 \times \cos 34 =$$

$= -13{,}14 + 14{,}92 = 1{,}78$ kN/cm^2



b.) Glavna naprezanja djeluju u ravnini u kojoj iščezavaju posmična naprezanja; kut koji zatvara normala ravnine s osi x izračuna se kao :

$$\operatorname{tg} 2\alpha_G = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 18}{47} = 0.76696 \rightarrow \quad 2 \times \alpha_G = 37{,}45° \quad ; \quad \alpha_G = 18{,}73°$$

$$\sigma_1 = \sigma_x \times \cos^2 \alpha_G + \sigma_y \times \sin^2 \alpha_G + \tau_{xy} \times \sin 2\alpha_G$$

$$= 65 \times \cos^2 18{,}73 + 18 \times \sin^2 18{,}73 + 18 \times \sin 37{,}45$$

$= 58{,}30 + 1{,}86 + 10{,}95 =$ <u>71,11 kN/cm^2</u>

$$\sigma_2 = \sigma_x \times \sin^2 \alpha_G + \sigma_y \times \cos^2 \alpha_G - \tau_{xy} \times \sin 2\alpha_G$$

$$= 65 \times \sin^2 18{,}73 + 18 \times \cos^2 18{,}73 - 18 \times \sin 37{,}45$$

$= 6{,}70 + 16{,}14 - 10{,}95 =$ <u>11,89 kN/cm^2</u>

Kontrola : $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

$65 + 18 = 71{,}11 + 11{,}89$

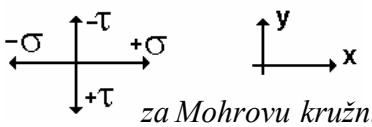
c.) $\tau_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{11{,}89 - 71{,}11}{2} = -29{,}61$ kN/cm$^2 \rightarrow$ zatvaraju kut od $\frac{\pi}{4}$ s glavnim naprezanjima

$$\sigma_3 = \sigma_4 = \sigma_s = \frac{\sigma_2 + \sigma_1}{2} = = \frac{11{,}89 + 71{,}11}{2} = \frac{83}{2} = 41{,}5 \text{ kN/cm}^2$$

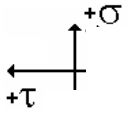
d.) Crtanje Mohrove kružnice naprezanja

- pretpostavka $\tau_{xy} = -\tau_{yx}$,zadano $\sigma_x, \sigma_y, \tau_{xy}$

1. Koordinatni sustavi



za Mohrovu kružnicu



na poprecnom presjeku

2. Točka $N_x(\sigma_x, \tau_{xy})$ i $N_y(\sigma_y, \tau_{yx})$, dužina N_xN_y je promjer Mohrove kružnice naprezanja.

3. Kroz N_x povlačimo paralelu sa osi x (σ), a kroz N_y paralelu sa osi y (τ), sjecište pravaca nam određuje točku **P** tj. **pol** , koja ima svojstvo da se u njemu sijeku zrake usporedne s normalama promatranog skupa.

4. Ako kroz **P** povučemo paralelu s normalom $\vec{n}$, ona će Mohrovu kružnicu sjeći u točki **Nn** $\rightarrow$ koordinate točke **Nn** određuju $\boldsymbol{\sigma_u}$ **i** $\boldsymbol{\tau_{uv}}$ u ravnini čija normala zatvara kut α s osi x .

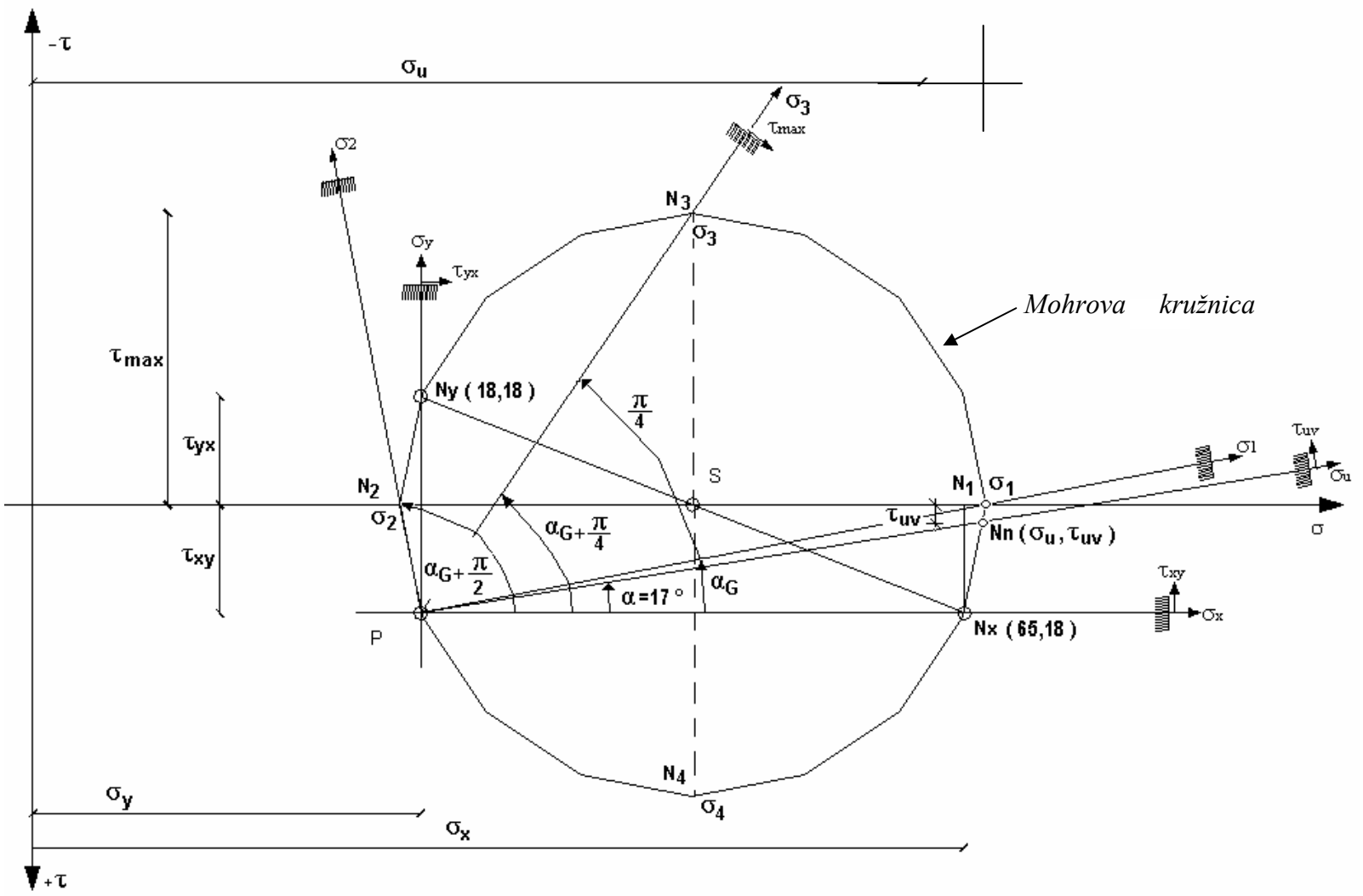
5. Sjecište N_1 i N_2 Mohrove kružnice s apscisom σ određuju veličine σ_1 i σ_2 tj. **glavna naprezanja**. Spojnica P sa N_1 određuje smjer σ_1 ,a spojnica P i N_2 određuje smjer σ_2 , tj smjerove glavnih naprezanja.

6. Točka N_3 određuje **ekstremno posmično naprezanje** $\boldsymbol{\tau_{max}}$. Spojnica točke P i točke N_3 određuje smjer normale presjeka u kojem djeluje najveće posmično naprezanje $\boldsymbol{\tau_{max}}$. Presjeci u kojima se pojavljuje maksimalno posmično naprezanje zatvara kut $\alpha = \frac{\pi}{4}$ s presjecima glavnih naprezanja.

7. U presjecima max posmičnih naprezanja postoje i normalna naprezanja

$$\sigma_3 = \sigma_4 = \sigma_s = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

Mohrova kruznica naprezanja za ravninsko stanje naprezanja
- pretpostavka za Mohrovu kruznicu naprezanja $\tau_{xy} = -\tau_{yx}$



ZADATAK 52:

Zadana su naprezanja σ_x=100N/cm^2, σ_y=50 N/cm^2 , τ_{xy}= 10 N/cm^2 , potrebno je odrediti :

- **a.) Napone σ_u i τ_{uv} ako normala proizvoljnog presjeka zatvara sa osi x α=30°**
- **b.) Glavne napone σ_1 i σ_2**
- **c.) Maksimalni posmični napon τ_{max} i njemu odgovarajući normalni napon**
- **d.) sve provjeriti na Mohrovoj kružnici napona**

a.) $$\sigma_u = \sigma_x \times \cos^2\alpha + \sigma_y \times \sin^2\alpha + \tau_{xy} \times \sin 2\alpha$$

$$\tau_{uv} = \left(-\sigma_x + \sigma_y\right) \times \frac{\sin 2\alpha}{2} + \tau_{xy} \times \cos 2\alpha$$

$$\sigma_u = 100 \times \cos^2 30 + 50 \times \sin^2 30 + 10 \times \sin 2 \times 30 = 96{,}16 \text{ N/cm}^2$$

$$\tau_{uv} = \left(-100 + 50\right) \times \frac{\sin 2 \times 30}{2} + 10 \times \cos 2 \times 30 = -16.7 \text{ N/cm}^2$$

b.) $$tg2\alpha_G = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 10}{100 - 50} = 0.4 \rightarrow \qquad 2 \times \alpha_G = 21.8° \;; \qquad \alpha_G = 10{,}9°$$

$$\sigma_1 = \sigma_x \times \cos^2\alpha_G + \sigma_y \times \sin^2\alpha_G + \tau_{xy} \times \sin 2\alpha_G$$

$$\sigma_2 = \sigma_x \times \sin^2\alpha_G + \sigma_y \times \cos^2\alpha_G - \tau_{xy} \times \sin 2\alpha_G$$

$$\sigma_1 = 100 \times \cos^2 10.9 + 50 \times \sin^2 10.9 + 10 \times \sin 2 \times 10.9 = 101.9 \text{ N/cm}^2$$

$$\sigma_2 = 100 \times \sin^2 10.9 + 50 \times \cos^2 10.9 - 10 \times \sin 2 \times 10.9 = 48.1 \text{ N/cm}^2$$

Kontrola : $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

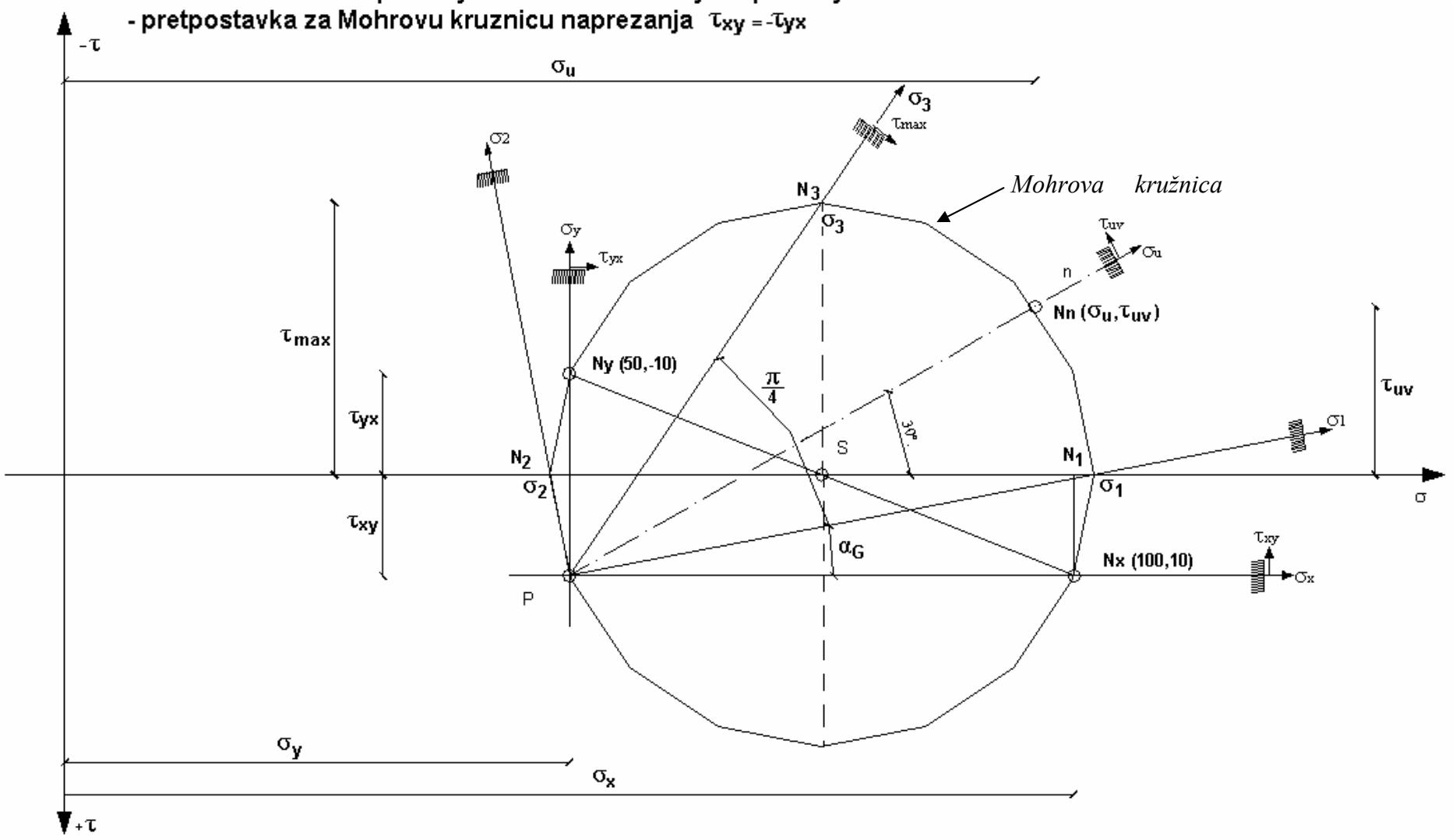
$100 + 50 = 101.9 + 48.1$

c.) $$\tau_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{48.1 - 101{,}9}{2} = -26.9 \text{ N/cm}^2$$

$$\sigma_3 = \sigma_4 = \sigma_s = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2} = \frac{48{,}1 + 101{,}9}{2} = 75{,}0 \text{ N/cm}^2$$

d.) Crtanje Mohrove kružnice naprezanja

Mohrova kruznica naprezanja za ravninsko stanje naprezanja
- pretpostavka za Mohrovu kruznicu naprezanja $\tau_{xy} = -\tau_{yx}$

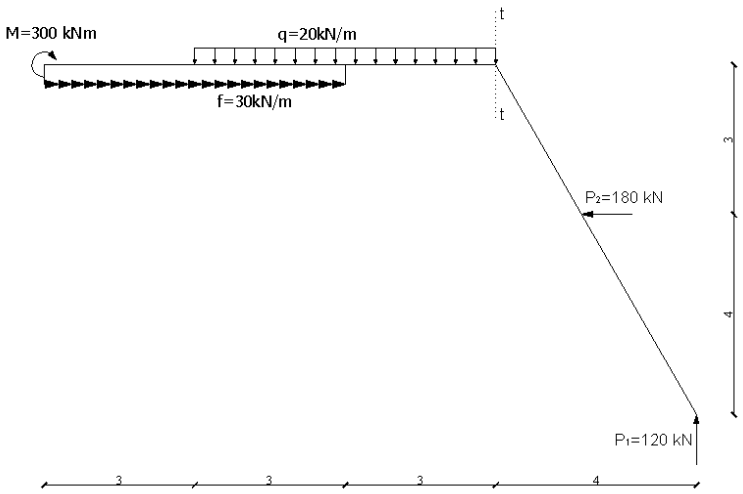


ODREĐIVANJE NAPREZANJA U PROIZVOLJNOM PRESJEKU ŠTAPA

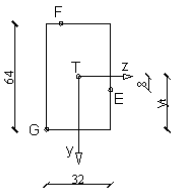
- za djelovanje uzdužne sile N_x, poprečne sile T_y i momenta savijanja M_z u poprečnom presjeku štapa javljaju se:
  - normalno naprezanje $\sigma = \frac{N}{F} \pm \frac{|M_z|}{I_z} \cdot y$
  - posmično naprezanje $\tau = \frac{|T| \cdot S_z}{I_z \cdot b}$

ZADATAK 53:

Za prikazani sustav, u presjeku t-t, potrebno je odrediti vrijednosti normalnog i posmičnog naprezanja za točke E, F i G.

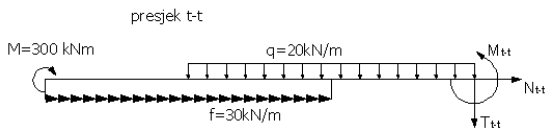


M= 300kNm
P_1= 120kN
P_2= 180kN
q= 20kN/m
f= 30kN/m
poprečni presjek



Postupak:

Izračun unutarnjih sila u presjeku t-t:

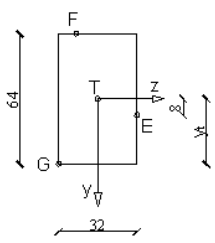


Σx=0 N_{t-t}= -f · 6= -180kN

Σy=0 T_{t-t}= -q · 6= -120kN

$\Sigma M_{(t-t)}$=0 M_{t-t}= -60kNm (vlak gore)

Izračun karakteristika poprečnog presjeka



$F = 32 \cdot 64 = 2048 cm^2$

$$I_z = \frac{32 \cdot 64^3}{12} = 699050,7 cm^4$$

$y_G = y_T$= 32cm y_E= 8cm y_F= 32cm (očitamo

Normalna naprezanja

$$\sigma_E = \frac{N_{t-t}}{F} - \frac{|M_{t-t}|}{I_z} \cdot y_E = \frac{-180}{2048} - \frac{\overbrace{6000}^{kNcm}}{699050,7} \cdot 8 = -0,1565 kN/cm^2$$

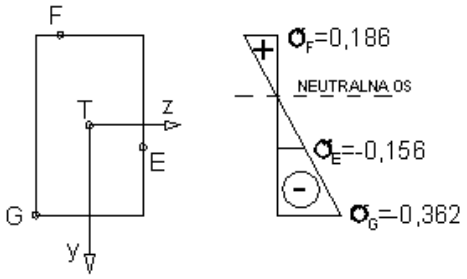
vlak je na gornjoj strani, a točka E je ispod težišta gdje je tlak, zato je minus

-

$$\sigma_F = \frac{N_{t-t}}{F} + \frac{|M_{t-t}|}{I_z} \cdot y_F = \frac{-180}{2048} + \frac{\overbrace{6000}^{kNcm}}{699050,7} \cdot 32 = 0,1865 kN/cm^2$$

$$\sigma_G = \frac{N_{t-t}}{F} - \frac{|M_{t-t}|}{I_z} \cdot y_G = \frac{-180}{2048} - \frac{\overbrace{6000}^{kNcm}}{699050,7} \cdot 32 = -0,3623 kN/cm^2$$

vlak je na gornjoj strani, a točka G je ispod težišta gdje je tlak, zato je minus



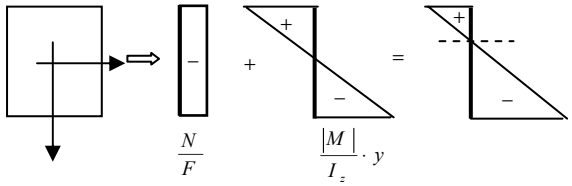
$\sigma_{\max} \rightarrow$ uvijek na vlačnoj strani

$\sigma_{\min} \rightarrow$ uvijek na tlačnoj strani

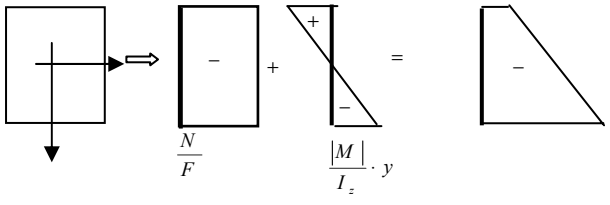
Funkcija promjene normalnog naprezanja σ po visini poprečnog presjeka je **linearna funkcija** i za svaki poprečni presjek postoje dvije ekstremne vrijednosti

$$\Rightarrow \sigma_{\max} \; i \; \sigma_{\min}$$

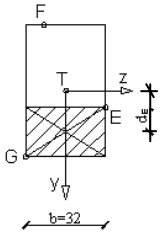
Grafički prikaz "*zbrajanja*" naprezanja.



Ukoliko je utjecaj uzdužne sile veći od utjecaja momenata savijanja može se dogoditi da cijeli poprečni presjek bude u istom predznaku *(kod ovog primjera cijeli presjek je u tlaku)*



Posmična naprezanja $\tau = \dfrac{|T| \cdot S_z}{I_z \cdot b}$



S_z – statički moment površine na os z.
(biramo onu površinu u kojoj se ne nalazi težište T)

$F_E = 32 \cdot 24 = 768cm^2$

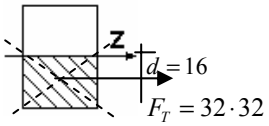
$d_E = 32-(24/2)=32-12= 20cm$

$S_z^E = F_E \cdot d_E = 768 \cdot 20 = 15360cm^3$

$$\tau_E = \frac{|120| \cdot 15360}{699050,7 \cdot 32} = 0,0824 kN/cm^2$$

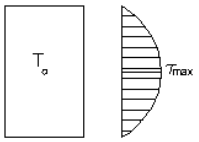
$S_z^G = S_z^F = 0$ $\quad F^F = F^G = 0$ $\quad \Rightarrow \tau_F = \tau_G = 0$

$S_{zmax} = 32 \cdot 32 \cdot 16 = 16384cm^3$



$$\tau_{\max} = \frac{|120| \cdot 16384}{699050,7 \cdot 32} = 0,08789 kN/cm^2 \Rightarrow$$

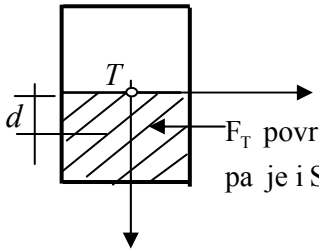
maksimalno posmično naprezanje je u **težištu** poprečnog presjeka jer je tamo S_z maksimalan



Pojašnjenje uz statički moment površine

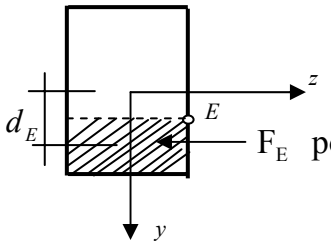
$$S_z = F \cdot d$$

maksimalni statički moment površine S_{zmax}



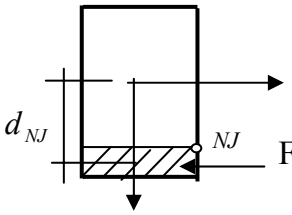
F_T povrsina ispod tezista je maksimalna povrsina pa je i S_T ujedno i S_{max}, a $S_{z\max} = F_T \cdot d$

statički moment površine neke točke E



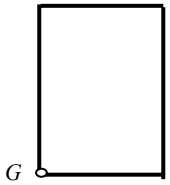
F_E povrsina ispod tocke E $S^E{}_z = F_E \cdot d_E$

statički moment površine neke točke NJ



F_{NJ} povrsina ispod tocke NJ $S^{NJ}{}_z = F_{NJ} \cdot d_{NJ}$

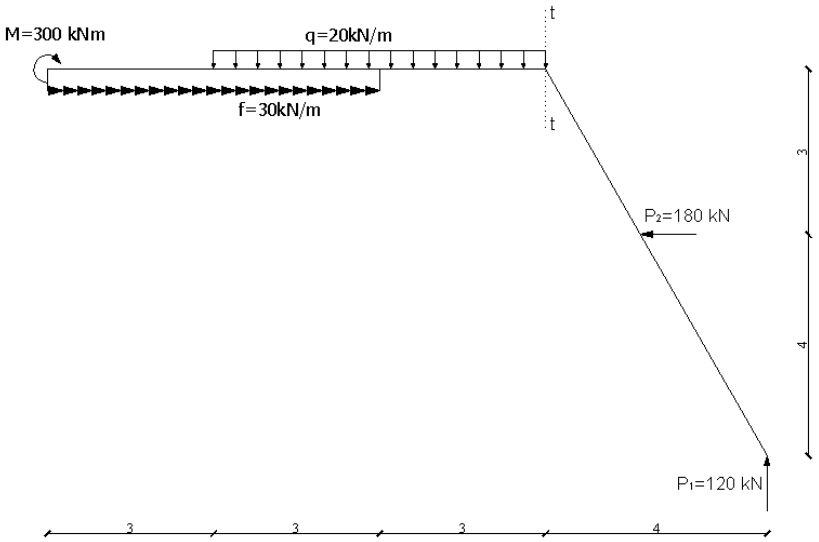
statički moment površine neke točke G (točka na rubu poprečnog presjeka)



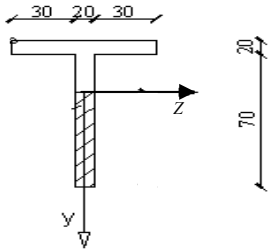
F_G povrsina ispod tocke G jednaka je nuli pa je i $S^G{}_z = 0$,
$tj.\ S^G{}_z = F_G \cdot d_G = 0 \Rightarrow \tau_G = 0$

ZADATAK 54:

Za prikazani sustav, u presjeku t-t, potrebno je odrediti ekstremne vrijednosti normalnih i posmičnih naprezanja.

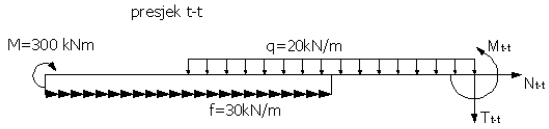


M= 300kNm
P_1= 120kN
P_2= 180kN
q= 20kN/m
f= 30kN/m
poprečni presjek



Postupak:

Izračun unutarnjih sila u presjeku t-t:



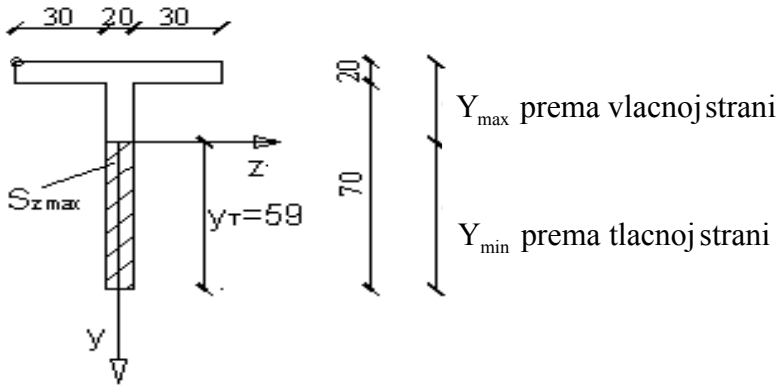
Σx=0 **N_{t-t}= -f · 6= -180kN**

Σy=0 **T_{t-t}= -q · 6= -120kN**

$\Sigma M_{(t-t)}$=0 **M_{t-t}= -60kNm**
(vlak gore)

Izračun karakteristika poprečnog presjeka:

F= 3000cm^2
I_z= 2137000cm^4
y_t= 59cm
S_z= 34810cm^3

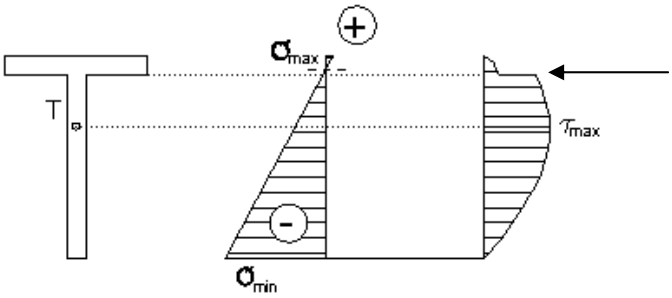


$$\sigma_{\max} = \frac{N_{t-t}}{F} + \frac{|M_{t-t}|}{I_z} \cdot y_{\max} = \frac{-180}{3000} + \frac{6000}{2137000} \cdot 31 = 0{,}027 kN/cm^2$$

$$\sigma_{\min} = \frac{N_{t-t}}{F} - \frac{|M_{t-t}|}{I_z} \cdot y_{\min} = \frac{-180}{3000} - \frac{6000}{2137000} \cdot 59 = -0{,}226 kN/cm^2$$

$$\tau_{\max} = \frac{|T| \cdot S_{z\max}}{I_z \cdot b} = \frac{|120| \cdot 34810}{2137000 \cdot 20} = 0{,}0977 kN/cm^2$$

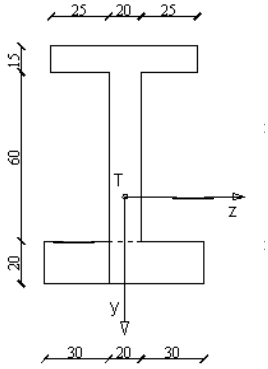
$$S_{z\max} = 59 \cdot 20 \cdot \frac{59}{2} = 34810 cm^3$$



Skok zbog promjene širine b poprečnog presjeka. (gore je širina b=80 cm, na "vratu" je b=20cm)

ZADATAK 55:

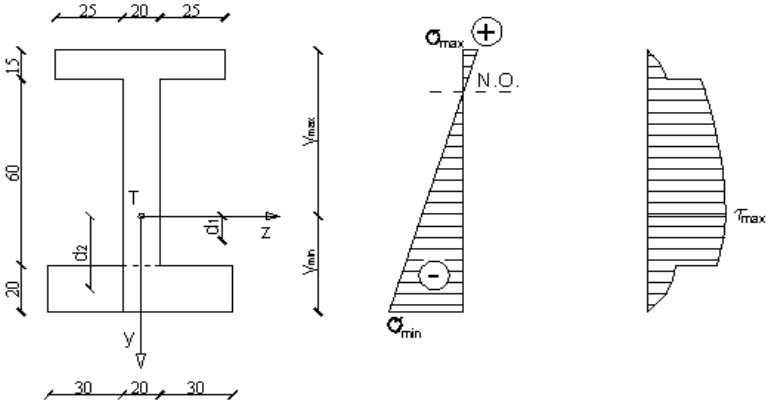
Za sustav kao u zadacima 53. i 54., ali za novi poprečni presjek,potrebno je odrediti ekstremne vrijednosti normalnih i posmičnih naprezanja.



Postupak:
Izračunali smo u zadacima 5. i 6.:
N= -180kN T= -120kN M= -60kNm= -6000kNcm (vlak gore)

Izračun karakteristika poprečnog presjeka:

$F= 3850cm^2$
$y_t= 43,6cm \quad y_{max}= 51,4cm \quad y_{min}= y_T= 43,6cm$
$I_z= 4312079,33\ cm^4$
$d_1= (43,6-20)/2=11,8cm; \quad d_2= 43,6-10= 33,6cm$
$F_1= (43,6-20)\cdot 20= 472cm^2; \quad F_2= 80\cdot 20= 1600cm^2$
$S_{z\,max}= F_1\cdot d_1 + F_2\cdot d_2= 472\cdot 11,8 + 1600\cdot 33,6= 59329,6cm^3$



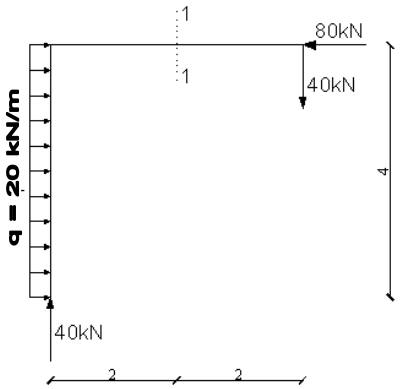
$$\sigma_{\max}=\frac{N}{F}+\frac{|M|}{I_z}\cdot y_{\max}=\frac{-180}{3850}+\frac{6000}{4312079}\cdot 51,4=0,02477kN/cm^2$$

$$\sigma_{\min}=\frac{N}{F}-\frac{|M|}{I_z}\cdot y_{\min}=\frac{-180}{3850}-\frac{6000}{4312079}\cdot 43,6=-0,10742kN/cm^2$$

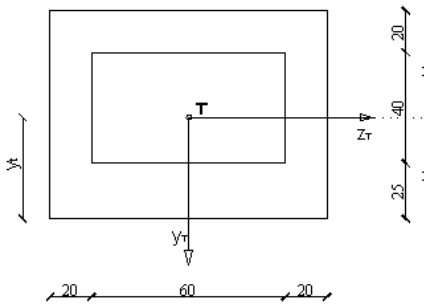
$$\tau_{\max}=\frac{|T|\cdot S_{z\max}}{I_z\cdot b}=\frac{|120|\cdot 59329,6}{4312079\cdot 20}=0,08255kN/cm^2$$

ZADATAK 56:

Za prikazani uravnoteženi sustav u presjeku 1-1 potrebno je odrediti ekstremne vrijednosti normalnih i posmičnih naprezanja.



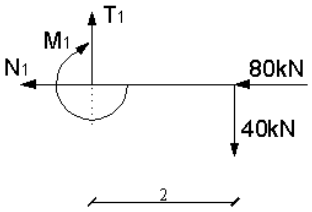
Poprečni presjek:



Postupak:

Izračun unutarnjih sila u presjeku 1-1:

Presjek 1-1



Σx=0 **N_1**= -80kN

Σy=0 **T_1**= 40kN

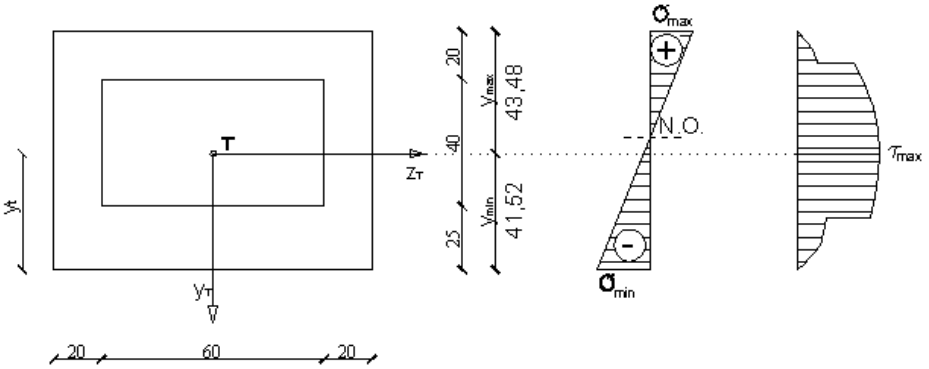
$\Sigma M_{(1-1)}=0$ $-M_1-40 \cdot 2=0$

M_1= -80kNm (vlak gore)

Izračun karakteristike poprečnog presjeka:

F= 6100cm^2 y_t= 41,52cm = y_{min} y_{max}= 43,48cm

I_z= 4776807cm^4 S_z= 78008,2cm^3



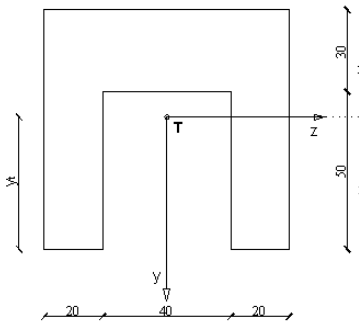
$$\sigma_{\max} = \frac{N_1}{F} + \frac{|M_1|}{I_z} \cdot y_{\max} = \frac{-80}{6100} + \frac{8000}{4776807} \cdot 43{,}48 = 0{,}05971 kN/cm^2$$

$$\sigma_{\min} = \frac{N_1}{F} - \frac{|M_1|}{I_z} \cdot y_{\min} = \frac{-80}{6100} - \frac{8000}{4776807} \cdot 41{,}52 = -0{,}08265 kN/cm^2$$

$$\tau_{\max} = \frac{|T| \cdot S_{z\max}}{I_z \cdot b} = \frac{40 \cdot 78008{,}2}{4776807 \cdot 40} = 0{,}1633 kN/cm^2$$

ZADATAK 57:

Za sustav kao u zadatku 56., ali za novi poprečni presjek,potrebno je odrediti ekstremne vrijednosti normalnih i posmičnih naprezanja.



Postupak:

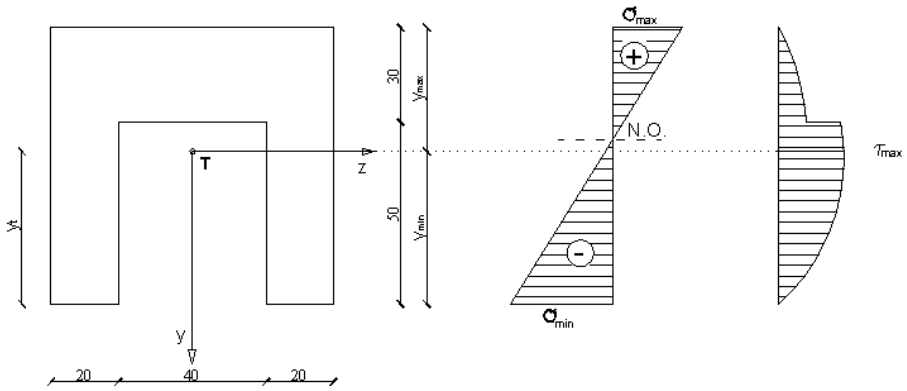
Izračun unutarnjih sila u presjeku 1-1:

N= -80kN T= 40kN M= -80kNm= -8000kNcm (vlak gore)

Izračun karakteristika poprečnog presjeka:

$F = 4400cm^2$ $y_t = 46,82cm = y_{min}$ $y_{max} = 33,18cm$

$I_z = 2342121,3cm^4$ $S_z = 43842,2cm^3$



$$\sigma_{\max} = \frac{N}{F} + \frac{|M|}{I_z} \cdot y_{\max} = \frac{-80}{4400} + \frac{8000}{2342121} \cdot 33,18 = 0,09515 kN/cm^2$$

$$\sigma_{\min} = \frac{N}{F} - \frac{|M|}{I_z} \cdot y_{\min} = \frac{-80}{4400} - \frac{8000}{2342121} \cdot 46,82 = -0,1781 kN/cm^2$$

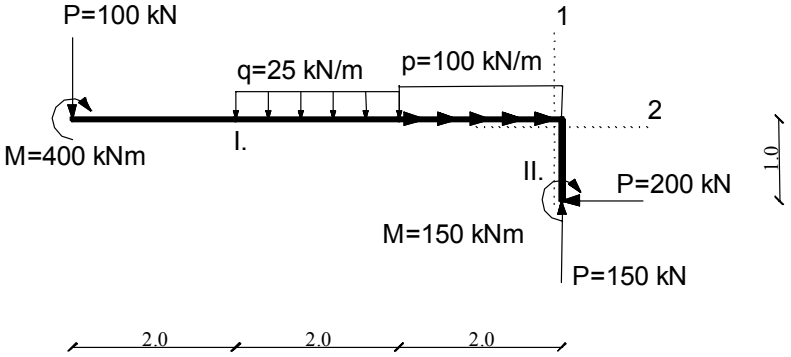
$$\tau_{\max} = \frac{|T| \cdot S_{z\max}}{I_z \cdot b} = \frac{40 \cdot 43842,2}{2342121,3 \cdot 40} = 0,01872 kN/cm^2$$

ZADATAK 58:

Za prikazani sustav :

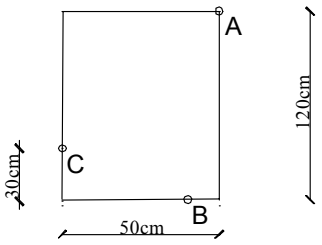
1.) **u presjeku 1-1 potrebno je odrediti vrijednosti normalnog i posmičnog naprezanja za točke A, B i C.**

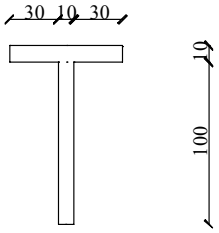
2.) **u presjeku 2-2 potrebno je odrediti vrijednosti maksimalnog normalnog i posmičnog naprezanja**



Poprečni presjeci:

štap I. **štap II.**





Postupak:

1.) presjek 1-1

Izračun unutarnjih sila:

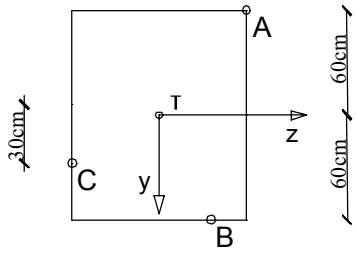
$\sum x = 0$ $N_1+100\times2 = 0$ **N_1= - 200 kN**

$\sum y = 0$ $-T_1-100-25\times2=0$; **T_1= - 150 kN**

$\sum M = 0$ $M_1-400+100\times6+25\times2.0\times3.0=0$; **$M_1$= - 350 kNm M_1=-35000kNcm**

(vlak gore)

Izračun karakteristika poprečnog presjeka:



$F = 50\times120 = 6000\ cm^2$

$I_z = \frac{50\times120^3}{12} = 7\ 200\ 000\ cm^4$

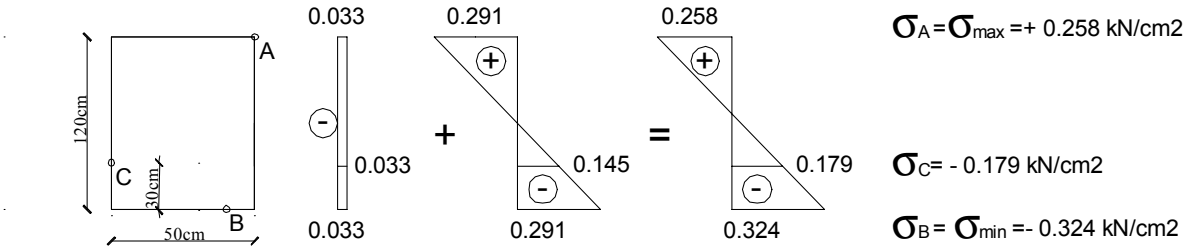
y_A= 60 cm
y_B= 60 cm
y_C= 30 cm

Normalna naprezanja

$$\sigma_A = \frac{N_1}{F} \pm \frac{M_1}{I_z} \times y_A = -\frac{200}{6000} + \frac{35000}{7200000} \times 60 = -0.033 + 0.291 = 0.258 \text{ kN/cm}^2$$

$$\sigma_B = \frac{N_1}{F} \pm \frac{M_1}{I_z} \times y_B = -\frac{200}{6000} - \frac{35000}{7200000} \times 60 = -0.033 - 0.291 = -0.324 \text{ kN/cm}^2$$

$$\sigma_C = \frac{N_1}{F} \pm \frac{M_1}{I_z} \times y_C = -\frac{200}{6000} - \frac{35000}{7200000} \times 30 = -0.033 - 0.145 = -0.179 \text{ kN/cm}^2$$



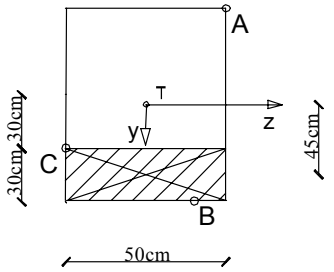
Funkcija promjene normalnog naprezanja **σ** po visini presjeka je linearna funkcija. Za svaki presjek postoje dvije ekstremne vrijednosti σ_{min} i σ_{max}.

σ_{min} – je uvijek na tlačnoj strani

σ_{max} – je uvijek na vlačnoj strani

Posmična naprezanja

$$\tau = \frac{|T| \times S_z}{I_z \times b}$$



$$S_z^C = F_C \times d_C = 150 \times 45 = 6750\,\text{cm}^3$$

$$F_C = 50 \times 30 = 150\,\text{cm}$$

$$d_C = 60 - 15 = 45\text{cm}$$

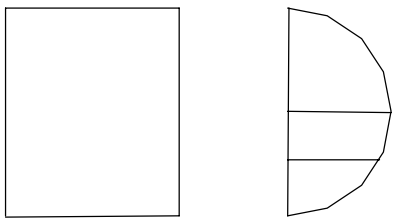
$$\tau_E = \frac{|T| \times S_z}{I_z \times b} = \frac{150 \times 6750}{7200000 \times 50} = 0.028 \text{ kN/cm}^2$$

$S_Z^A = S_Z^B = 0$ jer je $F_A = F_B = 0$ ➔ $\tau_A = \tau_B = 0$

Ekstremna vrijednost za τ je u težištu jer je tamo S_Z maksimalan.

$$S_z^{max} = F_{max} \times d = (50 \times 60) \times 30 = 9000\,\text{cm}^3$$

$$\tau_{max} = \frac{150 \times 9000}{7200000 \times 50} = 0.00375 \text{ kN/cm}^2$$



τ_{max} = 0.0375 kN/cm2

τ_C= 0.028 kN/cm2

2.) presjek 2-2

Izračun unutarnjih sila:

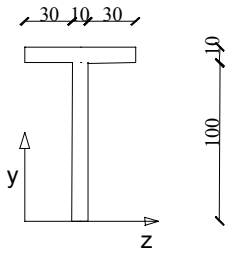
$\sum x = 0$ T_2 - 200 = 0 **T_2= 200 kN**

$\sum y = 0$ N_2 + 150 = 0 ; **N_2= -150 kN**

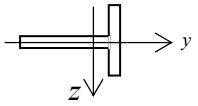
$\sum M = 0$ $-M_2$ -150-200×1=0; **M_2= - 350 kNm M_1=-35000kNcm**

(vlak desno)

Izračun karakteristika poprečnog presjeka



F = 10×100 + 70×10 = 1700 cm^2



y_1 = 50cm

y_2 = 105cm *(položaj presjeka u konstrukciji)*

$$y_T = \frac{F_1 \times y_1 + F_2 \times y_2}{F} = \frac{(10\times100)\times50 + (70\times10)\times105}{1700} = 72.6\text{cm}$$

$d_1 = y_1 - y_T$ = 50-72.6 = -22.6 cm

$d_2 = y_2 - y_T$ = 105-72.6 = 32.4 cm

S_{max} = 10×72.6×72.6/2 = 26 353.8 cm^3

S_{max} = 70×10×32.4+10×(100-72.6)×(100-72.6) / 2 = **26 433.8 cm^3**

(greška zaokruživanja)

$$I_Z = I_{z1} + I_{Z2} = \frac{70\times10^3}{12} + 700\times32.4^2 + \frac{10\times100^3}{12} + 1000\times22.6^2 = \textit{2 084 758,6 cm}^4$$

Izračun maksimalnih normalnih naprezanja

y_{max} = 37.4 cm (prema vlačnoj strani)

y_{min} = 72.6 cm (prema tlačnoj strani)

$$\sigma_{max} = \frac{N_2}{F} \pm \frac{M_2}{I_z} \times y_{max} = -\frac{150}{1700} + \frac{35000}{2084758,6} \times 37.4 = \text{-0.0088+0.627} = \underline{\textbf{0.618 kN/cm}^2}$$

$$\sigma_{min} = \frac{N_2}{F} \pm \frac{M_2}{I_z} \times y_{min} = -\frac{150}{1700} - \frac{35000}{2084758,6} \times 72.6 = \text{-0.0088–1.21} = \underline{\textbf{- 1.22 kN/cm}^2}$$

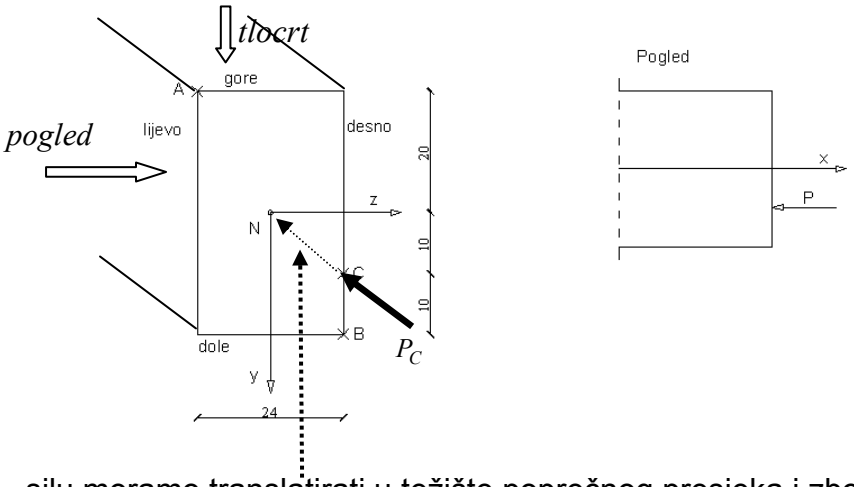
Izračun maksimalnog posmičnog naprezanja:

$$\tau_{max} = \frac{|T| \times S_z}{I_z \times b} = \frac{200\times26353.8}{2084758,6\times10} = \underline{\textbf{0.25 kN/cm}^2}$$

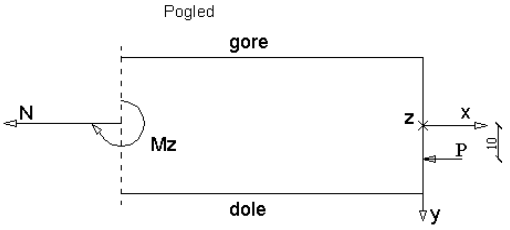
KOSO SAVIJANJE

ZADATAK 59:

Za djelovanje *tlačne* sile u točki C, P_c=80kN, potrebno je izračunati naprezanja za točke A i B. Potrebno je izračunati i položaj neutralne osi.



- silu moramo translatirati u težište poprečnog presjeka i zbog te translacije pojavit će se koncentrirani momenti My i Mz.
- Oko osi x nema momenata jer je sila P paralelna s osi x.

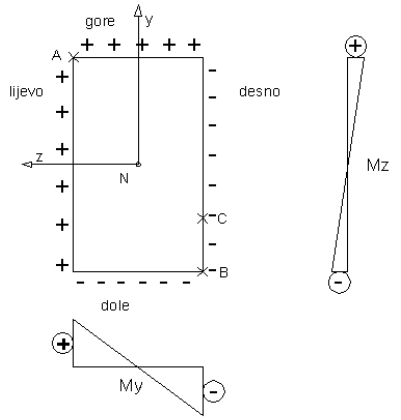


Σx=0 -N-P=0 $\rightarrow$ **N**= -P= **-80kN**

$\Sigma M_{(z)}=0$ -P·10-M_z=0 $\rightarrow$ **M_z** = -P·10= -80·10= **-800kNcm**
(vlak na gornjoj strani)

Σx=0 -N-P=0 $\rightarrow$ **N**= -P= **-80kN**

$\Sigma M_{(y)}=0$ -M_y+P·12=0 $\rightarrow$ **M_y** = P·12= 80·12= **960kNcm**
(vlak je lijevo)



$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z$$

Izračunamo: F= 24·40= 960cm^2;

$$I_z = \frac{24\cdot 40^3}{12} = 128000 cm^4$$

$$I_y = \frac{40\cdot 24^3}{12} = 46080 cm^4$$

$y_a = y_b$ = 20cm; $z_a = z_b$ = 12cm

$$\sigma_A = \frac{-80}{960} + \frac{800}{128000} \cdot 20 + \frac{960}{46080} \cdot 12 = -0{,}083 + 0{,}125 + 0{,}25 = 0{,}292 kN/cm^2$$

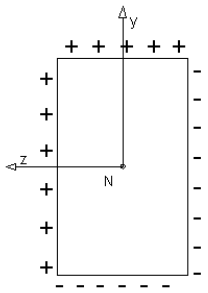
$$\sigma_B = \frac{-80}{960} - \frac{800}{128000} \cdot 20 - \frac{960}{46080} \cdot 12 = -0{,}083 - 0{,}125 - 0{,}25 = -0{,}458 kN/cm^2$$

Izračunavanje položaja neutralne osi

Uvjet za neutralnu os: $\sigma = 0$

$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z} \cdot y \pm \frac{|M_y|}{I_y} \cdot z = 0$$

Koordinatne osi poprečnog presjeka orijentirali smo prema vlačnim stranama presjeka (proizvoljno)



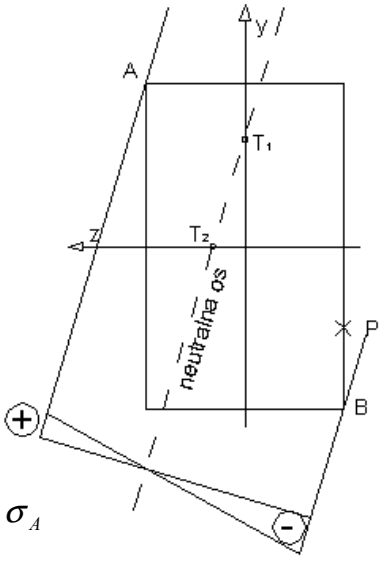
$$\frac{-80}{960} + \frac{800}{128000} \cdot y + \frac{960}{46080} \cdot z = 0$$

$$-0{,}0833 + 0{,}00625 \cdot y + 0{,}02083 \cdot z = 0$$

$$0{,}00625 y = 0{,}0833 - 0{,}02083 \cdot z \,/ : 0{,}00625$$

$$y = 13{,}33 - 3{,}33 z$$

jednadžba neutralne osi



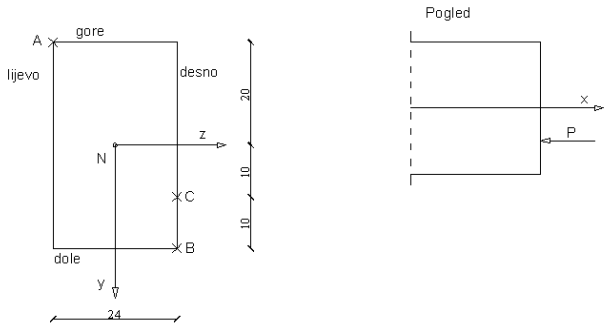
σ_A

σ_B

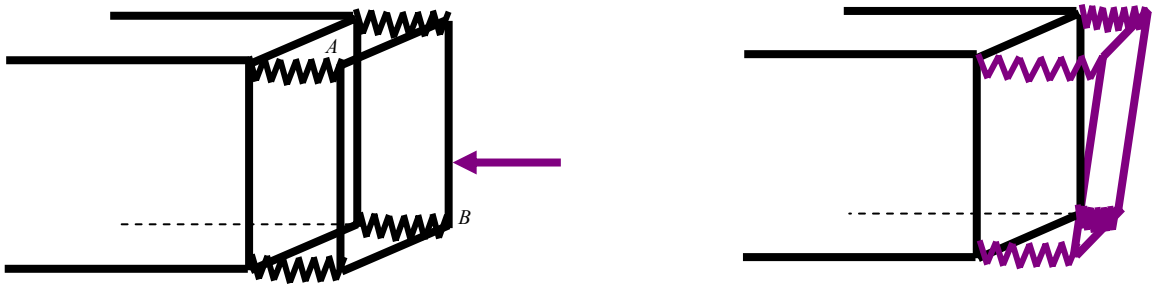
1.točka $T_1 \rightarrow$ z_1=0
y_1= 13,33cm

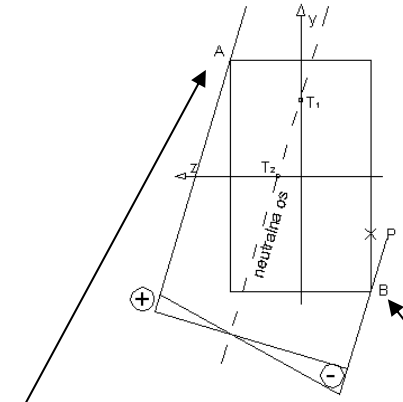
2.točka $T_1 \rightarrow$ z_2= 4cm
y_2= 0cm

Malo pojašnjenje uz zadatak 59:



Pogled ⇒ prije djelovanja sile P **nakon djelovanja sile P**





Opruga u točki **A** Opruga u točki **B**

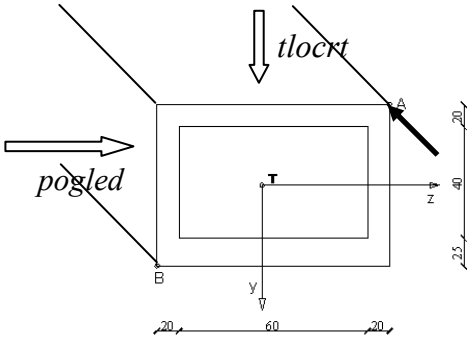
prije djelovanja sile

poslije djelovanja sile

vlačno naprezanje **tlačno** naprezanje

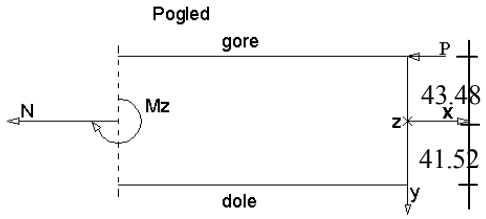
ZADATAK 60:

Za zadani poprečni presjek i zadanu *tlačnu* silu u točki A, P_A=100kN, treba izračunati normalna naprezanja u točkama A i B. Treba izračunati i položaj neutralne osi.



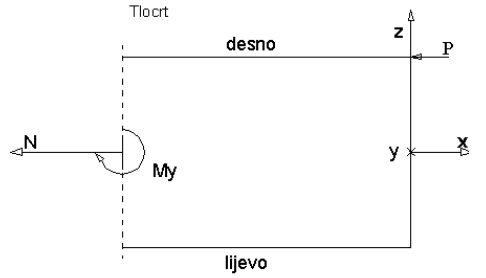
Izračunamo:
F= 6100cm^2
I_z= 4 776 807cm^4
I_y= 6 363 333cm^4
y_t= 41,52cm = y_B
y_A= 43,48cm
z_T= 50cm

Sila P_A ne djeluje u težištu poprečnog presjeka pa ju moramo tamo translatirati. Zbog te translacije pojavit će se momenti My i Mz. Oko osi x nema momenata jer je sila P_A paralelna s osi x.



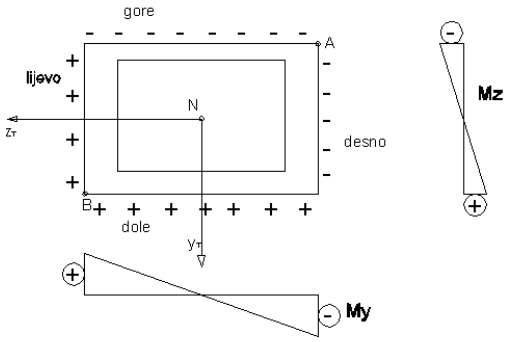
$\Sigma x=0$ -N-P=0 $\rightarrow$ **N**= -P= **-100kN**

$\Sigma M_{(z)}=0$ P·43,48-M_z=0 $\rightarrow$
M_z = P·43,48=100·43,48= **4348kNcm**
(vlak na donjoj strani)



$\Sigma x=0$ -N-P=0 $\rightarrow$ **N**= -P= **-100kN**

$\Sigma M_{(y)}=0$ -M_y+P·50= $\rightarrow$ M_y = P·50=
=100·50= **5000kNcm**
(vlak je lijevo)



$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z$$

$$\sigma_A = \frac{-100}{6100} - \frac{4348}{4776807}\cdot 43{,}48 - \frac{5000}{6363333}\cdot 50 =$$
$$= -0{,}01639 - 0{,}0395 - 0{,}03929 = -0{,}0952 kN/cm^2$$

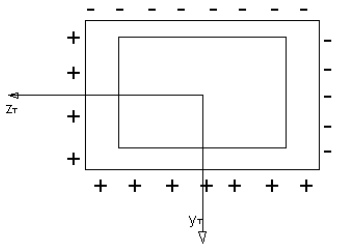
$$\sigma_B = \frac{-100}{6100} + \frac{4348}{4776807}\cdot 41{,}52 + \frac{5000}{6363333}\cdot 50 =$$
$$= -0{,}01639 + 0{,}03779 + 0{,}03929 = 0{,}0607 kN/cm^2$$

Izračunavanje položaja neutralne osi

Uvjet za neutralnu os: $\sigma = 0$

$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z = 0$$

Koordinatne osi poprečnog presjeka orijentirali smo prema vlačnim stranama presjeka (proizvoljno)



$$\frac{-100}{6100} + \frac{4348}{4776807}\cdot y + \frac{5000}{6363333}\cdot z = 0$$

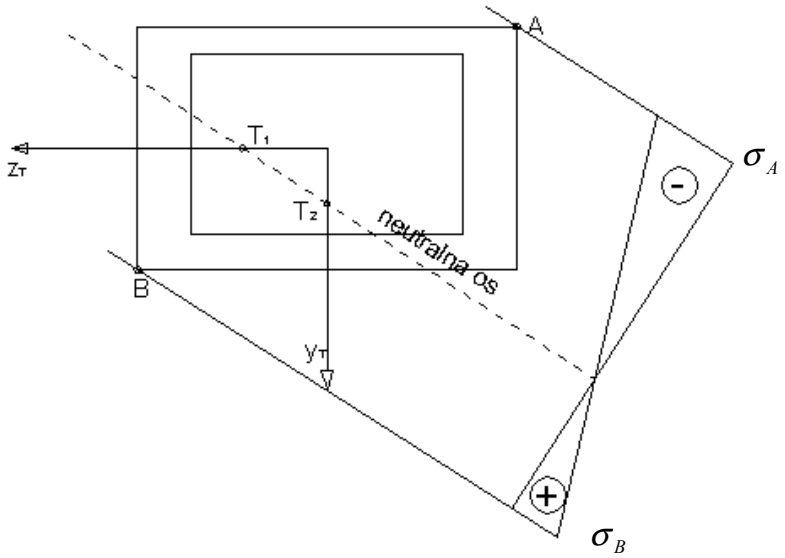
$$-0{,}0164 + 0{,}000909\cdot y + 0{,}0007858\cdot z = 0$$

$$0{,}000909\,y = 0{,}0164 - 0{,}0007858\cdot z \,/:0{,}000909$$

$$y = 18{,}04 - 0{,}864z$$

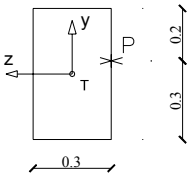
jednadžba neutralne osi

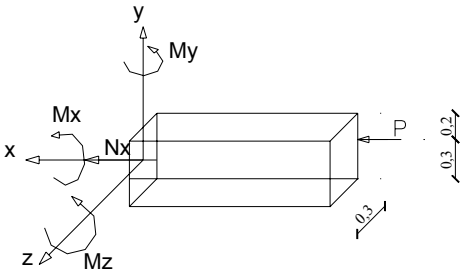
1.točka $T_1 \rightarrow$ z_1=18,04/0,864=20,9cm
y_1= 0
2.točka $T_1 \rightarrow$ z_2= 0cm
y_2= 18,04cm



ZADATAK 61:

Za zadani presjek potrebno je odrediti vrijednosti ekstremnih normalnih naprezanja za djelovanje sile P=100 kN te odrediti položaj neutralne osi.





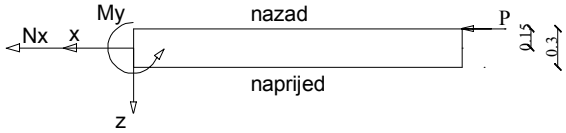
Postupak:

pogled u projekciji x-y

$\sum x = 0$ -N-100 = 0 **N_x = - 100 kN**

$\sum M = 0$ M_z+100×0.05 = 0; **M_z= - 5.0 kNm** ; **M_z=-500 kNcm**
(vlak dolje)

pogled u projekciji x-z

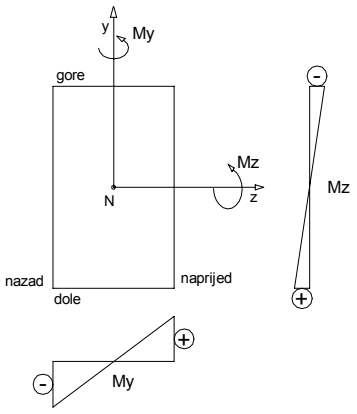


$\sum x = 0$ -N-100 = 0 **N_x = - 100 kN**

$\sum M = 0$ M_y+100×0.15 = 0; **M_y= -15.0 kNm** ; **M_y=-1500 kNcm**
(vlak naprijed)

M_x= 0.0 kNm ; -zato što je sila paralelna sa osi

Izračun karakteristika poprečnog presjeka:



$I_z = \frac{30 \times 50^3}{12} = 312\,500 \text{ cm}^4$

$I_y = \frac{50 \times 30^3}{12} = 112\,500 \text{ cm}^4$

$F = 30 \times 50 = 1500 \text{cm}^2$

Izračun naprezanja:

$$\sigma = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y$$

Maksimalno naprezanje:

$$\sigma_A = -\frac{N}{F} + \frac{M_y}{I_y} \times z + \frac{M_z}{I_z} \times y$$

$$\sigma_A = -\frac{100}{1500} + \frac{1500}{112500} \times 15 + \frac{500}{312500} \times 25 = 0.534 \text{ kN/cm}^2$$

Minimalno naprezanje:

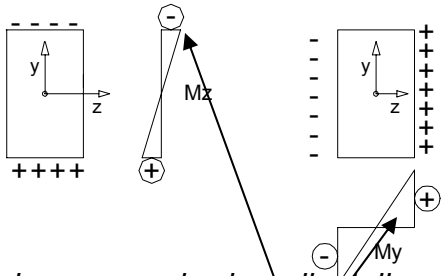
$$\sigma_B = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y$$

$$\sigma_B = -\frac{100}{1500} - \frac{1500}{112500} \times 15 - \frac{500}{312500} \times 25 = -0.667 \text{ kN/cm}^2$$

Izračunavanje položaja neutralne osi:

$\sigma = 0$ → uvjet za neutralnu os

$$0 = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y$$



(pogled s desna na presjek)

s obzirom na smjer koordinatnih osi očitavamo +/- kod momenata, a prema gornjoj skici

$$\sigma = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y = 0$$

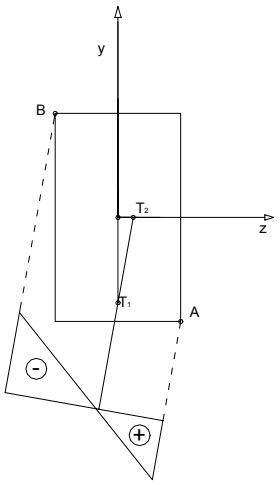
$$\sigma = -\frac{100}{1500} + \frac{1500}{112500} \times z - \frac{500}{312500} \times y = 0$$

$$\sigma = -0.0667 + 0.01333 \times z - 0.0016 \times y = 0$$

$$y = 8.3z - 41.7$$

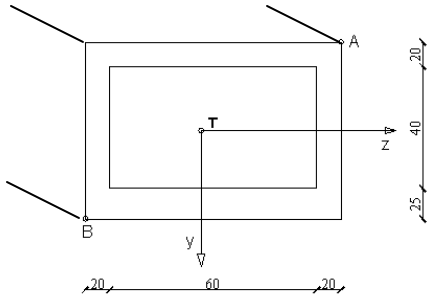
T_1 ➔ z = 0 $\qquad y = -41.7$ cm

T_2 ➔ y = 0 $\qquad z = \frac{41.7}{8.3} = 7.32$ cm



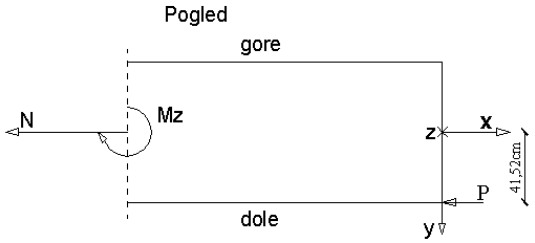
ZADATAK 62:

U točki A prikazanog poprečnog presjeka zadano je *vlačno* naprezanje σ_A= 1000kN/m^2. Treba izračunati koliko iznosi sila u točki B koja odgovara tom naprezanju.



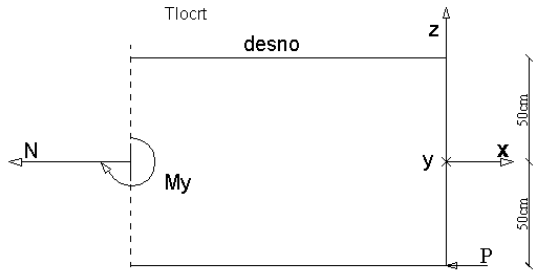
Izračunamo:
F= 6100cm^2
I_z= 4 776 807cm^4
I_y= 6 363 333cm^4
y_t= 41,52cm = y_B
y_A= 85-41,52= 43,48cm
z_T= 50cm

Pretpostavili smo *tlačnu* silu u točki B



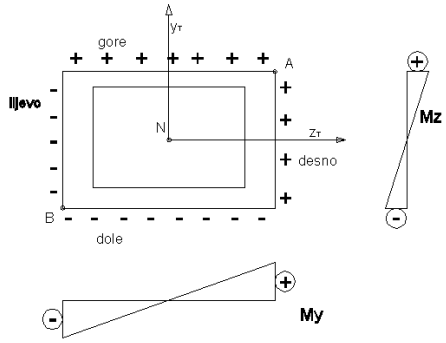
Σx=0 -N-P=0 → **N= -P**

$\Sigma M_{(z)}$=0 -P·41,52-M_z=0 →
M_z = -P·41,52
(vlak na gornjoj strani)



Σx=0 -N-P=0 → **N= -P**

$\Sigma M_{(y)}$=0 -M_y-P·50=0 →
M_y = -50P
(vlak je desno)



$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z$$

$$1000 = \frac{-P}{0,61} + \frac{0,4152P\cdot 0,4348}{0,04776807} + \frac{0,50P\cdot 0,50}{0,0636333} =$$

$$1000 = -1,639P + 3,779P + 3,9287P$$

$$1000 = 6,0687P \rightarrow P = 164,8kN$$

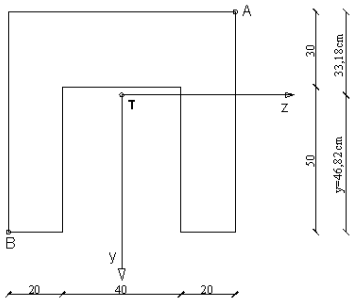
Kontrola M_y= P·z/2= 164,8·0,50= 82,40kNm (vlak desno)
M_z= P·0,4152= 164,8·0,4152= 68,42kNm (vlak gore)

$$\sigma_A = \frac{-164,8}{0,61} + \frac{68,42}{0,04776807}\cdot 0,4348 + \frac{82,4}{0,0636333}\cdot 0,50 =$$

$$= -270,16 + 623,32 + 647,46 = 1000kN/m^2$$

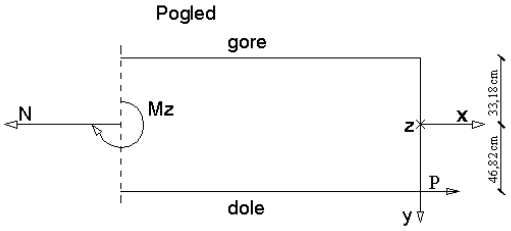
ZADATAK 63:

U točki A prikazanog poprečnog presjeka zadano je tlačno naprezanje σ_A= -1000kN/m^2. Treba izračunati koliko iznosi sila u točki B koja odgovara tom naprezanju.



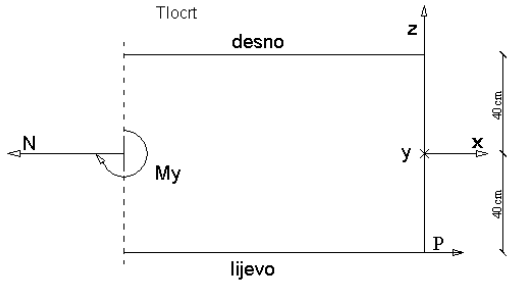
Izračunamo:
F= 4400cm^2
I_z= 2 342 121cm^4
I_y= 3,146 667cm^4
y_t= 46,82cm = y_B
y_A= 80-46,82= 33,18cm
z_A= z_B= 40cm

Pretpostavili smo vlačnu silu u točki B



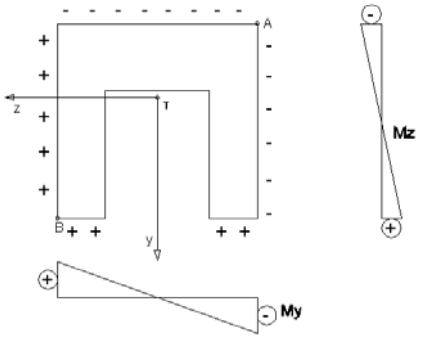
Σx=0 -N+P=0 $\rightarrow$ **N= P**

$\Sigma M_{(z)}=0$ $P\cdot 46{,}82-M_z=0 \rightarrow$
$M_z = P\cdot 46{,}82$
(vlak na donjoj strani)



Σx=0 -N+P=0 $\rightarrow$ **N= P**

$\Sigma M_{(y)}=0$ $-M_y+P\cdot 40=0 \rightarrow$
$M_y = 40P$
(vlak je lijevo)



$$\sigma = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z$$

$$-1000 = \frac{P}{0{,}44} - \frac{0{,}4682P\cdot 0{,}3318}{0{,}0234121} - \frac{0{,}40P\cdot 0{,}40}{0{,}031467} =$$

$$-1000 = 2{,}273P - 6{,}635P - 5{,}085P$$

$$-1000 = -9{,}45P \rightarrow P = 105{,}8kN$$

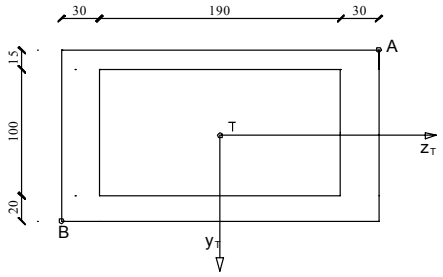
Kontrola

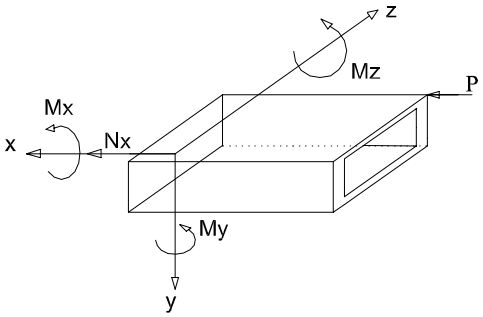
$$\sigma_A = \frac{105{,}8}{0{,}44} - \frac{0{,}4682\cdot 105{,}8}{0{,}0234121}\cdot 0{,}3318 - \frac{0{,}40\cdot 105{,}8}{0{,}031467}\cdot 0{,}40 =$$

$$= 240{,}45 - 702{,}02 - 537{,}96 = -999{,}53kN/m^2$$

ZADATAK 64:

Za zadani poprečni presjek i zadanu tlačnu silu P = 300 kN koja djeluje u točki A, izračunajte normalna naprezanja u točki B. ***(promijenjen koordinatni sustav)***

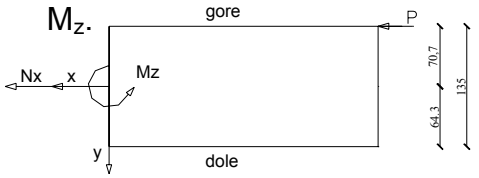




Postupak:

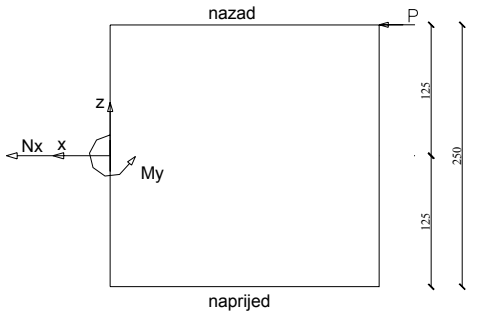
Izračunamo karakteristike poprečnog presjeka:
F = 14 750 cm^2 ; z_T =125cm; y_T =64.3cm
I_Z = 35 152 769,2 cm^4
I_y =118 622 916.67 cm^4

- sila izvan težišta paralelna s osi x, izaziva uzdužnu silu N_x i momente oko osi M_y i M_z.



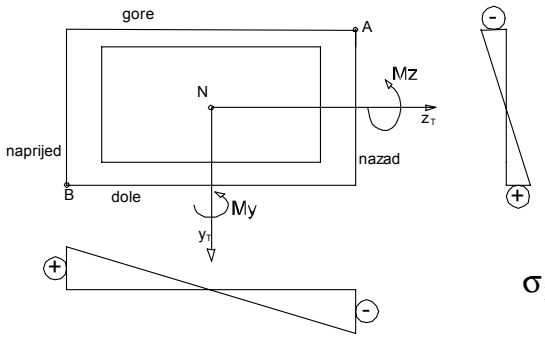
$\sum x = 0$ -N-300 = 0 **N_x = - 300 kN**

$\sum M = 0$ M_z+70.7×300 = 0; **M_z=-21210 kNcm** (vlak dolje)



$\sum x = 0$ -N-300 = 0 **N_x = - 300 kN**

$\sum M = 0$ M_y+125×300 = 0; **M_y=-37500 kNcm** (vlak naprijed)

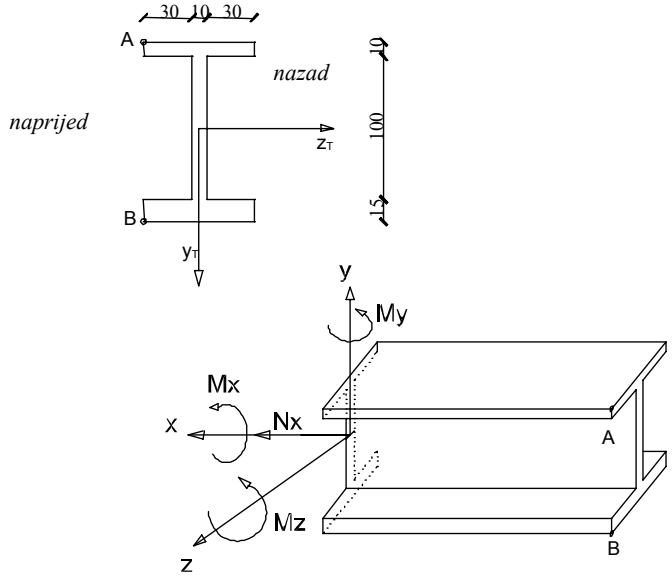


$$\sigma_B = \pm\frac{N}{F} \pm \frac{M_y}{I_y}\times z \pm \frac{M_z}{I_z}\times y$$

$$\sigma_B = -\frac{300}{14750} + \frac{37500}{118\,622\,916.67}\times 125 + \frac{21210}{35152769,2}\times 64.3 = 0.0188 \text{ kN/cm}^2$$

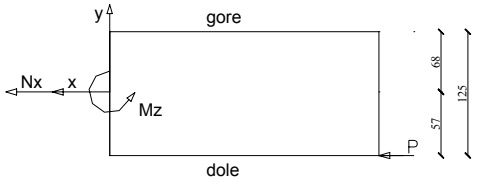
ZADATAK 65:

U točki A prikazanog poprečnog presjeka zadano je *tlačno* naprezanje σ_A= - 1000 kN/cm^2 . Potrebno je izračunati koliko iznosi sila P u točki B koja izaziva to naprezanje.



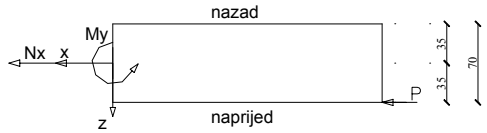
Izračunamo:
F=2750 cm^2
$z_T = 35$cm; y_T =57cm
I_Z = 6 273922 cm^4
I_y =722 916,67 cm^4

- **pretpostavljamo tlačnu** silu u točki B.



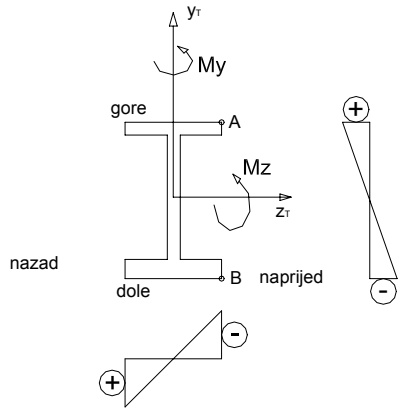
$\sum x = 0$ $\quad -N_x - P = 0$ $\quad$ **N_x = - P**

$\sum M = 0$ $\quad M_z - 57 \times P = 0;$ $\quad$ **M_z= 57P**



$\sum x = 0$ $\quad -N_x - P = 0$ $\quad$ **N_x = - P**

$\sum M = 0$ $\quad M_y - 35 \times P = 0;$ $\quad$ **M_y=35P**



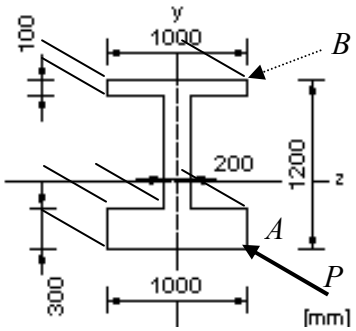
$$\sigma_A = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y$$

$$-1000 = -\frac{P}{2750} - \frac{35P}{722916{,}67} \times 35 + \frac{57P}{6\,273922} \times 68$$

<u>P = 694 268 kN</u>

ZADATAK 66:

Za zadani poprečni presjek i zadanu *tlačnu* silu u točki A, P_A=250kN, treba izračunati normalna naprezanja u točkama A i B.

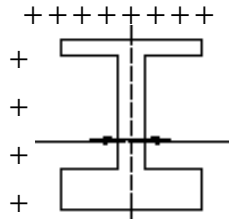


Izračunamo:

Karakteristike poprečnih presjeka:

F= 0,56 m^2 Y_T=48,6 cm ili 0,486 m = Y_A; Y_B= 72,4 cm ili 0,724 m
I_z=0,09615238 m^4 I_y=0,03386667 m^4

Sile:



N=-P=-250 kN
M_z = P*Y_A=250*0,486=121,5 kNm (vlak gore)
M_y= P* 0,5= 125 kNm (vlak lijevo)

$$\sigma_A = \frac{N}{F} - \frac{M_z}{I_Z} y_A - \frac{M_y}{I_y}.z_A$$

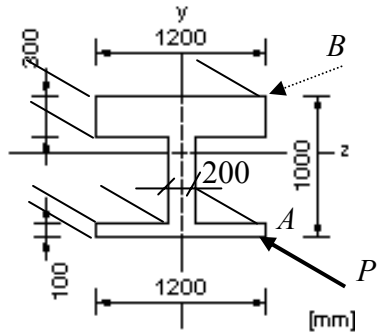
$$\sigma_A = \frac{-250}{0,56} - \frac{121,5}{0,09615238}0,486 - \frac{125}{0,03386667}.0,5 = -446,4 - 614,1 - 1845,5 = -2906 kN/m^2$$

$$\sigma_B = \frac{N}{F} + \frac{M_z}{I_Z} y_B - \frac{M_y}{I_y}.z_B$$

$$\sigma_B = \frac{-250}{0,56} + \frac{121,5}{0,09615238}0,724 - \frac{125}{0,03386667}.0,5 = -446,4 + 914,9 - 1845,5 = -1377 kN/m^2$$

ZADATAK 67:

Za zadani poprečni presjek i zadanu *tlačnu* silu u točki A, P_A=250kN, treba izračunati normalna naprezanja u točkama A i B.

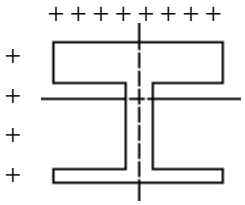


Izračunamo:

Karakteristike poprečnog presjeka:

F= 0,6 m^2 Y_T=60 cm ili 0,6 m = Y_A; Y_B= 40 cm ili 0,4 m
I_z=0,07 m^4 I_y=0,058 m^4

Sile:



N=-P=-250 kN
$M_z = P*y_A$=250*0,6=150 kNm (vlak gore)
M_y= P* 0,6= 150 kNm (vlak lijevo)

$$\sigma_A = \frac{N}{F} - \frac{M_z}{I_Z} y_A - \frac{M_y}{I_y}.z_A$$

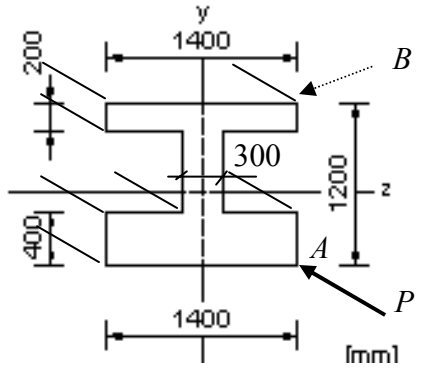
$$\sigma_A = \frac{-250}{0,6} - \frac{150}{0,07}0,6 - \frac{150}{0,058}.0,6 = -416,7 - 1285,7 - 1551,7 = -3254,1 kN/m^2$$

$$\sigma_B = \frac{N}{F} + \frac{M_z}{I_Z} y_B - \frac{M_y}{I_y}.z_B$$

$$\sigma_B = \frac{-250}{0,6} + \frac{150}{0,07}0,4 - \frac{150}{0,058}.0,6 = -416,7 + 857,1 - 1551,7 = -1111,3 kN/m^2$$

ZADATAK 68:

Za zadani poprečni presjek i zadanu *tlačnu* silu u točki A, P_A=250kN, treba izračunati normalna naprezanja u točkama A i B.

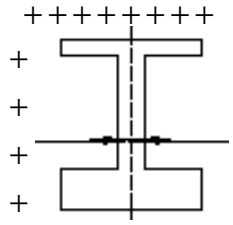


Izračunamo:

Karakteristike poprečnog presjeka:
F= 1,02 m^2 Y_T=53,5 cm ili 0,535 m = Y_A; Y_B= 66,5 cm ili 0,665 m
I_z=0,170929 m^4 I_y=0,13855 m^4

Sile:

N=-P=-250 kN
M_z = P*0,Y_A=250*0,535=133,75 kNm (vlak gore)
M_y= P* 0,7= 175 kNm (vlak lijevo)



$$\sigma_A = \frac{N}{F} - \frac{M_z}{I_Z} y_A - \frac{M_y}{I_y}.z_A$$

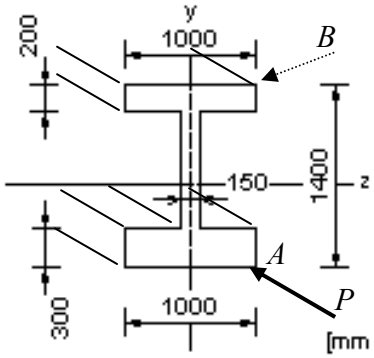
$$\sigma_A = \frac{-250}{1,02} - \frac{133,75}{0,170929}0,535 - \frac{175}{0,13855}.0,7 = -245,1 - 418,6 - 884,2 = -1547,9 kN/m^2$$

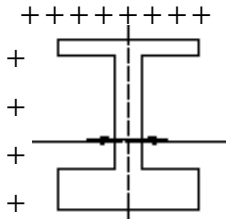
$$\sigma_B = \frac{N}{F} + \frac{M_z}{I_Z} y_B - \frac{M_y}{I_y}.z_B$$

$$\sigma_B = \frac{-250}{1,02} + \frac{133,75}{0,170929}0,665 - \frac{175}{0,13855}.0,7 = -245,1 + 520,4 - 884,2 = -608,9 kN/m^2$$

ZADATAK 69:

Za zadani poprečni presjek i zadanu *tlačnu* silu u točki A, P_A=250kN, treba izračunati normalna naprezanja u točkama A i B.





Izračunamo:

Karakteristike poprečnog presjeka:

F= 0,635 m^2 Y_T=64 cm ili 0,64 m = Y_A; Y_B= 76 cm ili 0,76 m

I_z=0,172816 m^4 I_y=0,041979 m^4

Sile:

N=-P=-250 kN
M_z = P*0,Y_A=250*0,64=160 kNm (vlak gore)
M_y= P* 0,5= 125 kNm (vlak lijevo)

$$\sigma_A = \frac{N}{F} - \frac{M_z}{I_Z} y_A - \frac{M_y}{I_y}.z_A$$

$$\sigma_A = \frac{-250}{0,635} - \frac{160}{0,1728126} 0,64 - \frac{125}{0,041979}.0,5 = -393,7 - 592,5 - 1488,8 = -2475 kN/m^2$$

$$\sigma_B = \frac{N}{F} + \frac{M_z}{I_Z} y_B - \frac{M_y}{I_y}.z_B$$

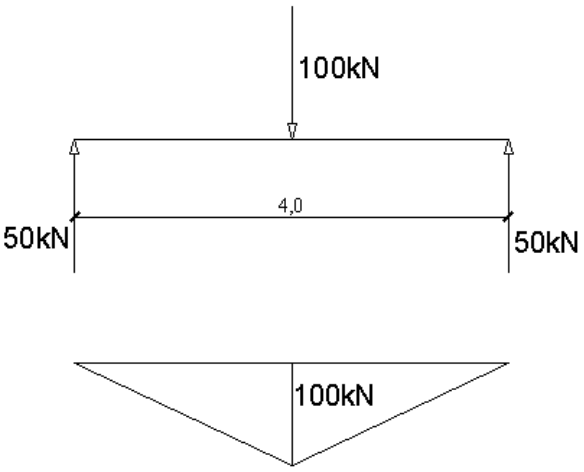
$$\sigma_B = \frac{-250}{0,635} + \frac{160}{0,1728126} 0,76 - \frac{125}{0,041979}.0,5 = -393,7 + 703,6 - 1488,8 = -1178,9 kN/m^2$$

PRIMJERI ISPITNIH ZADATAKA

ZADATAK 70:

Za gredu kvadratnog presjeka opterećenu silom od 100kN odredi potrebne dimenzije da naprezanje uzrokovano savijanjem ne pređe dopuštenu nosivost σ_{dop} =20N/mm^2.

Postupak:



- prvo odredimo M dijagram

- iz njega očitamo M_{max} ,
M_{max}= 50·2= 100kNm= 100·10^{6}Nmm

$$\sigma = \frac{M}{I_z} \cdot y$$

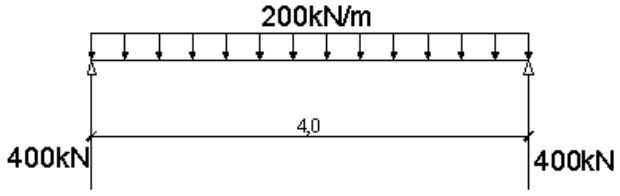
- I_z= a^4/12, y=a/2

$$20 = \frac{100 \cdot 10^6}{\frac{a^4}{12}} \cdot \frac{a}{2}$$

$$20 = \frac{100 \cdot 10^6 \cdot 6}{a^3} \Rightarrow a^3 = \frac{100 \cdot 10^6 \cdot 6}{20} = 30 \cdot 10^6 \Rightarrow a = 310mm = 31\,cm$$

ZADATAK 71:

Za gredu presjeka (b/h) opterećenu kontinuiranim opterećenjem odredi širinu presjeka ako je h=60cm da normalno naprezanje ne pređe dopuštenu nosivost σ_{dop} =20N/mm^2.



odredimo M_{max} iz uvjeta T_x= 0

T_x= 400-q·x= 0 → q·x=400 → **x=2m**

ili znamo

M_{max}= q·l^2/8= 400kNm → 400·10^{6}Nmm

$$\sigma = \frac{M}{I_z} \cdot y$$

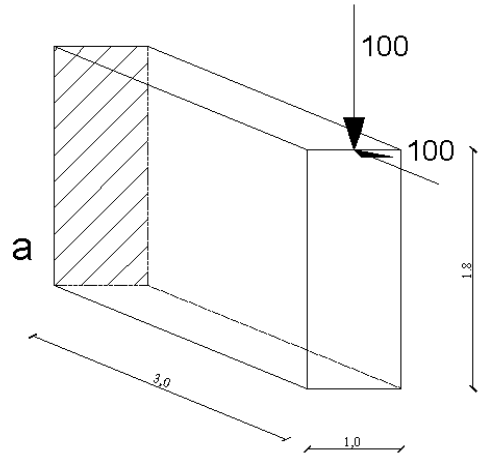
$$20 = \frac{400 \cdot 10^6}{\frac{b \cdot h^3}{12}} \cdot \frac{h}{2}$$

$$20 = \frac{200 \cdot 12 \cdot 10^6}{b \cdot 600^2}$$

$$b = \frac{200 \cdot 12 \cdot 10^6}{600^2 \cdot 20} = 333{,}33mm = 33{,}3cm$$

ZADATAK 72:

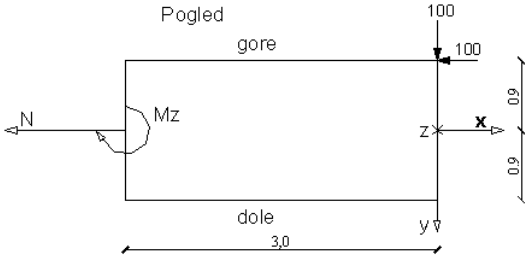
Odredite vrijednost normalnog naprezanje u plohi a.



Izračunamo: $F= 1,0 \cdot 1,8= 1,8m^2$

$I_z= 1,0 \cdot 1,8^3/12= 0,486m^4$

$I_y= 1,8 \cdot 1,0^3/12= 0,15m^4$

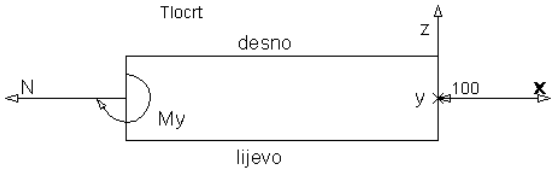


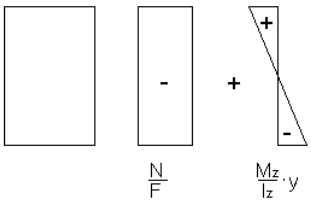
Σx=0 -N-100=0 $\rightarrow$ **N= -100kN**

$\Sigma M_{(z)}$=0 $-M_z+100 \cdot 0,9-100 \cdot 3,0=0 \rightarrow$

M_z = 90-300= **-210kNm**

(vlak na gornjoj strani)





$$\sigma_{gore} = \frac{N}{F} + \frac{M_z}{I_z} \cdot 0,9 = \frac{-100}{1,8} + \frac{210}{0,486} \cdot 0,9 =$$

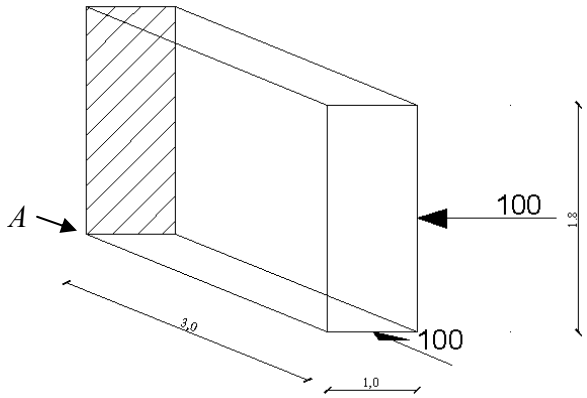
$$= -55,56 + 388,89 = 333,33 kN/m^2$$

$$\sigma_{dole} = \frac{N}{F} - \frac{M_z}{I_z} \cdot 0,9 = \frac{-100}{1,8} - \frac{210}{0,486} \cdot 0,9 =$$

$$= -55,56 - 388,89 = -444,45 kN/m^2$$

ZADATAK 73:

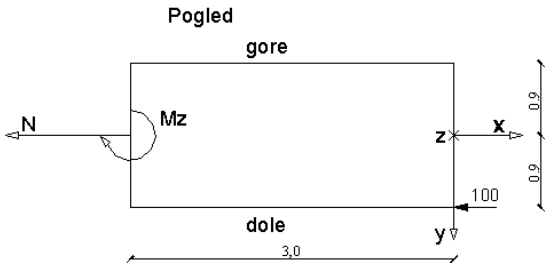
Odredite normalno naprezanje u točki A.



Izračunamo: $F= 1,0 \cdot 1,8= 1,8m^2$

$I_z= 1,0 \cdot 1,8^3/12= 0,486m^4$

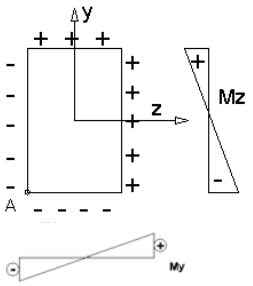
$I_y= 1,8 \cdot 1,0^3/12= 0,15m^4$



$\Sigma x=0$ -N-100=0 $\rightarrow$ **N= -100kN**

$\Sigma M_{(z)}=0$ -Mz-100·0,9=0 $\rightarrow$
M_z = -90kNm
(vlak na gornjoj strani)

$\Sigma M_{(y)}=0$ -M_y-100·3=0 $\rightarrow$
M_y = -300kNm
(vlak je desno)



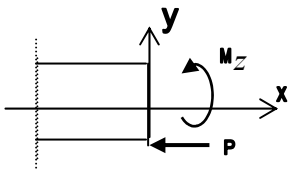
$$\sigma_A = \pm\frac{N}{F} \pm \frac{|M_z|}{I_z}\cdot y \pm \frac{|M_y|}{I_y}\cdot z$$

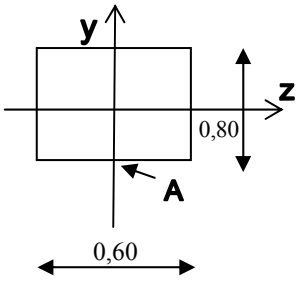
$$\sigma_A = \frac{-100}{1,8} - \frac{90}{0,486}\cdot 0,9 - \frac{300}{0,15}\cdot 0,50 =$$

$$= -55,56 - 166,67 - 1000 = -1222,2 kN/m^2$$

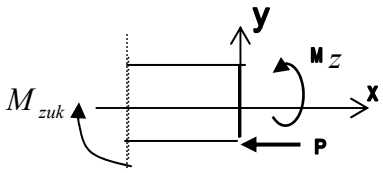
ZADATAK 74:

U točki A poprečnog presjeka zadana je tlačna sila P=800 kN i moment savijanja M_z=100kNm. Potrebno je odrediti i položaj neutralne osi.





Postupak rješavanja:



$$\sum M_z = 0, \cdots -M_{zuk} + M_z - P \cdot 0{,}4 = 0$$

$$M_{zuk} = M_z - P \cdot 0{,}4 = 100 - 320 = -220kNm \quad (vlak\ gore)$$

$$\sigma = \frac{N}{F} \pm \frac{M_z}{I} y$$

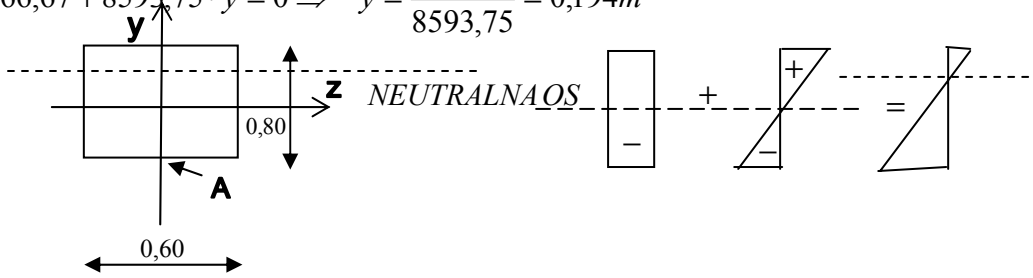
Izračunamo:

$F=0{,}48m^2$; $I_z=0{,}0256m^4$

Položaj neutralne osi iz uvjeta $\sigma = 0$

$$\sigma = \frac{N}{F} \pm \frac{M_z}{I} y = -\frac{800}{0{,}48} + \frac{220}{0{,}0256} \cdot y = 0$$

$$-1666{,}67 + 8593{,}75 \cdot y = 0 \Rightarrow \quad y = \frac{1666{,}67}{8593{,}75} = 0{,}194m$$



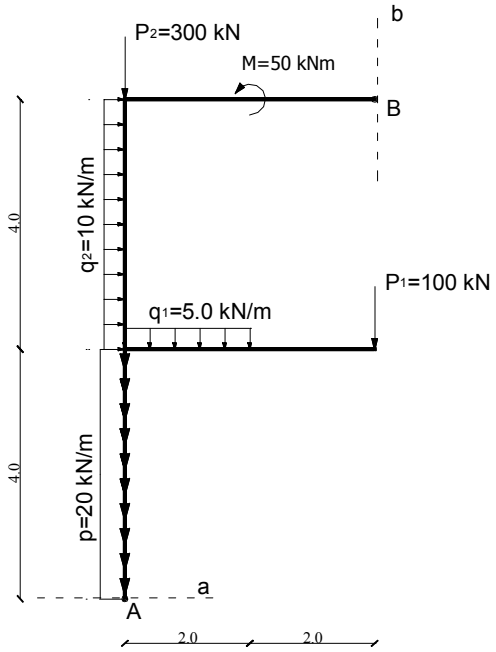
RIJEŠENI ISPITNI ROKOVI

ISPITNI ROK 22. 01. 2002.

TEHNIČKA MEHANIKA

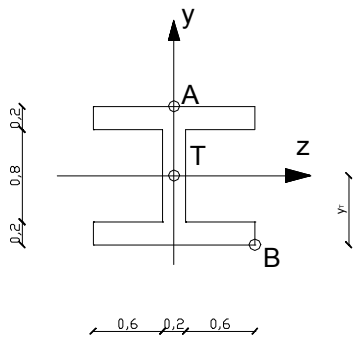
(ime i prezime ; matični broj)

3.) **Zadani sustav u ravnini uravnotežiti silom u točki *"A"* na pravcu *"a"*, silom u točki *"B"* i na pravcu *"b"*, i momentom u točki *"A"*.**



4.) Za uravnoteženi štap iz prvog zadatka izračunati i nacrtati dijagrame unutarnjih sila.

5.) Za zadani poprečni presjek i zadano naprezanje u točki A σ_A = -1000 kN/m^2(tlak) izračunati potrebnu silu u točci B.



1). ODREĐIVANJE URAVNOTEŽAVAJUĆIH SILA

$$\sum y = 0$$

$-P_1 - q_1 \times 2.0 - P_2 - p \times 4.0 + B = 0$

$-100 - 5.0 \times 2.0 - 300 - 20 \times 4.0 + B = 0$

B = 490.0 kN

$$\sum M_{(A)} = 0$$

$M - q_1 \times 2.0 \times 1.0 - q_2 \times 4.0 \times 6.0 - P_1 \times 4.0 + M_A + B \times 4.0 = 0$

$50 - 5.0 \times 2.0 \times 1.0 - 10 \times 4.0 \times 6.0 - 100 \times 4.0 + M_A + 490 \times 4.0 = 0$

M_A=1360.0 kNm

$$\sum x = 0$$

$q_2 \times 4.0 + A = 0$

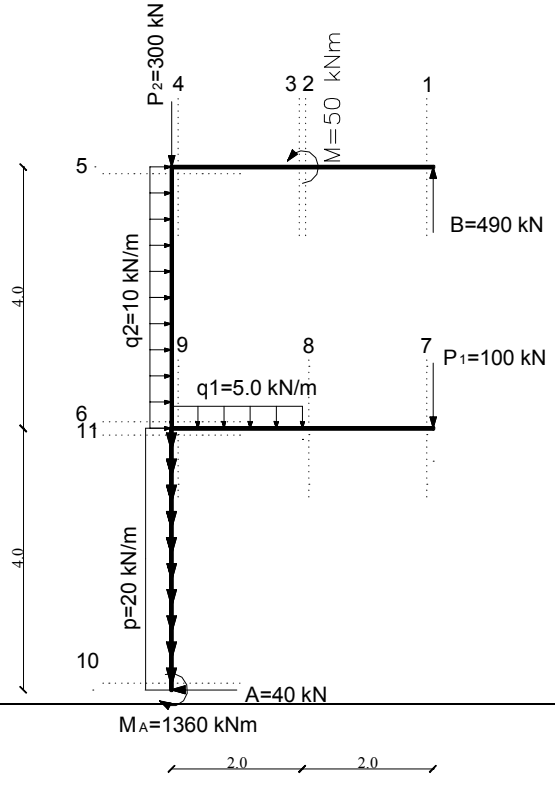
$10 \times 4.0 + A = 0$

A = -40.0 kN

Kontrola :

$\sum M_{(X)} = 0$ *- suma momenata na bilo koju točku*

2). PRORAČUN UNUTARNJIH SILA I DIJAGRAMI



Presjek 1-1

$\sum x = 0$	$-N_1+0 = 0$	**$N_1=0$**
$\sum y = 0$	$+T_1+490=0$;	**$T_1=-490$ kN**
$\sum M = 0$	$-M_1+0=0$;	**$M_1=0$**

Presjek 2-2

$\sum x = 0$	$-N_2+0= 0$	**$N_2=0$**
$\sum y = 0$	$+T_2+490=0$;	**$T_2=-490$ kN**
$\sum M = 0$	$-M_2+490\times2.0=0$;	**$M_2=980$ kNm**

Presjek 3-3

$\sum x = 0$	$-N_3+0= 0$	**$N_3=0$**
$\sum y = 0$	$+T_3+490=0$;	**$T_3=-490$ kN**
$\sum M = 0$	$-M_3+490\times2.0+50=0$;	**$M_3=1030$ kNm**

Presjek 4-4

$\sum x = 0$	$-N_4+0= 0$	**$N_4=0$**
$\sum y = 0$	$+T_4+490=0$;	**$T_4=-490$ kN**
$\sum M = 0$	$-M_4+490\times4.0+50=0$;	**$M_4=2010$ kNm**

Presjek 5-5

$\sum x = 0$	$-T_5+0 = 0$	**$T_5=0$**
$\sum y = 0$	$-N_5+490-300 = 0$	**$N_5=190$ kN**
$\sum M = 0$	$-M_5+490\times4.0+50=0$;	**$M_5=2010$ kNm**

Presjek 6-6

$\sum x = 0$	$-T_6+10\times4.0 = 0$	**$T_6=40$ kN**
$\sum y = 0$	$-N_6+490-300 = 0$	**$N_6=190$ kN**
$\sum M = 0$	$-M_6+490\times4.0+50-10\times4.0\times2.0=0$;	**$M_6=1930$ kNm**

Presjek 7-7

$\sum x = 0$	$-N_7+0=0$	**$N_7=0$**
$\sum y = 0$	$+T_7-100=0$	**$T_7=100$ kN**
$\sum M = 0$	$-M_7+0=0$;	**$M_7=0$**

Presjek 8-8

$\sum x = 0$	$-N_8+0=0$	**$N_8=0$**
$\sum y = 0$	$+T_8-100=0$	**$T_8=100$ kN**
$\sum M = 0$	$-M_8-100\times2.0 = 0$	**$M_8=-200$ kN**

Presjek 9-9

$\sum x = 0$	$-N_9+0=0$	**$N_9=0$**
$\sum y = 0$	$+T_9-100-5\times2.0=0$	**$T_9=110$ kN**
$\sum M = 0$	$-M_9-100\times4.0-5.0\times2.0\times1.0=0$	**$M_9=-410$ kN**

Presjek 10-10

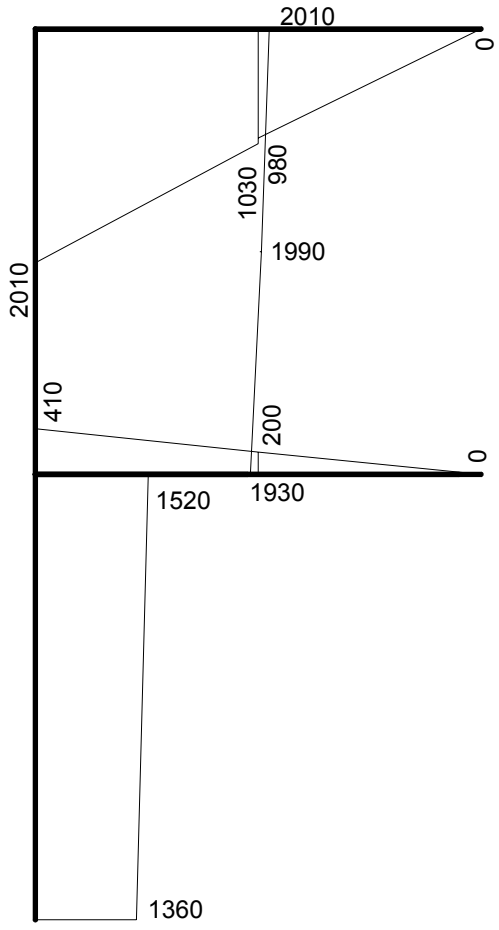
$\sum x = 0$	$T_{10}-40=0$;	**$T_{10}=40$ kN**
$\sum y = 0$	$N_{10}+0=0$	**$N_{10}=0$**
$\sum M = 0$	$M_{10}-1360=0$;	**$M_{10}=1360$ kNm**

Presjek 11-11

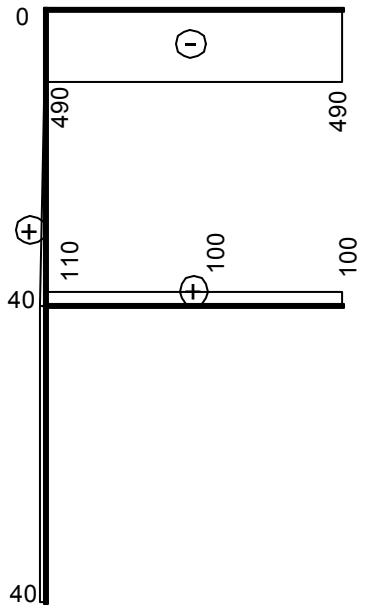
$\sum x = 0$	$T_{11}-40=0$;	**$T_{11}=40$ kN**
$\sum y = 0$	$N_{11}-20\times4.0=0$	**$N_{11}=80$ kN**
$\sum M = 0$	$M_{11}-1360-40\times4.0=0$;	**$M_{11}=1520$ kNm**

Dijagrami :

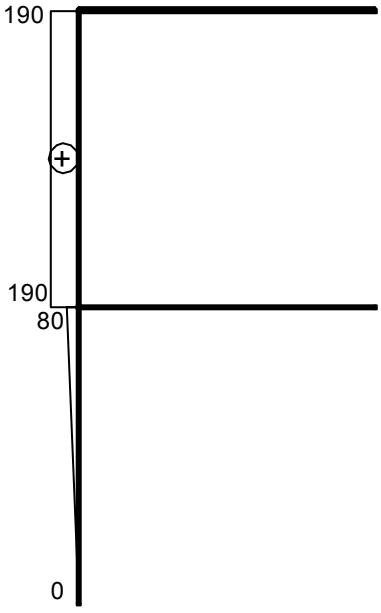
M



T

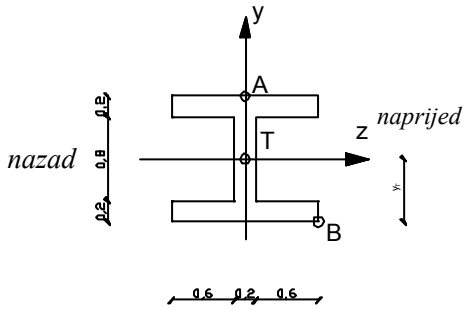


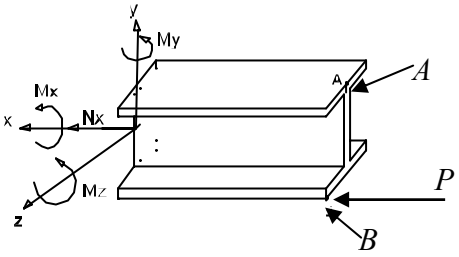
N



3). NAPREZANJA

U točki A zadanog poprečnog presjeka zadano je naprezanje σ_A= - 1000 kN/m^2 . Potrebno je izračunati koliko iznosi sila P u točki B koja izaziva to naprezanje.





Izračunamo:

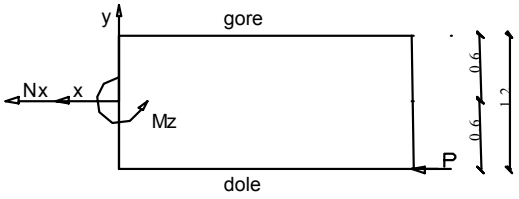
F=0.72 m^2

$z_T = 0.7$ m; y_T =0.6 m

I_Z = 0.1504 m^4

I_y =0.092 m^4

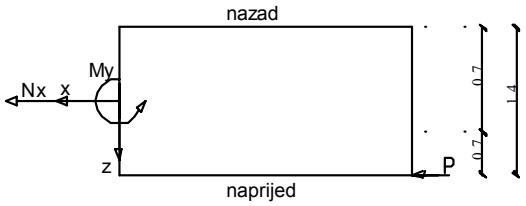
- pretpostavljamo tlačnu silu u točki B.



$\sum x = 0$ $\quad -N_x - P = 0$ $\quad$ **N_x = - P**

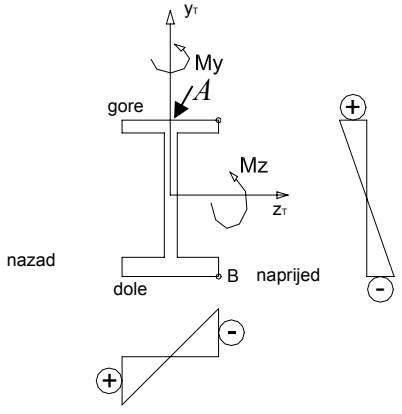
$\sum M = 0$ $\quad M_z - 0.6 \times P = 0;$ $\quad$ **M_z = 0.6 P**

(vlak gore)



$\sum x = 0$ $\quad -N_x - P = 0$ $\quad$ **N_x = - P**

$\sum M = 0$ $\quad M_y - 0.7 \times P = 0;$ $\quad$ **M_y= 0.7 P**



$$\sigma_A = \pm\frac{N}{F} \pm \frac{M_y}{I_y} \times z \pm \frac{M_z}{I_z} \times y$$

$$-1000 = -\frac{P}{0.72} + \frac{0.6P}{0.1504} \times 0.6$$

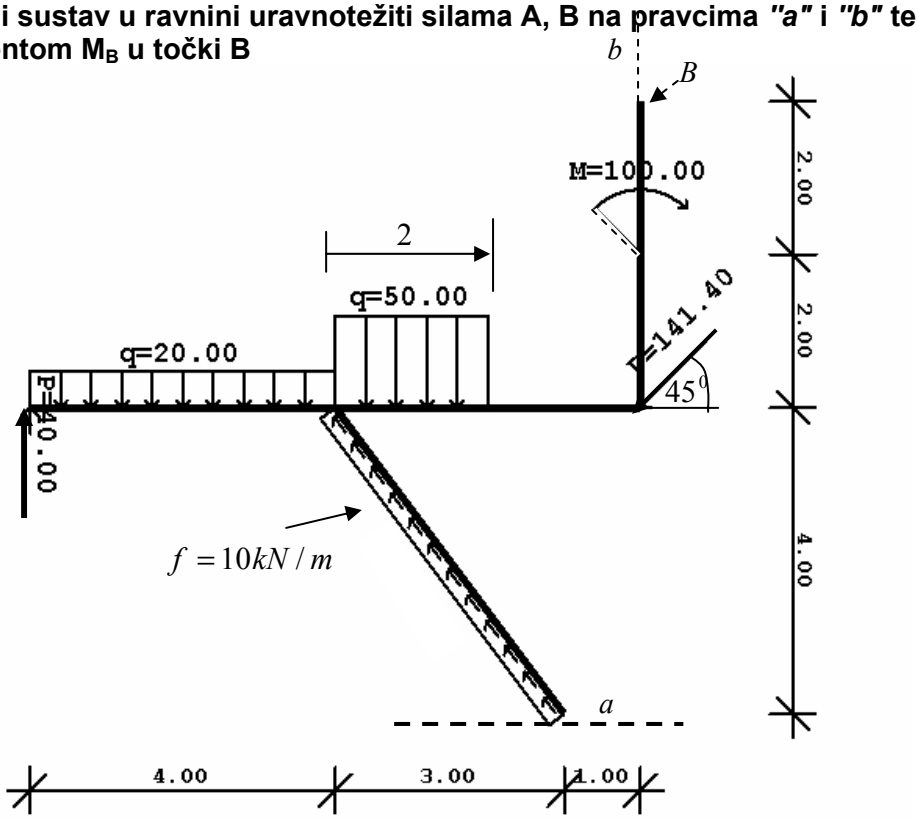
P = -995 kN

ISPITNI ROK 14. 2. 2006.

grupa **A**

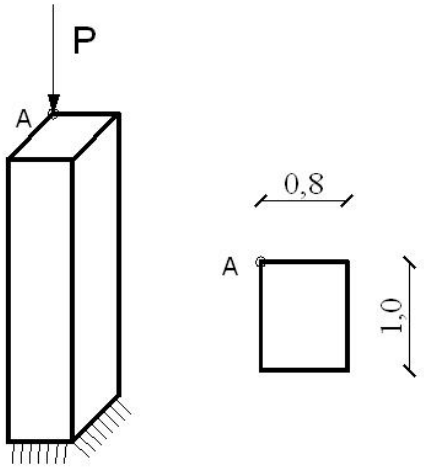
(ime i prezime ; matični broj)

1.) **Zadani sustav u ravnini uravnotežiti silama A, B na pravcima *"a"* i *"b"* te momentom M_B u točki B**



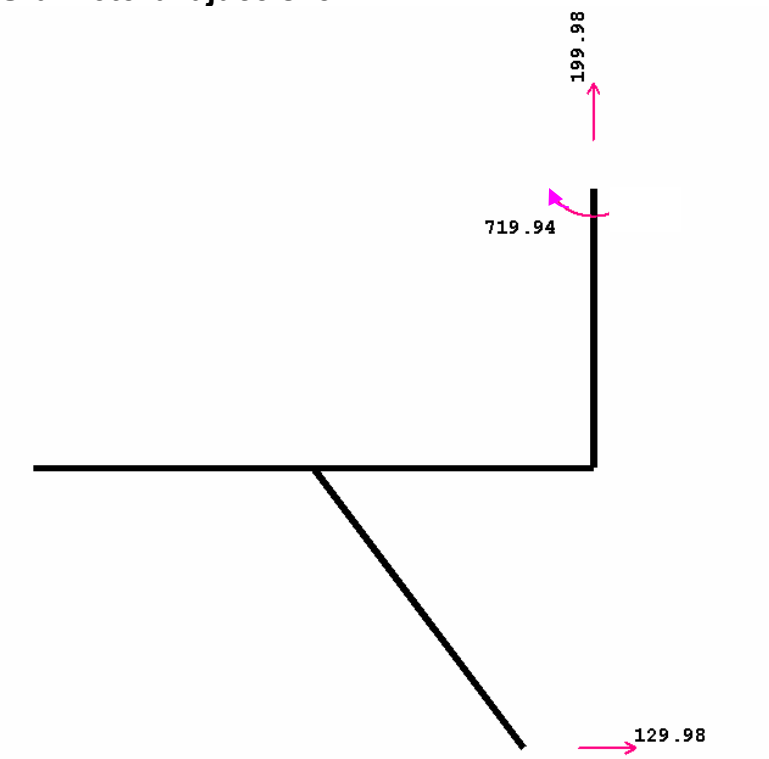
2.) Za uravnoteženi sustav iz prvog zadatka izračunati i nacrtati dijagrame unutarnjih sila.

3.) Stup, zadanog poprečnog presjeka, visok 3m je opterećen silom P= 400 kN. Odredi točku maksimalnog vlačnog naprezanja i izračunaj njegovu vrijednost.

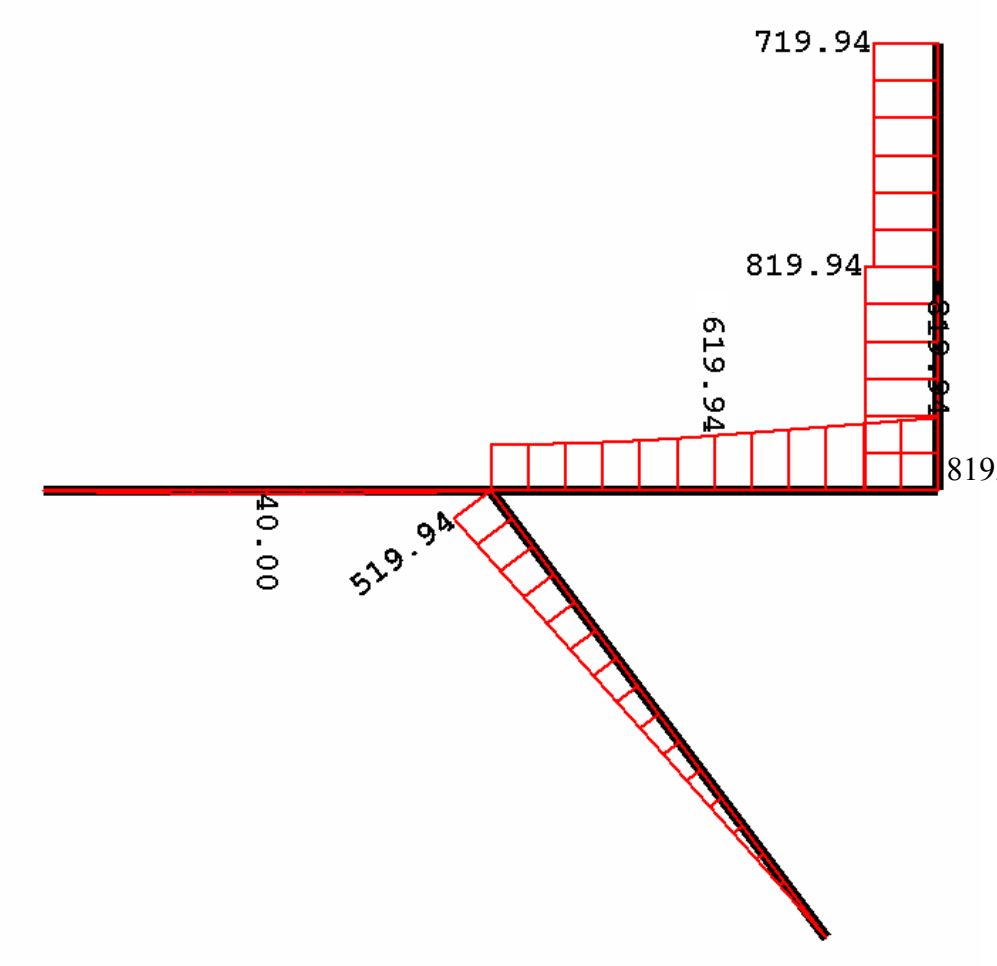


Napomena: Za izlazak na usmeni dio ispita potrebno je sakupiti najmanje 50 bodova na pismenom dijelu, ali pod uvjetom da u 2. zadatku treba imati dobar M dijagram.

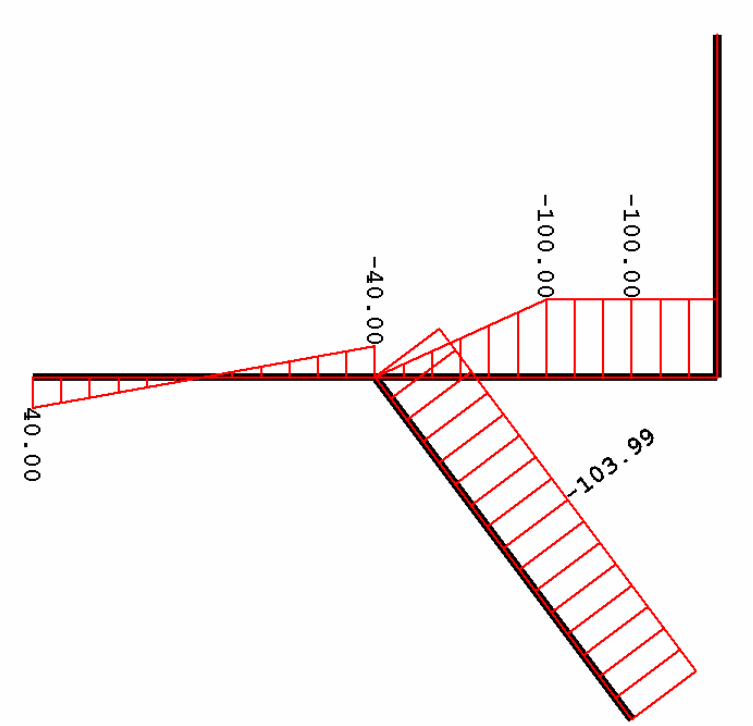
Rješenja
Uravnotežavajuće sile



M dijagram

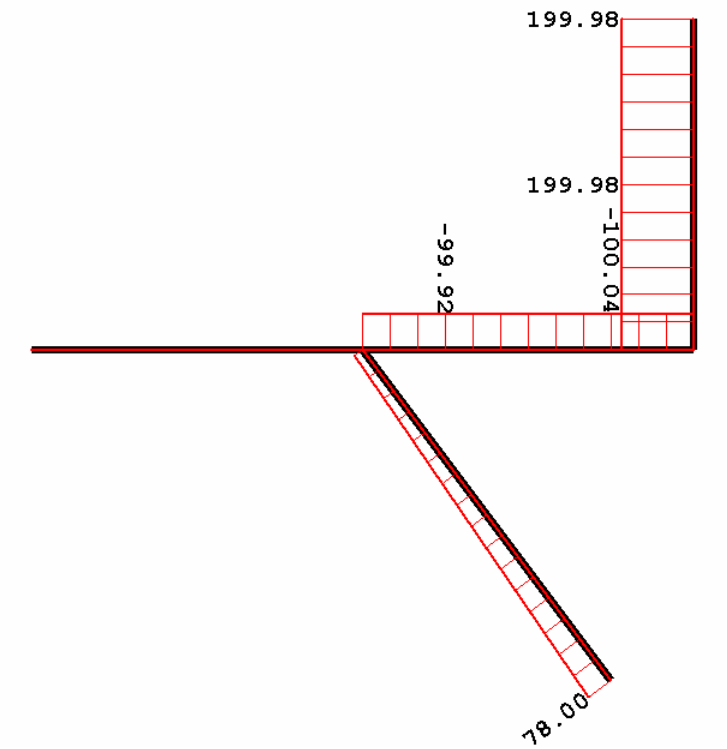


T dijagram

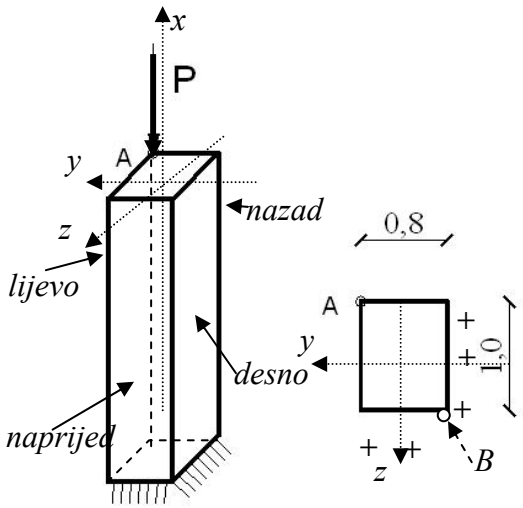


***Napomena**:ako želite dijagram možete zrcaliti oko uzdužne osi svakog elementa tako da odgovara konvenciji koju smo naučili u Tehničkoj mehanici*

N dijagram

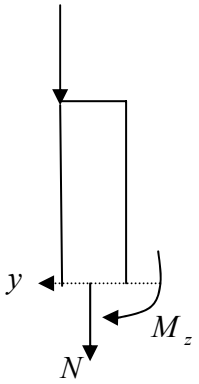


3.

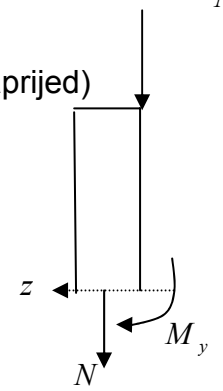


$$I_y = \frac{0.8 \cdot 1^3}{12} = 0.067 m^4 \quad I_z = \frac{1 \cdot 0.8^3}{12} = 0.0427 m^4$$ F=0.8m^2

$M_z = P * 0.4 = 160 kNm$ (vlak desno)



$M_y = P * 0.5 = 200 kNm$ (vlak naprijed)



$$\sigma_B = \pm\frac{N}{F} \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z = -\frac{400}{0.8} + \frac{160}{0.0427} 0.4 + \frac{200}{0.067} 0.5 = 2490 kN/m^2 = 0.25 kN/cm^2$$

$\sigma_{\max} = 2490 kN/m^2$, *točka B je po dijagonali od točke A*