

## OPERACIJSKA ISTRAŽIVANJA 1

## DEMONSTRATURE 9

## TRANSPORTNI PROBLEM

✓ problem linearnog programiranja koji rješava problem transporta/prijevoza tereta

- ✓ iz više ishodišta -  $i = 1 \dots m$  (redova)
  - imaju fiksnu ponudu ( $a_i, i=1,2,\dots,m$ )
  - kapacitet/količina skladišta -  $a$
  - odgovarajuća količina robe u pojedinom ishodištu
  - troškovi proizvodnje
- ✓ u više odredišta -  $j = 1 \dots n$  (stupaca)
  - imaju fiksnu potražnju ( $b_j, j=1,2,\dots,n$ )
  - kapacitet/potrebe odredišta -  $b$
  - potražnja za robom određenog korisnika
  - trošak prerade

	$O_1$	$O_2$	...	$O_n$	$a_i$
$I_1$	$C_{ij}$ $x_{ij}$				$a_1$
$I_2$					$a_2$
.					.
.					.
.					.
$I_m$					$a_m$
$b_j$	$b_1$	$b_2$	...	$b_n$	

✓ SVRHA: minimalizacija troškova na relacijama između ishodišta i odredišta uz uvjet da se zadovolje potrebe odredišta i u potpunosti iskoriste ponude/kapacitete ishodišta

$$Z = \sum_{j=0}^n \sum_{i=0}^m C_{ij} \cdot x_{ij} \rightarrow \min$$

✓  $C_{ij}$  = stvarni trošak po jedinici tereta na relaciji  $i - j$  (+, 0)

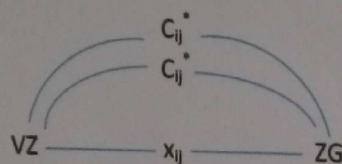
$C_{ij}^*$  = relativni trošak prijevoza (+, -, 0 → smjer promjene)

- količina promjene, za koliko će se pokvariti (-)/poboljšati (+) fja cilja

- za koliko će se uvećati/umanjiti troškovi programa

- **NEGATIVAN:** pokazuje za koliko su novčanih jedinica po jedinici tereta ukupni troškovi smanjeni zbog toga što je neka relacija, za koju se izračunava relativni trošak, na ruti transporta
- **POZITIVAN:** pokazuje za koliko novčanih jedinica po jedinici tereta su ukupni troškovi uvećani zbog toga što neka relacija, za koju se izračunava relativni trošak, nije na ruti transporta

✓  $X_{ij}$  = oznaka količine tereta od određenog tereta  $a_i$  do određenog odredišta  $b_j$  (+, 0)



✓ DVOINDEKSNI  $\Rightarrow$  2 parametra 2 uvjeta  $\rightarrow$  svaki za sebe zaseban skup rješenja



#### Zatvoreni:

- ✓ Original zatvorenog transportnog problema je kanonski problem za minimum
- ✓ Dual je standardni problem za max bez uvjeta nenegativnosti
- ✓ rang sustava: broj linearno nezavisnih vektora koji opisuju sustav
  - ✓ nezadovoljen rang  $\rightarrow$  DEGENERACIJA

#### Nove rute:

- ✓ prvo treba odrediti točku oko koje bi se iteracija kretala: najveći poz. relativni trošak prijevoza
- ✓ količina koja se premješta: najmanji od onih polja na kojima je minus

#### Nova iteracija:

- ✓ prepisat prvo što se nije mijenjalo
- ✓ promjena na poljima:  $c_{ij}^*$  koji je poz na izabranu količinu koja se premješta staviti s - od tog  $c_{ij}^*$  krenuti po rutama s + (onog kojei se premješta)



$$\begin{aligned}
 Z = & C_{11}x_{11} + C_{12}x_{12} + \dots + C_{1n}x_{1n} + \\
 & C_{21}x_{21} + C_{22}x_{22} + \dots + C_{2n}x_{2n} + \\
 & \vdots \\
 & C_{n1}x_{n1} + C_{n2}x_{n2} + \dots + C_{mn}x_{mn} \rightarrow \min
 \end{aligned}$$

$$\begin{aligned}
 x_{11} + x_{12} + \dots + x_{1n} &= a_1 \\
 x_{21} + x_{22} + \dots + x_{2n} &= a_2 \\
 x_{n1} + x_{n2} + \dots + x_{mn} &= a_m
 \end{aligned}$$

$$x_{ij} \geq 0$$

$$\begin{aligned}
 x_{11} + x_{21} + \dots + x_{m1} &= b_1 \\
 x_{12} + x_{22} + \dots + x_{m2} &= b_2 \\
 x_{1m} + x_{2m} + \dots + x_{mn} &= b_n
 \end{aligned}$$

$$x_{ij} \geq 0$$

$$\begin{aligned}
 Z^d = & a_1u_1 + a_2u_2 + \dots + a_mu_m + \\
 & b_1v_1 + b_2v_2 + \dots + b_nv_n \rightarrow \max
 \end{aligned}$$

$$u_1 + v_1 \leq C_{11}$$

$$u_1 + v_2 \leq C_{12}$$

$$\vdots$$

$$u_1 + v_n \leq C_{1n}$$

$$u_2 + v_1 \leq C_{21}$$

$$u_2 + v_2 \leq C_{22}$$

$$\vdots$$

$$u_2 + v_n \leq C_{2n}$$

$$\dots$$

$$u_m + v_1 \leq C_{m1}$$

$$u_m + v_2 \leq C_{m2}$$

$$\vdots$$

$$u_m + v_n \leq C_{mn}$$

NEMA UVJETA NEGATIVNOSTI!

## OPERACIJSKA ISTRAŽIVANJA 1

## DEMONSTRATURE 11

## TRANSPORTNI PROBLEM

DEGENERACIJA:

I: 20, 40, 30, 40

trošak lokacije: 1, 1, 0, 2

O: 20, 30, 60, 20

$$C_{ij} = \begin{pmatrix} 2 & 6 & 1 & 3 \\ 4 & 7 & 0 & 2 \\ 9 & 4 & 5 & 2 \\ 8 & 0 & 4 & 5 \end{pmatrix} + \begin{matrix} +1 \\ +1 \\ +0 \\ +2 \end{matrix} = \begin{pmatrix} 3 & 7 & 2 & 4 \\ 5 & 8 & 1 & 3 \\ 9 & 4 & 5 & 2 \\ 10 & 2 & 6 & 7 \end{pmatrix}$$

$$Z = 3x_{11} + 7x_{12} + 2x_{13} + 4x_{14} + \\ 5x_{21} + 8x_{22} + 1x_{23} + 3x_{24} + \\ 9x_{31} + 4x_{32} + 5x_{33} + 2x_{34} + \\ 10x_{41} + 2x_{42} + 6x_{43} + 7x_{44} + \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 20 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 40 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 30 \\ x_{41} + x_{42} + x_{43} + x_{44} &= 40 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} &= 20 \\ x_{12} + x_{22} + x_{32} + x_{42} &= 30 \\ x_{13} + x_{23} + x_{33} + x_{43} &= 60 \\ x_{14} + x_{24} + x_{34} + x_{44} &= 20 \end{aligned}$$

$$x_{ij} \geq 0$$

$$x_{ij} \geq 0$$

$$Z^d = 20u_1 + 40u_2 + 30u_3 + 40u_4 + \\ 20v_1 + 30v_2 + 60v_3 + 20v_4 \rightarrow \max$$

$$\begin{aligned} u_1 + v_1 &\leq 3 \\ u_1 + v_2 &\leq 7 \\ u_1 + v_3 &\leq 2 \\ u_1 + v_4 &\leq 4 \end{aligned}$$

$$\begin{aligned} u_2 + v_1 &\leq 5 \\ u_2 + v_2 &\leq 8 \\ u_2 + v_3 &\leq 1 \\ u_2 + v_4 &\leq 3 \end{aligned}$$

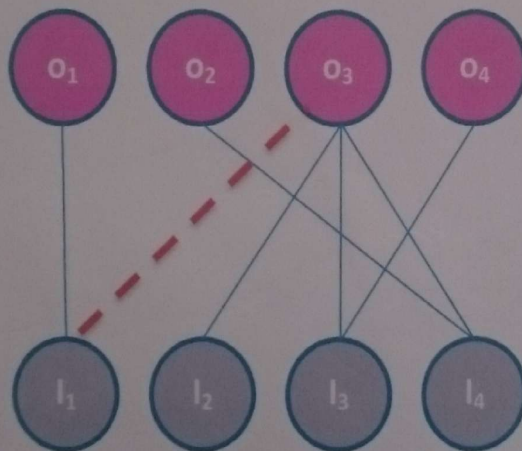
$$\begin{aligned} u_3 + v_1 &\leq 9 \\ u_3 + v_2 &\leq 4 \\ u_3 + v_3 &\leq 5 \\ u_3 + v_4 &\leq 2 \end{aligned}$$

$$\begin{aligned} u_4 + v_1 &\leq 10 \\ u_4 + v_2 &\leq 2 \\ u_4 + v_3 &\leq 6 \\ u_4 + v_4 &\leq 7 \end{aligned}$$



	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$	$r_i$	$u_i$
$I_1$	3 20	7 -9	2 0	4 -5	20	1,1,1,1	0
$I_2$	5 -3	8 -11	1 40	3 -5	40	2,2,4	-1
$I_3$	9 -3	4 -3	5 10	2 20	30	1,3,4,4	3
$I_4$	10 -3	2 30	6 10	7 -4	40	4,1,4,4	4
$b_j$	20	30	60	20	130 130		
$r_j$	2,2,2,6	2,	1,1,1,1	1,1			
$v_i$	3	-2	2	-1			

RANG!  $\rightarrow$  degeneracija  $\rightarrow 4 + 4 - 1 = 7$



2.06.2016.

OTVORENI PROBLEM:

$$I: 10, 15, 30, 25 \quad 80$$

$$O: 14, 26, 17 \quad 57$$

$$C_{ij} = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 2 & 3 \end{pmatrix}$$

Original:

$$Z = 4x_{11} + 2x_{12} + 1x_{13} + \\ 3x_{21} + 1x_{22} + 3x_{23} + \\ 2x_{31} + 1x_{32} + 4x_{33} + \\ 3x_{31} + 2x_{32} + 3x_{33} \rightarrow \min$$

$$x_{11} + x_{12} + x_{13} \leq 10$$

$$x_{21} + x_{22} + x_{23} \leq 15$$

$$x_{31} + x_{32} + x_{33} \leq 30$$

$$x_{41} + x_{42} + x_{43} \leq 25$$

$$x_{ij} \geq 0$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 14$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 26$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 17$$

$$x_{ij} \geq 0$$

Kanonski:

$$Z = 4x_{11} + 2x_{12} + 1x_{13} + 0x_{14} \\ 3x_{21} + 1x_{22} + 3x_{23} + 0x_{24} \\ 2x_{31} + 1x_{32} + 4x_{33} + 0x_{34} \\ 3x_{41} + 2x_{42} + 3x_{43} + 0x_{44} \rightarrow \min$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 10$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 15$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 30$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 25$$

$$x_{ij} \geq 0$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 14$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 26$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 17$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 23$$

$$x_{ij} \geq 0$$



Dual:

$$Z^d = 10u_1 + 15u_2 + 30u_3 + 25u_4 + 14v_1 + 16v_2 + 17v_3 + 23v_4 \rightarrow \max$$

$$\begin{array}{llll} u_1 + v_1 \leq 4 & u_2 + v_1 \leq 3 & u_3 + v_1 \leq 2 & u_4 + v_1 \leq 3 \\ u_1 + v_2 \leq 2 & u_2 + v_2 \leq 1 & u_3 + v_2 \leq 1 & u_4 + v_2 \leq 2 \\ u_1 + v_3 \leq 1 & u_2 + v_3 \leq 3 & u_3 + v_3 \leq 4 & u_4 + v_3 \leq 3 \\ u_1 + v_4 \leq 0 & u_2 + v_4 \leq 0 & u_3 + v_4 \leq 0 & u_4 + v_4 \leq 0 \end{array}$$

Z = 95

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
I <sub>1</sub>	4	2	1	0	10
I <sub>2</sub>	3	1	3	0	15
I <sub>3</sub>	2	1	4	0	30
I <sub>4</sub>	3	2	3	0	25
b <sub>j</sub>	14	26	17	23	80

$$\begin{aligned} C_{11}^* &= -4 + 1 - 3 + 0 - 0 + 3 = -3 \\ C_{12}^* &= -2 + 1 - 3 + 0 - 0 + 3 - 2 + 1 = -2 \\ C_{14}^* &= -0 + 0 - 3 + 1 = -2 \\ C_{21}^* &= -3 + 3 - 0 + 0 = 0 \\ C_{22}^* &= -1 + 1 - 2 + 3 - 0 + 0 = 1 \\ C_{33}^* &= -4 + 2 - 3 + 0 - 0 + 3 = -2 \\ C_{34}^* &= -0 + 0 - 3 + 2 = -1 \\ C_{42}^* &= -2 + 3 - 2 + 1 = 0 \\ C_{43}^* &= -3 + 0 - 0 + 3 = 0 \end{aligned}$$

RANG ✓

Z = 87

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
I <sub>1</sub>	4	2	1	0	10
I <sub>2</sub>	3	1	3	0	15
I <sub>3</sub>	2	1	4	0	30
I <sub>4</sub>	3	2	3	0	25
b <sub>j</sub>	14	26	17	23	80

$$\begin{array}{cccccc} \boxed{1} & -26 & +4 & -10 & +15 & -8 \\ 8 & 18 & 12 & 2 & 23 & \boxed{-1} \end{array}$$

$$\begin{aligned} C_{11}^* &= -4 + 1 - 3 + 1 - 1 + 2 = -4 \\ C_{12}^* &= -2 + 1 - 3 + 1 = -3 \\ C_{14}^* &= -0 + 0 - 3 + 2 - 1 + 1 - 3 + 1 = -3 \\ C_{21}^* &= -3 + 1 - 1 + 2 = -1 \\ C_{33}^* &= -4 + 3 - 1 + 1 = -1 \\ C_{34}^* &= -0 + 0 - 3 + 2 = -1 \\ C_{42}^* &= -2 + 3 - 2 + 1 = 0 \\ C_{43}^* &= -3 + 3 - 2 + 1 - 1 + 3 = 1 \end{aligned}$$

$Z = 85$

	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$
$I_1$	4	2	1	0	10
$I_2$	3	1	3	0	15
$I_3$	2	1	4	0	30
$I_4$	3	2	3	0	25
$b_j$	14	26	17	23	80

$$\begin{matrix} 1 & -2 & +12 & -18 & +8 & -7 \\ 2 & -1 & 14 & 16 & 10 & 5 \end{matrix}$$

$$C_{11}^* = -4 + 1 - 3 + 1 - 1 + 2 = -4$$

$$C_{12}^* = -2 + 1 - 3 + 1 = -3$$

$$C_{14}^* = -0 + 0 - 3 + 1 = -2$$

$$C_{21}^* = -3 + 1 - 1 + 2 = -1$$

$$C_{24}^* = -0 + 0 - 3 + 3 = 0$$

$$C_{33}^* = -4 + 3 - 1 + 1 = -1$$

$$C_{34}^* = -0 + 0 - 3 + 3 - 1 + 1 = 0$$

$$C_{42}^* = -2 + 3 - 3 + 1 = -1$$

$Z = 85$

	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$
$I_1$	4	2	1	0	10
$I_2$	3	1	3	0	15
$I_3$	2	1	4	0	30
$I_4$	3	2	3	0	25
$b_j$	14	26	17	23	80

$$\begin{matrix} 0 & -23 & +2 & -5 \\ -5 & 18 & 7 & 0 \end{matrix}$$

$$C_{11}^* = -4 + 1 - 3 + 0 - 0 + 1 - 1 + 2 = -4$$

$$C_{12}^* = -2 + 1 - 3 + 1 = -3$$

$$C_{14}^* = -0 + 0 - 3 + 1 = -2$$

$$C_{21}^* = -3 + 1 - 1 + 2 = -1$$

$$C_{24}^* = -0 + 0 - 3 + 3 = 0$$

$$C_{33}^* = -4 + 3 - 1 + 1 = -1$$

$$C_{34}^* = -0 + 0 - 3 + 3 - 1 + 1 = 0$$

$$C_{42}^* = -2 + 3 - 3 + 1 = -1$$



$$I = 50, 30, 10$$

$$O = 25, 15, 22, 28$$

$$Z = 1x_{11} + 5x_{12} + 8x_{13} + 2x_{14} + 2x_{21} + 6x_{22} + 3x_{23} + 4x_{24} + 7x_{31} + 9x_{32} + 11x_{33} + 3x_{34} \rightarrow \min$$

$$C_{ij} = \begin{Bmatrix} 1 & 5 & 8 & 2 \\ 2 & 6 & 3 & 4 \\ 7 & 9 & 11 & 3 \end{Bmatrix}$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 50 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 30 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 10 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} &= 25 \\ x_{12} + x_{22} + x_{32} &= 15 \\ x_{13} + x_{23} + x_{33} &= 22 \\ x_{14} + x_{24} + x_{34} &= 28 \end{aligned}$$

$$x_{ij} \geq 0$$

$$Z^d = 50u_1 + 30u_2 + 10u_3 + 25v_1 + 15v_2 + 22v_3 + 28v_4 \rightarrow \max$$

$$\begin{aligned} u_1 + v_1 &\leq 1 & u_2 + v_1 &\leq 2 & u_3 + v_1 &\leq 7 \\ u_1 + v_2 &\leq 5 & u_2 + v_2 &\leq 6 & u_3 + v_2 &\leq 9 \\ u_1 + v_3 &\leq 8 & u_2 + v_3 &\leq 3 & u_3 + v_3 &\leq 11 \\ u_1 + v_4 &\leq 2 & u_2 + v_4 &\leq 4 & u_3 + v_4 &\leq 3 \end{aligned}$$

METODA SZ KUTA!

METODA MIN. TROŠKOVA!

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
I <sub>1</sub>	1 25	5 15	8 10	2	50
I <sub>2</sub>	2	6	3 17	4 18	30
I <sub>3</sub>	7	9	11	3 10	10
b <sub>j</sub>	25	15	22	28	30

$$Z = 1 \cdot 25 + 5 \cdot 15 + 8 \cdot 10 + (2 \cdot 3 + 4 \cdot 18) + 3 \cdot 10 = 318$$

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
I <sub>1</sub>	1 25	5	8	2 15	50
I <sub>2</sub>	2	6 8	3 22	4	30
I <sub>3</sub>	7 -1	9 7	11 -1	3 5	10
b <sub>j</sub>	25	15	22	28	30

$$Z = 1 \cdot 25 + 2 \cdot 25 + 6 \cdot 8 + 3 \cdot 22 + 9 \cdot 7 + 3 \cdot 3 = 261$$



# VOGELOVA APROKSIMATIVNA METODA!

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>	r <sub>i</sub>
I <sub>1</sub>	1 25	5 7	9 8	2 13	50	1,1,1,1 4
I <sub>2</sub>	2	6 8	3 22	4	30	1,2,2,1 4
I <sub>3</sub>	7	9	11	3 10	10	4,4
b <sub>j</sub>	25	15	22	28	90	30
r <sub>j</sub>	1,1,1 1	1,1,1 1	6	1,1,2		

$$z = 1 \cdot 25 + 5 \cdot 7 + 2 \cdot 11 + 6 \cdot 8 + 3 \cdot 22 + 3 \cdot 10 = 220$$

40

Seminar 16.05.2015.

Zad. ① MODI METODA! → izračunavanje optimalnog rešenja

$$u_i + v_j = C_{ij}$$

$$u_i = C_{ij} - v_j$$

$$v_j = C_{ij} - u_i$$

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>	u <sub>i</sub>
I <sub>1</sub>	1 25	5 7	8 -6	2 13	50	0
I <sub>2</sub>	2 0	6 8	3 22	4 -1	30	1
I <sub>3</sub>	7 -5	9 -3	11 -8	3 10	10	1
b <sub>j</sub>	25	15	22	28	90	30
v <sub>j</sub>	1	5	2	2		

rang:  $n + m - 1 = 3 + 4 - 1 = 6$  (✓)

$$C_{ij}^* = (u_i + v_j) - C_{ij} \Rightarrow \text{nezauzeta polja}$$

- $C_{13} = 0 + 2 - 8 = -6$
- $C_{21} = 1 + 1 - 2 = 0$
- $C_{24} = 1 + 2 - 4 = -1$
- $C_{31} = 1 + 1 - 7 = -5$
- $C_{32} = 1 + 5 - 9 = -3$
- $C_{33} = 1 + 2 - 11 = -8$



# METODA RELATIVNA TROŠKOVA 1.

	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$
$I_1$	1	5	8	7	50
$I_2$	2	6	3	4	30
$I_3$	7	9	11	3	10
$b_j$	25	15	22	28	30

$$C_{13} = -8 + 5 - 6 + 3 = -6$$

$$C_{21} = -2 + 6 - 5 + 1 = 0$$

$$C_{24} = -4 + 6 - 5 + 2 = -1$$

$$C_{31} = -7 + 1 - 2 + 3 = -5$$

$$C_{32} = -9 + 5 - 2 + 3 = -3$$

$$C_{33} = -11 + 3 - 6 + 5 - 2 + 3 = -8$$

Zad. 2.

- 82 Cnt -

	$O_1$	$O_2$	$O_3$	$a_i$
$I_1$	5	10	15	45
$I_2$	12	4	8	50
$I_3$	7	3	9	90
$I_4$	14	16	1	45
$b_j$	20	40	70	230

$$Z = 5 \cdot 45 + 12 \cdot 50 + 7 \cdot 25 + 3 \cdot 40 + 9 \cdot 25 + 1 \cdot 35 = 1390$$

a) original

$$Z = 5x_{11} + 10x_{12} + 15x_{13} + 12x_{21} + 4x_{22} + 8x_{23} + 7x_{31} + 3x_{32} + 9x_{33} + 14x_{41} + 16x_{42} + 1x_{43} \rightarrow \min$$

Ograničavaju ispravlja:

Ograničavaju ostališta:

$$\left. \begin{aligned} x_{11} + x_{12} + x_{13} &= 45 \\ x_{21} + x_{22} + x_{23} &= 50 \\ x_{31} + x_{32} + x_{33} &= 90 \\ x_{41} + x_{42} + x_{43} &= 45 \end{aligned} \right\} u_i$$

$$\left. \begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} &= 120 \\ x_{12} + x_{22} + x_{32} + x_{42} &= 40 \\ x_{13} + x_{23} + x_{33} + x_{43} &= 70 \end{aligned} \right\} v_j$$

$$x_{ij} \geq 0$$

b) dual

$$Z^d = 45u_1 + 50u_2 + 90u_3 + 45u_4 + 120v_1 + 40v_2 + 70v_3$$

$$\begin{aligned} u_1 + v_1 &\leq 5 & u_2 + v_1 &\leq 12 & u_3 + v_1 &\leq 7 & u_4 + v_1 &\leq 14 \\ u_1 + v_2 &\leq 10 & u_2 + v_2 &\leq 4 & u_3 + v_2 &\leq 3 & u_4 + v_2 &\leq 16 \\ u_1 + v_3 &\leq 15 & u_2 + v_3 &\leq 8 & u_3 + v_3 &\leq 9 & u_4 + v_3 &\leq 1 \end{aligned}$$



# MIN. TROŠKOVA

$-4 \rightarrow 40 + 50 - 25$   
 $(15) \rightarrow 75$

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	a <sub>i</sub>
I <sub>1</sub>	5 (45)	10 -9	15 -14	45
I <sub>2</sub>	12 (25)	4 4	8 (25)	50
I <sub>3</sub>	7 (50)	3 (10)	9 -6	90
I <sub>4</sub>	14 -8	16 -15	1 (45)	45
b <sub>j</sub>	120	40	70	230 230

$z = 5 \cdot 45 + 12 \cdot 25 + 8 \cdot 25 + 7 \cdot 50 + 3 \cdot 40 + 45 = 1240$

# VOGELOVA

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	a <sub>i</sub>	u <sub>i</sub>
I <sub>1</sub>	5 (45)	10	15	45	5, 5
I <sub>2</sub>	12	4 (25)	8 (25)	50	4, 4, 4
I <sub>3</sub>	7 (75)	3 (15)	9	90	4, 4, 4, 6
I <sub>4</sub>	14	16	1 (45)	45	13
b <sub>j</sub>	120	40	70	230 230	
v <sub>j</sub>	2, 2, 5	1, 1, 1, 1	7, 1, 1, 1		

$z = 5 \cdot 45 + 4 \cdot 25 + 8 \cdot 25 + 7 \cdot 75 + 3 \cdot 15 + 45 = 1140$

# MODI METODA

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	a <sub>i</sub>	u <sub>i</sub>
I <sub>1</sub>	5 (45)	10 -9	15 -10	45	0
I <sub>2</sub>	12 -4	4 (25)	8 (25)	50	3
I <sub>3</sub>	7 (75)	3 (15)	9 -2	90	2
I <sub>4</sub>	14 -13	16 -19	1 (45)	45	-4
b <sub>j</sub>	120	40	70	230 230	
v <sub>j</sub>	5	1	5		

# METODA REL. TROŠKOVA

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	a <sub>i</sub>
I <sub>1</sub>	5 (45)	10 -9	15 -10	45
I <sub>2</sub>	12 -4	4 (25)	8 (25)	50
I <sub>3</sub>	7 (75)	3 (15)	9 -2	90
I <sub>4</sub>	14 -13	16 -19	1 (45)	45
b <sub>j</sub>	120	40	70	230 230

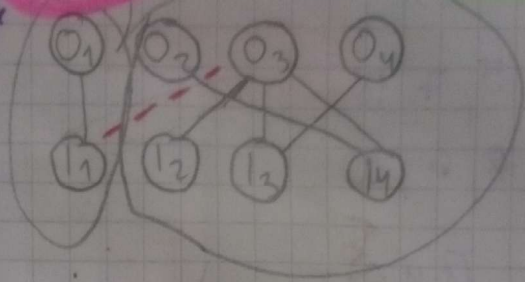
-5                      -6



Seminar 29.5.2015.

ili troškovi proizvodnje

RANG:  $4+4-1=7 \Rightarrow$  DEGENERACIJA



Zad 1)

1: 20, 40, 30, 40

0: 20, 30, 60, 20

trošak lobanje: 1, 1, 0, 2

trošak prerade

$$C_{ij} = \begin{Bmatrix} 2 & 6 & 1 & 3 \\ 4 & 7 & 0 & 2 \\ 9 & 4 & 5 & 2 \\ 8 & 0 & 4 & 5 \\ 1 & 1 & 0 & 2 \end{Bmatrix} \begin{matrix} +1 \\ +1 \\ +0 \\ +2 \end{matrix}$$

$$\begin{Bmatrix} 3 & 7 & 2 & 4 \\ 5 & 8 & 1 & 3 \\ 9 & 4 & 5 & 2 \\ 10 & 2 & 6 & 7 \end{Bmatrix}$$

$$Z = 3x_{11} + 7x_{12} + 2x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 1x_{23} + 3x_{24} + 9x_{31} + 4x_{32} + 5x_{33} + 2x_{34} + 10x_{41} + 2x_{42} + 6x_{43} + 7x_{44} \rightarrow \min$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 20$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 40 \quad x_{ij} \geq 0$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 30$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 40$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 20$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 30 \quad x_{ij} \geq 0$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 60$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 20$$

$$Z^d = 20u_1 + 40u_2 + 30u_3 + 40u_4 + 20v_1 + 30v_2 + 60v_3 + 20v_4 \rightarrow \max$$

$$u_1 + v_1 \leq 3 \quad u_2 + v_1 \leq 5$$

$$u_1 + v_2 \leq 7 \quad u_2 + v_2 \leq 8$$

$$u_1 + v_3 \leq 2 \quad u_2 + v_3 \leq 1$$

$$u_1 + v_4 \leq 4 \quad u_2 + v_4 \leq 3$$

$$u_3 + v_1 \leq 9 \quad u_4 + v_1 \leq 10$$

$$u_3 + v_2 \leq 4 \quad u_4 + v_2 \leq 2$$

$$u_3 + v_3 \leq 5 \quad u_4 + v_3 \leq 6$$

$$u_3 + v_4 \leq 2 \quad u_4 + v_4 \leq 7$$

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>	v <sub>i</sub>	u <sub>i</sub>
I <sub>1</sub>	3 20	7 -9	2 0	4 -5	20	1,1,1,1	0
I <sub>2</sub>	5 -3	8 -11	1 40	3 -5	40	2,2,4	-1
I <sub>3</sub>	9 -3	4 -3	5 10	2 10	30	2,3,4,4	3
I <sub>4</sub>	10 -3	2 30	6 10	7 -4	40	4,1,4,4	4
b <sub>j</sub>	20	30	60	20	130		
v <sub>j</sub>	2,2,2,4 6	2	1,1,1,1 1	1,1			
v <sub>j</sub>	3	-2	2	-1			



$$4+4-1=7$$



Zad. (2)

l: 5, 7, 18

o: 8, 6, 10

$$c_{ij} = \begin{Bmatrix} 1 & 4 & 6 \\ 8 & 2 & 5 \\ 3 & 7 & 9 \end{Bmatrix}$$

	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$	
$l_1$	1	4	6	0	5	0
$l_2$	8	2	5	0	7	-2
$l_3$	3	7	9	0	18	2
$b_j$	8	6	10	6	30	

Original:

$$Z = 1x_{11} + 4x_{12} + 6x_{13} + 8x_{21} + 2x_{22} + 5x_{23} + 3x_{31} + 7x_{32} + 9x_{33} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} &\leq 5 \\ x_{21} + x_{22} + x_{23} &\leq 7 \\ x_{31} + x_{32} + x_{33} &\leq 18 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} &= 8 \\ x_{12} + x_{22} + x_{32} &= 6 \\ x_{13} + x_{23} + x_{33} &= 10 \end{aligned}$$

$$x_{ij} \geq 0$$

$$x_{ij} \geq 0$$

$$\begin{aligned} &-9 \\ &+3 \\ &\boxed{-5} \end{aligned}$$

Kanowski:

$$Z = 1x_{11} + 4x_{12} + 6x_{13} + 0x_{14} + 8x_{21} + 2x_{22} + 5x_{23} + 0x_{24} + 3x_{31} + 7x_{32} + 9x_{33} + 0x_{34} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 5 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 7 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 18 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} &= 8 \\ x_{12} + x_{22} + x_{32} &= 6 \\ x_{13} + x_{23} + x_{33} &= 10 \\ x_{14} + x_{24} + x_{34} &= 6 \end{aligned}$$

$$x_{ij} \geq 0$$

Dual:

$$Z = 5u_1 + 7u_2 + 18u_3 + 8v_1 + 6v_2 + 10v_3 + 6v_4 \rightarrow \max$$

$$\begin{aligned} u_1 + v_1 &\leq 1 & u_2 + v_1 &\leq 8 & u_3 + v_1 &\leq 3 \\ u_1 + v_2 &\leq 4 & u_2 + v_2 &\leq 2 & u_3 + v_2 &\leq 7 \\ u_1 + v_3 &\leq 6 & u_2 + v_3 &\leq 5 & u_3 + v_3 &\leq 9 \\ u_1 + v_4 &\leq 0 & u_2 + v_4 &\leq 0 & u_3 + v_4 &\leq 0 \end{aligned}$$

	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$
$l_1$	1	4	6	0	5
$l_2$	8	2	5	0	7
$l_3$	3	7	9	0	18
$b_j$	8	6	10	6	30

$$Z = 6 \cdot 5 + 2 \cdot 6 + 5 \cdot 1 + 3 \cdot 8 + 9 \cdot 4 + 0 \cdot 6 =$$



Seminari

5.6.2015.

RANG:  $4+3-1=6$  ✓

Zad ①

l 22, 19, 31

o: 30, 25, 20

$$C_{ij} = \begin{Bmatrix} 15 & 10 & 20 \\ 7 & 15 & 12 \\ 18 & 8 & 10 \end{Bmatrix}$$

original :

$$Z = 15x_{11} + 10x_{12} + 20x_{13} + 7x_{21} + 15x_{22} + 12x_{23} + 18x_{31} + 8x_{32} + 10x_{33} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 22 \\ x_{21} + x_{22} + x_{23} &= 19 \\ x_{31} + x_{32} + x_{33} &= 31 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} &\leq 30 \\ x_{12} + x_{22} + x_{32} &\leq 25 \\ x_{13} + x_{23} + x_{33} &\leq 20 \end{aligned}$$

$$Z = 15 \cdot 8 + 10 \cdot 11 + 7 \cdot 19 + 18 \cdot 11 + 10 \cdot 20 + 0 \cdot 3 = 681$$

Kanonskii :

$$x_{ij} \geq 0$$

$$x_{ij} \geq 0$$

$$Z = 15x_{11} + 10x_{12} + 20x_{13} + 7x_{21} + 15x_{22} + 12x_{23} + 18x_{31} + 8x_{32} + 10x_{33} + 0x_{u1} + 0x_{u2} + 0x_{u3} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 22 \\ x_{21} + x_{22} + x_{23} &= 19 \\ x_{31} + x_{32} + x_{33} &= 31 \\ x_{u1} + x_{u2} + x_{u3} &= 3 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{12} + x_{31} + x_{u1} &= 30 \\ x_{12} + x_{22} + x_{32} + x_{u2} &= 25 \\ x_{13} + x_{23} + x_{33} + x_{u3} &= 20 \end{aligned}$$

$$x_{ij} \geq 0$$

Dual:

$$Z^d = 22u_1 + 19u_2 + 31u_3 + 3u_4 + 30v_1 + 25v_2 + 20v_3 \rightarrow \max$$

$$\begin{aligned} u_1 + v_1 &\leq 15 \\ u_1 + v_2 &\leq 10 \\ u_1 + v_3 &\leq 20 \end{aligned}$$

$$\begin{aligned} u_2 + v_1 &\leq 7 \\ u_2 + v_2 &\leq 15 \\ u_2 + v_3 &\leq 12 \end{aligned}$$

$$\begin{aligned} u_3 + v_1 &\leq 18 \\ u_3 + v_2 &\leq 8 \\ u_3 + v_3 &\leq 10 \end{aligned}$$

$$\begin{aligned} u_4 + v_1 &\leq 0 \\ u_4 + v_2 &\leq 0 \\ u_4 + v_3 &\leq 0 \end{aligned}$$

	$O_1$	$O_2$	$O_3$	$a_i$
$I_1$	15 ③	10 ④	20 -8	22
$I_2$	7 ⑤	15 -13	12 -8	19
$I_3$	18 -5	8 ⑥	10 ⑦	31
$I_4$	0 ⑧	0 -5	0 -3	3
$b_j$	30	25	20	75

5, 5, 5

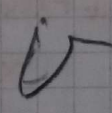
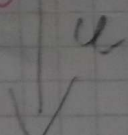
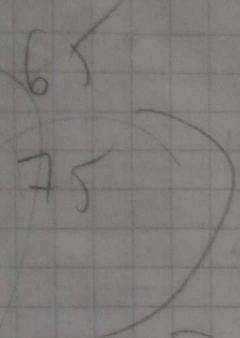
5

2, 2, 10

8, 3, 3    2, 2, 3    2, 10

72    75

više





zad ②

-80

l: 10, 15, 30, 25

o: 14, 26, 17

$$C_{ij} = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 2 & 3 \end{pmatrix}$$

Original:

$$z = 4x_{11} + 2x_{12} + 1x_{13} + 3x_{21} + 1x_{22} + 3x_{23} + 2x_{31} + 1x_{32} + 4x_{33} + 3x_{41} + 2x_{42} + 3x_{43} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} &\leq 10 \\ x_{21} + x_{22} + x_{23} &\leq 15 \\ x_{31} + x_{32} + x_{33} &\leq 30 \\ x_{41} + x_{42} + x_{43} &\leq 25 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} &\leq 14 \\ x_{12} + x_{22} + x_{32} + x_{42} &\leq 16 \\ x_{13} + x_{23} + x_{33} + x_{43} &\leq 17 \end{aligned}$$

$x_{ij} \geq 0$

$x_{ij} \geq 0$

Kanonski:

$$z = 4x_{11} + 2x_{12} + 1x_{13} + 0x_{14} + 3x_{21} + 1x_{22} + 3x_{23} + 0x_{24} + 2x_{31} + 1x_{32} + 4x_{33} + 0x_{34} + 3x_{41} + 2x_{42} + 3x_{43} + 0x_{44} \rightarrow \min$$

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 10 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 15 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 30 \\ x_{41} + x_{42} + x_{43} + x_{44} &= 25 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} &= 14 \\ x_{12} + x_{22} + x_{32} + x_{42} &= 16 \\ x_{13} + x_{23} + x_{33} + x_{43} &= 17 \\ x_{14} + x_{24} + x_{34} + x_{44} &= 23 \end{aligned}$$

$x_{ij} \geq 0$

$x_{ij} \geq 0$

Dual:

$$z^d = 10u_1 + 15u_2 + 30u_3 + 25u_4 + 14v_1 + 16v_2 + 17v_3 + 23v_4 \rightarrow \max$$

$$\begin{aligned} u_1 + v_1 &\leq 4 & u_2 + v_1 &\leq 3 & u_3 + v_1 &\leq 2 \\ u_1 + v_2 &\leq 2 & u_2 + v_2 &\leq 1 & u_3 + v_2 &\leq 1 \\ u_1 + v_3 &\leq 1 & u_2 + v_3 &\leq 3 & u_3 + v_3 &\leq 4 \\ u_1 + v_4 &\leq 0 & u_2 + v_4 &\leq 0 & u_3 + v_4 &\leq 0 \end{aligned}$$

$$\begin{aligned} u_4 + v_1 &\leq 3 \\ u_4 + v_2 &\leq 2 \\ u_4 + v_3 &\leq 3 \\ u_4 + v_4 &\leq 0 \end{aligned}$$

$z = 85$

$z^d = 4$

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
l <sub>1</sub>	4	2	1	0	10
l <sub>2</sub>	3	1	3	0	15
l <sub>3</sub>	2	1	4	0	30
l <sub>4</sub>	3	2	3	0	25
b <sub>j</sub>	14	26	17	23	80

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
l <sub>1</sub>	4	2	1	0	10
l <sub>2</sub>	3	1	3	0	15
l <sub>3</sub>	2	1	4	0	30
l <sub>4</sub>	3	2	3	0	25
b <sub>j</sub>	14	26	17	23	80

	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	a <sub>i</sub>
l <sub>1</sub>	4	2	1	0	10
l <sub>2</sub>	3	1	3	0	15
l <sub>3</sub>	2	1	4	0	30
l <sub>4</sub>	3	2	3	0	25
b <sub>j</sub>	14	26	17	23	80



	$O_1$	$O_2$	$O_3$	$O_4$	$a_i$
$I_1$	4	2	1	0	10
$I_2$	-4	-3	10	-2	15
$I_3$	3	1	3	0	30
$I_4$	-1	10	0	5	25
$b_j$	44	26	17	25	80

~~8~~  
~~80~~ ~~8~~ + 15  
~~8~~ = 10 - 8  
+ 4 + 8  
~~26~~ - 8