

## Integrali - zadatci

Tablica neodređenih integrala

$g(x)$	$\int g(x)dx$
$c(\text{const.})$	0
$x^n, n \in \mathbb{R} \setminus \{-1\}$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x  + C$
$e^x$	$e^x + C$
$a^x$	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\frac{1}{\cos^2 x}$	$\text{tg } x + C$
$-\frac{1}{\sin^2 x}$	$\text{ctg } x + C$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\arcsin \frac{x}{a} + C$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \arctg \frac{x}{a} + C$

1. Odredite sljedeće integrale koristeći tablicu:

$$\begin{aligned}
 (1) \int \left( \frac{3}{x} + \sqrt[4]{x} + 1 \right) dx & \quad (2) \int \frac{dx}{\cos^2 x \sin^2 x} & (3) \int \left[ \frac{1}{\sqrt{9-x^2}} - \frac{1}{\sqrt{4+x^2}} \right] dx, \\
 (4) \int \frac{(\sqrt{x}-1)^3}{x} dx & \quad (5) \int e^x \left( 1 - \frac{e^{-x}}{x^2} \right) dx & (6) \int \frac{e^x + 1}{e^x} dx \\
 (7) \int \frac{1 - \sin^3 x}{\sin^2 x} dx & \quad (8) \int \frac{x^4}{1+x^2} dx
 \end{aligned}$$

(1)  $3 \ln|x| + \frac{4}{5} x^{5/4} + x + C$ , (2)  $-\text{ctg } x + \text{tg } x + C$ , (3)  $\arcsin \frac{x}{3} - \frac{1}{2} \arctg \frac{x}{2} + C$ , (4)  $\frac{2x^{3/2}}{3} - 3x + 6\sqrt{x} - \ln x + C$ , (5)  $e^x + \frac{1}{x} + C$ ,  
 (6)  $x - e^{-x} + C$ , (7)  $\cos x - \text{ctg } x + C$ , (8)  $\frac{x^3}{3} - x + \arctg x + C$

2. Odredite sljedeće integrale koristeći metodu supstituciju:

$$\begin{aligned}
 (1) \int 2xe^{x^2} dx & \quad (2) \int \frac{dx}{2x+5} & (3) \int x\sqrt{x+2} dx & (4) \int \frac{\ln^2 x}{x} dx \\
 (5) \int \text{tg } x dx & \quad (6) \int \sin^4 x \cos x dx & (7) \int (5-2x)^9 dx & (8) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \\
 (9) \int e^{-x^2} x dx & \quad (10) \int \frac{\sqrt{1+\ln x}}{x} dx
 \end{aligned}$$

(1)  $e^{x^2} + C$ , (2)  $\frac{1}{2} \ln|2x+5| + C$ , (3)  $\frac{2}{5} \sqrt{(x+2)^5} - \frac{4}{3} \sqrt{(x+2)^3} + C$ , (4)  $\frac{1}{3} \ln^3 x + C$ , (5)  $-\ln|\cos x| + C$ , (6)  $\frac{1}{5} \sin^5 x + C$ ,  
 (7)  $-\frac{1}{20} (5-2x)^{10} + C$ , (8)  $2e^{\sqrt{x}} + C$ , (9)  $-\frac{e^{-x^2}}{2} + C$ , (10)  $\frac{2}{3} (\ln x + 1)^{3/2} + C$

3. Odredite sljedeće integrale koristeći metodu parcijalne integracije:

$$\begin{aligned}
 (1) \int \ln(1+x^2) dx & \quad (2) \int \arctg x dx & (3) \int \cos(\ln x) dx & (4) \int \sin(\ln x) dx \\
 (5) \int e^x \cos x dx & \quad (6) \int e^x \sin x dx & (7) \int \frac{\ln x}{x^2} dx
 \end{aligned}$$

(1)  $x \ln(1 + x^2) - 2x + 2 \operatorname{arctg} x + C$ , (2)  $x \operatorname{arctg} x - \frac{1}{2} \ln(1 + x^2) + C$ , (3)  $\frac{1}{2} x \cos(\ln x) + \frac{1}{2} x \sin(\ln x) + C$ , (4)  $\frac{1}{2} x \sin(\ln x) - \frac{1}{2} x \cos(\ln x) + C$ , (5)  $\frac{1}{2} e^x (\sin x + \cos x) + C$ , (6)  $\frac{1}{2} e^x (\sin x - \cos x) + C$ , (7)  $-\frac{\ln x + 1}{x} + C$

4. Odredite sljedeće integrale koristeći na rastav na parcijalne razlomke:

$$\begin{array}{lll} (1) \int \frac{dx}{x^2 + 3x + 2} & (2) \int \frac{x - 2}{x^2 + 2x + 5} dx & (3) \int \frac{x}{x^2 - x - 6} dx \\ (4) \int \frac{dx}{x^2(1 + x)} & (5) \int \frac{x^3 dx}{x^2 - x - 2} & (6) \int \frac{x^2 + x + 2}{x^2 - x - 2} dx \end{array}$$

(1)  $\ln \left| \frac{x+1}{x+2} \right| + C$ , (2)  $\frac{1}{2} \ln(x^2 + 2x + 5) - \frac{3}{2} \operatorname{arctg} \frac{x+1}{2} + C$ , (3)  $\frac{3}{5} \ln|x - 3| + \frac{2}{5} \ln|x + 2| + C$ , (4)  $-\ln|x| - \frac{1}{x} + \ln|1 + x| + C$ , (5)  $\frac{x^2}{2} + x + \frac{8}{3} \ln(x - 2) + \frac{1}{3} \ln(x + 1) + C$ , (6)  $x + \frac{8}{3} \ln|x - 2| - \frac{2}{3} \ln|x + 1| + C$

5. Odredite sljedeće integrale trigonometrijskih funkcija

$$(1) \int \sin^2 x \cos^3 x dx \quad (2) \int \sin^2 x \cos^4 x dx \quad (3) \int \sin^2 x \cos^2 x dx$$

(1)  $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$ , (2)  $\frac{1}{16} x - \frac{1}{64} \sin 4x + \frac{1}{48} \sin^3 2x + C$ , (3)  $\frac{x}{8} - \frac{1}{32} \sin(4x) + C$ ,

6. Odredite sljedeće integrale koristeći univerzalnu supstituciju  $t = \operatorname{tg} \frac{x}{2}$  ili  $t = \operatorname{tg} x$ :

$$\begin{array}{lll} (1) \int \frac{dx}{1 + \cos x} & (2) \int \frac{dx}{\sin x - 1} & (3) \int \frac{dx}{\sin^4 x} \\ (4) \int \frac{dx}{1 + 3 \sin^2 x} & (5) \int \frac{dx}{2 + \sin x \cos x} \end{array}$$

(1)  $\tan\left(\frac{x}{2}\right) + C$ , (2)  $-\frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)} + C$ , (3)  $-\frac{1}{3 \operatorname{tg}^3 x} - \frac{1}{\operatorname{tg} x} + C$ , (4)  $\frac{1}{2} \operatorname{arctg}(2 \operatorname{tg} x) + C$ , (5)  $\frac{2}{\sqrt{15}} \operatorname{arctg}\left(\frac{4 \operatorname{tg} x + 1}{\sqrt{15}}\right) + C$

7. Izračunajte određene integrale

$$\begin{array}{lll} (1) \int_1^3 x^3 dx & (2) \int_1^2 \left(x^2 + \frac{1}{x^4}\right) dx & (3) \int_1^4 \sqrt{x} dx \\ (4) \int_0^1 \sqrt{4 - x^2} dx & (5) \int_1^e \frac{dx}{x} & (6) \int_0^\pi \sin^2 x dx \\ (7) \int_0^{\pi/4} \sin 4x dx & (8) \int_0^1 \frac{e^x dx}{1 + e^{2x}} \end{array}$$

(1) 20, (2)  $\frac{21}{8}$ , (3)  $\frac{14}{3}$ , (4)  $\frac{\pi}{6}$ , (5) 1, (6)  $\frac{\pi}{2}$ , (7)  $\frac{1}{2}$ , (8)  $-\frac{\pi}{4} + \operatorname{arctg}(e)$

8. Odredite površinu omeđenu

- (a) lukom krivulje  $y = -x^2 + 2x$  i pravcem  $y = 0$ ,
- (b) lukom krivulje  $y^2 - 2x - 1 = 0$  i pravcem  $x - y - 1 = 0$ ,
- (c) parabolom  $y = -x^2 + x + 6$  i  $x$ -osi,
- (d) lukom krivulja  $y = x^2$  i  $y = 2 - x^2$ ,
- (e) lukom krivulja  $x^2 + y^2 = 8$  i  $y = \frac{1}{2}x^2$ .

(a)  $\frac{4}{3}$ , (b)  $\frac{16}{3}$ , (c)  $20\frac{5}{6}$ , (d)  $\frac{8}{3}$ , (e)  $2\pi + \frac{4}{3}$

## **Integrali - pitanja**

1. Definirajte pojam neodređenog integrala.
2. Navedite osnovna svojstva neodređenog integrala.
3. Definirajte pojam određenog integrala funkcije na segmentu.
4. Opišite vezu između određenog i neodređenog integrala.
5. Navedite osnovna svojstva određenog integrala.